

Exam
IN3070/IN4070
Autumn 2025

The maximum number of marks for the whole exam was 100.

The minimum number of marks required for each grade was as follows:
For E, min. 30. For D, min. 43. For C, min. 50. For B, min. 65. For A,
min. 85.

These boundaries are based on an alignment of the delivered work with
the grade definitions given here:

<https://www.uio.no/studier/eksamen/karakterer/>

The final grade for each candidate is based on an evaluation of the delivered
work as a whole, and may therefore in some cases deviate from the boundaries
given above.

Question 1 – True or False

[1 mark for a correct answer, -1 for incorrect, 0 for missing answer. Min 0
for the whole question]

- If a formula A is valid in classical logic, then any formula $B \rightarrow A$ is also valid.
- If the resolution procedure does not terminate, then the derived unit clauses provide a satisfying interpretation for the clause set.
- If $A \rightarrow B$ is valid in classical logic, then A must be satisfiable.
- If $A \rightarrow B$ is valid in classical logic, then B must be satisfiable.
- If $A \rightarrow B$ is valid in classical logic, and A is satisfiable, then B must be satisfiable. .

Question 2 – Sequent Calculi LK and LJ

Prove the validity of the following formulae using the given calculus. Note that the first two formulas are to be proven in the classical logic LK, while the last is to be proven in the modal logic K.

In this task you can submit a hand-written answer. Use the sketching paper handed to you in the exam room for this. See instructions in the link below the task bar.

A) $((p \vee q) \rightarrow (p \vee r)) \rightarrow (p \vee (q \rightarrow r))$ using propositional LK [5 marks]

Here a is the ‘dummy’ constant we have to instantiate with if there are no others to choose from. Then b, c, d, e are fresh constants introduced by the $\forall$ -r rule. They have to be fresh on their respective branch, so one can ‘reuse’ d, e on the right branch, but it’s also correct to introduce other constants.

Alternatively, the three instantiations can all be done on the same branch before applying $\rightarrow$ -left on $pb \rightarrow qc$.

It’s OK to abbreviate and show only one of the branches in full detail.

C) $\Box(p \rightarrow \Diamond q) \rightarrow (\Diamond p \rightarrow \Diamond \Diamond q)$ using the modal sequent calculus [8 marks]

It is also possible to omit $\Diamond$ -l on the right branch and leave $1 : \Diamond q$ on the branch. The branch can then be closed when $\Diamond$ -r introduces $1 : \Diamond q$ in the succedent.

Question 3 – Decidability

For each of the following logics, is validity decidable?

[1 mark for a correct answer, -1 for incorrect, 0 for missing answer. Min 0 for the whole question]

- Classical propositional logic
- Intuitionistic propositional logic
- Classical first-order logic
- Modal logic K
- Description Logic ALC

Question 4 – Classical first-order semantics

A) Construct interpretations $I = (D, \iota)$ that satisfy the formulas

1: $\forall x \forall y \forall z (p(x, y) \wedge p(x, z) \rightarrow \exists u (p(y, u) \wedge p(z, u)))$

2: $\exists x \exists y. p(x, y)$

3: $\exists x \exists y. \neg p(x, y)$

Formulas 2 and 3 ensure that your interpretations are not ‘trivial’ i.e. $p^\iota = \emptyset$ or $p^\iota = D \times D$ are not allowed.

[During the exam, it was clarified that each of the interpretations has to satisfy all three formulas]

Please give:

A1) an interpretation with a finite domain [6 marks], and

A2) an interpretation with an infinite domain (take e.g. the natural numbers) [6 marks]

B) Show that the same formula is not valid by giving a falsifying interpretation. [6 marks] An additional [2 marks] if the domain of your interpretation has only two elements.

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Question 5 – Resolution

Show that the following formula (which you have already seen in Question 2) is valid, by using the resolution calculus.

$\exists x. \forall y. \forall z. ((p(y) \rightarrow q(z)) \rightarrow (p(x) \rightarrow q(x)))$

Remember that resolution is a refutation calculus, i.e. you can derive that a set of clauses is unsatisfiable. Also remember that variables should be made disjoint before applying resolution.

- Arriving at a correct set of clauses: 5 marks
- Correct resolution proof: 5 marks

Question 6 – Description Logic

A) Given a vocabulary with atomic concepts C and D , and an atomic role R . Using the Direct (Model-theoretic) Semantics for ALC, show that if $\mathcal{I} \models C \sqsubseteq D$, then $\mathcal{I} \models \exists R.C \sqsubseteq \exists R.D$.

[5 marks]

B) Again using the direct semantics, show that the converse doesn't hold. I.e. give a counter-model in which $\mathcal{I} \models \exists R.C \sqsubseteq \exists R.D$ holds but not $\mathcal{I} \models C \sqsubseteq D$.

Remember to give the domain and the interpretation of all symbols. Explain why this is a counter-model.

[5 marks]

Question 7 – The Monotonicity Lemma

As explained in the lecture, the forcing relation of intuitionistic logic has the following ‘Monotonicity’ Property:

If a propositional formula A is forced in a world u of a J-structure, $u \Vdash A$, and v is reachable from u , $(u, v) \in R$, then A is also forced in v , $v \Vdash A$.

Prove this statement for **propositional** intuitionistic logic by structural induction over the formula A , using the Kripke semantics of intuitionistic logic. (The statement also holds for predicate logic, but you are not required to show this.)

In this task you can submit a hand-written answer.

[15 marks]

Hints:

- Remember that the definition of J-structures already tells you this for all atomic formulas p .
- The accessibility relation of an intuitionistic frame is transitive.
- All the definitions you need are in the attached slides.

1 Question 8 – The Disjunction Property

As explained in the lecture, intuitionistic logic has the following ‘Disjunction Property’:

If A and B are two formulas such that $A \vee B$ is intuitionistically valid, then either A is intuitionistically valid or B is.

Prove this statement for **propositional** intuitionistic logic using the Kripke semantics. (The statement also holds for predicate logic, but you do not need to show this)

In this task you can submit a hand-written answer.

[15 marks]

Hints.

- It is easiest to prove this by contraposition: Assume that A and B are two formulae that are not valid. I.e. for each of them there is a J-structure and a world in that structure that does not force the formula. Show that $A \vee B$ cannot be valid by combining the two structures to a new J-structure that has a (new) world that doesn't force $A \vee B$.
- You may use the monotonicity lemma from the previous question, i.e. for any formula C and world u , if $u \Vdash C$ and $(u, v) \in R$, then $v \Vdash C$