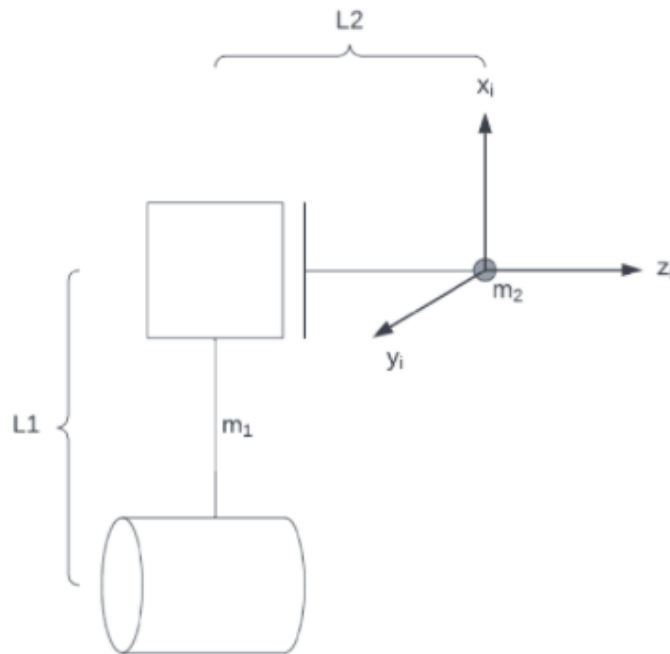


Exam Task 2022 Exercise 4.1:

4.1 Lagrangian



Figuren viser en forenklet utgave av manipulatoren fra forrige oppgave, der ledd 1 og link 1 ikke lenger er i bruk. Anta at massen til den nye link 1 er jevnt fordelt med inertia tensor:

$$I_1 = \begin{bmatrix} I_{1xx} & 0 & 0 \\ 0 & I_{1yy} & 0 \\ 0 & 0 & I_{1zz} \end{bmatrix}, \text{ og at link 2 har en punktmasse } m_2 \text{ plassert p  enden av roboten.}$$

Oppgaver:

Finn Lagrangian $\mathcal{L}$ for robotsystemet.

Maks poeng: 10

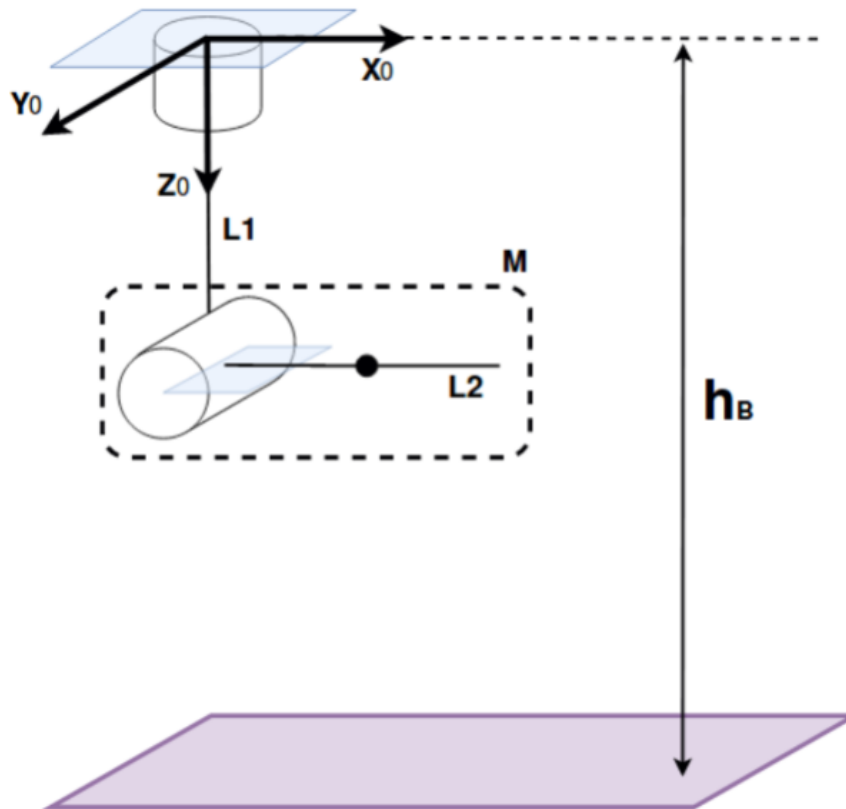
Jakobi

$$\begin{bmatrix} -L_1 \cdot \sin(\theta_1) + L_2 \cdot \sin(\theta_2) \cdot \cos(\theta_1) - L_3 \cdot \sin(\theta_1) & L_2 \cdot \sin(\theta_1) \cdot \cos(\theta_2) & \cos(\theta_1) \\ L_1 \cdot \cos(\theta_1) + L_2 \cdot \sin(\theta_1) \cdot \sin(\theta_2) + L_3 \cdot \cos(\theta_1) & -L_2 \cdot \cos(\theta_1) \cdot \cos(\theta_2) & \sin(\theta_1) \\ 0 & -L_2 \cdot \sin(\theta_2) & 0 \\ 0 & \cos(\theta_1) & 0 \\ 0 & \sin(\theta_1) & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Exam task 2019

The full Jacobian matrix:

$$J = \begin{bmatrix} -s_1(L_2c_2 + L_3^*s_2) & -c_1(L_2s_2 - L_3^*c_2) & c_1s_2 \\ c_1(L_2c_2 + L_3^*s_2) & -s_1(L_2s_2 - L_3^*c_2) & s_1s_2 \\ 0 & -L_2c_2 - L_3^*s_2 & c_2 \\ 0 & -s_1 & 0 \\ 0 & c_1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$



The image above shows a simplified version of the manipulator from the previous task. This manipulator has two degrees of freedom. Assume that the system has only one mass M - the mass of the second link (marked with the dashed line). Assume further that the mass is **uniformly distributed**, and that **its coordinate frame (in the initial position) is aligned in parallel with the frame of the second joint**. The distance between the ground level and the base of the manipulator is h_B (for this manipulator, $L_1 + L_2 \neq h_B$). Consider the inertia tensor to be as follows:

$$I = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}$$

a) 2% Produce the symbolic expression of the system's potential energy due to the ground level.

c) 12% Find the symbolic expression of the system's kinetic energy. You may use the following facts to assist your calculations:

$$R = R_1^0 R_{z, \theta_2} \quad \omega^T R = (R^T \omega)^T$$

d) Assemble the Lagrangian.