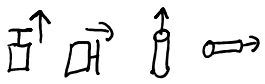
Forward kinematics

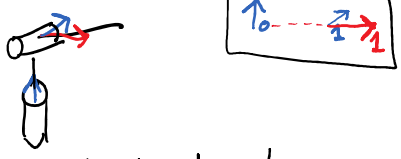
fredag 11. februar 2022 10:30

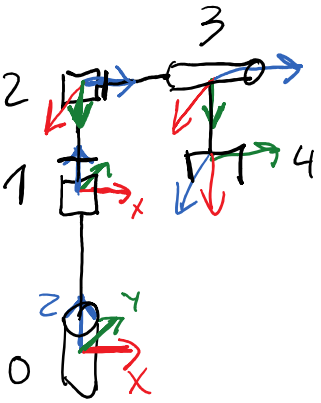
0 → last system 1 → new system

12 in the direction of motion



X_1 90° degrees on Z_0 and Z_1 (make it hit both vectors)

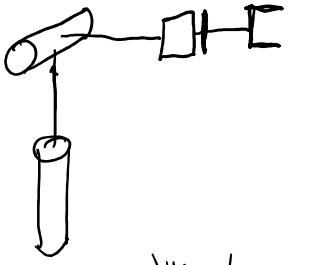


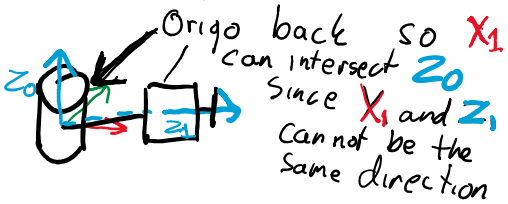


Y_1 right hand rule

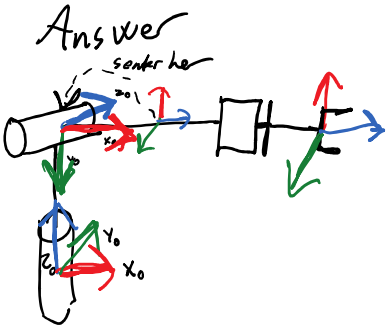
Task 1

fill in coordinate systems





Answer



Dh rules:

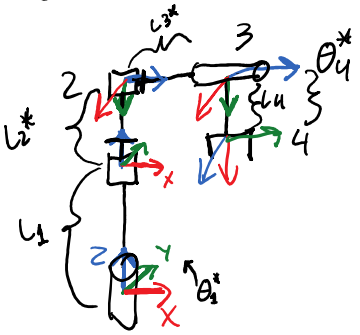
d → distance between X_0 and X_1 along the Z_0 axis + variable for prismatic joint

θ → rotation around the Z_0 axis to make X_0 and X_1 parallell

a → distance between Z_0 and Z_1 along the X_1 axis

α → rotation around the X_1 axis to make Z_0 and Z_1 parallell

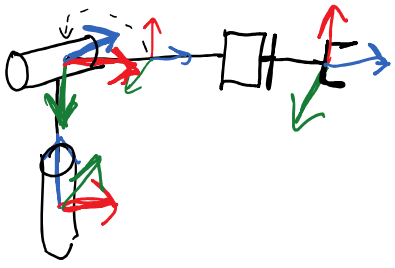
Example with table:



Link	d_i	θ_i	a_i	α_i
1	L_1	θ_1^*	0	0
2	L_2^*	$-90°$	0	$-90°$
3	L_3^*	0	0	0
4	0	$-90°+\theta_4^*$	L_4	$-90°$

Task 2

Fill in the Link table



answer:

Link	d	θ	a	α
1	L_1	θ_1^*	0	$-90°$
2	0	$-90°+\theta_2^*$	0	$-90°$
3	$L_3^*+L_2$	0	0	0

$A_i = Rot_{z,\theta_i}\ Trans_{z,d_i}\ Trans_{x,a_i}\ Rot_{x,\alpha_i}$

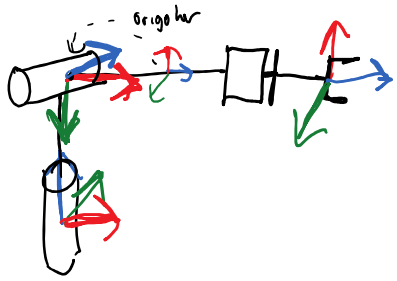
C = cos
S = sin

$$A_i = \begin{bmatrix} C\theta_i & -S\theta_i & 0 & 0 \\ S\theta_i & C\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\alpha_i & -S\alpha_i & 0 \\ 0 & S\alpha_i & C\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_i = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} Rot \end{bmatrix} & \begin{bmatrix} x \\ y \\ z \end{bmatrix} \\ 0\ 0\ 0 & 1 \end{bmatrix}$$

Matrix for one coordinate system to another

Fill in the matrix for this manipulator:



Link	d	θ	a	α
1	L_1	θ_1^*	0	$-90°$
2	0	$-90° + \theta_2^*$	0	$-90°$
3	$L_3^* + L_2$	0	0	0

$A_1 \times A_2 \times A_3$

C1 = Cos(θ_1)
S1 = Sin(θ_1)

$$\begin{bmatrix} C1 & 0 & -S1 & 0 \\ S1 & 0 & C1 & 0 \\ 0 & -1 & 0 & L1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} C(-90+\theta_2) & 0 & -S(-90+\theta_2) & C(-90+\theta_2^*)L_2 \\ S(-90+\theta_2) & 0 & C(-90+\theta_2) & S(-90+\theta_2^*)L_2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_3^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \begin{bmatrix} Rot \end{bmatrix} & \begin{bmatrix} x \\ y \\ z \end{bmatrix} \\ 0\ 0\ 0 & 1 \end{bmatrix}$$ → tool tip in World.
(AKA the forward kinematics)