IN3130 Undecidability Problem Set

Proposed solutions

Problem 9 from the compendium

Let
L_1 = {M | M writes a \$ for every input}
L_2 = {M | M writes a \$ for input '010'}
L_3 = {M | There is no *y* such that M writes a \$ for input *y*}
Show that L_1, L_2 and L_3 are undecidable.

All three proofs are simple modifications of the standard proof given at Pages 74-75 (Lecture 3, Slide 12) in the compendium.

For L1 and L2 the unmodified standard reduction will work – observe that the M' that this reduction produces does not look at its input; it simply halts for every input (and in particular for input '010') if the corresponding instance of the Halting Problem is a positive one (M halts on input x).
For L3 we only need to exchange the YES and NO in standard reduction.

Problem A

Consider L = {M : M skriver \$ etter < 100 skritt for ethvert input} Is L undecidable? Justify your answer (produce an informal proof).

L is decidable. The decision algorithm M_L is a modification of the Universal Turing Machine. M_L generates all possible inputs of length < 100 (notice that there are finitely many, since by convention the size of the input alphabet is constant). For each input M_L simulates M for at most 100 steps and answers YES and halts if M halts. If M does not halt for any of the inputs, M_L halts and answers NO.