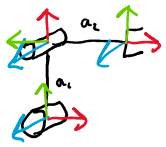
Inverse kinematics

torsdag 2. mars 2023 10:17

Forward kinematics → find X, Y, Z from joint variables.

Inverse kinematics → find joint variables from X, Y, Z.



Link	θ	d	a	α
1	$\theta_1 + 90$	0	a_1	0
2	$\theta_2 - 90$	0	a_2	0

$S_1 = \sin\theta_1$, $C_1 = \cos\theta_1$

$S_n = \sin\theta_n$, $C_n = \cos\theta_n$, $n = 1, 2, \ldots$

$S_{12} = \sin(\theta_1 + \theta_2)$, $C_{12} = \cos(\theta_1 + \theta_2)$

$\sin(\theta \pm 90^\circ) = \pm\cos\theta$

$\cos(\theta \pm 90^\circ) = \mp\sin\theta$

$$H_2^0 = \begin{bmatrix} C_1C_2 - S_1S_2 & -C_1S_2 - S_1C_2 & 0 & -a_1S_1 - a_2S_1S_2 + a_2C_1C_2 \\ C_1S_2 + S_1C_2 & C_1C_2 - S_1S_2 & 0 & a_1C_1 + a_2S_1C_2 + a_2C_1S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$

Algebra solution:

Use the x, y, z from the forward kinematics and try to reverse the function.

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$x = -a_1S_2 - a_2S_1S_2 + a_2C_1C_2$

$x = -a_1S_2 + a_2C_{12}$

$y = a_1C_1 + a_2S_1C_1 + a_2C_1S_2$

$y = a_1C_1 + a_2S_{12}$

$z = 0$

First we want to use the identity

$1 = \sin^2\theta + \cos^2\theta$

for this we will square x and y, then add them

↳ $x^2 + y^2$

$x^2 = a_1^2S_1^2 - 2a_1a_2S_1C_{12} + a_2^2C_{12}^2$

$y^2 = a_1^2C_1^2 + 2a_1a_2C_1S_{12} + a_2^2S_{12}^2$

θ_2:

$$x^2+y^2 = a_1^2\underbrace{(s_1^2+c_1^2)}_{1} + 2a_1a_2 s_1 c_{12} - 2a_1a_2 c_1 s_{12} + a_2^2\underbrace{(c_{12}^2+s_{12}^2)}_{1}$$

$$x^2+y^2 = a_1^2+a_2^2+2a_1a_2 s_1 c_{12} - 2a_1a_2 c_1 s_{12}$$

$\Rightarrow$ rewrite c_{12} and s_{12} to the full equations

$$\hookrightarrow x^2+y^2 = a_1^2+a_2^2+2a_1a_2 c_1(s_1c_2+c_1s_2) - 2a_1a_2 s_1(c_1c_2 - s_1s_2)$$

$$= a_1^2+a_2^2+2a_1a_2 s_1c_1c_2 + 2a_1a_2 c_1^2 s_2 + 2a_1a_2 s_1^2 s_2 - 2a_1a_2 s_1c_1c_2$$

$$= a_1^2+a_2^2+2a_1a_2 c_1^2 s_2 + 2a_1a_2 s_1^2 s_2$$

$$= a_1^2+a_2^2+2a_1a_2 s_2\underbrace{(c_1^2+s_1^2)}_{1}$$

$$x^2+y^2 = a_1^2+a_2^2+2a_1a_2 s_2$$

$$\hookrightarrow x^2+y^2-a_1^2-a_2^2 = 2a_1a_2 s_2$$

$$\hookrightarrow \frac{x^2+y^2-a_1^2-a_2^2}{2a_1a_2} = s_2$$

We want to use the function for atan2 since it knows the quadrant of the angle

$$\frac{x^2+y^2-a_1^2-a_2^2}{2a_1a_2} := D$$

$1 = \sin^2+\cos^2 \rightarrow \cos = \pm\sqrt{1-\sin^2}$

$$\hookrightarrow \theta_2 = \operatorname{atan2}\left(D, \pm\sqrt{1-D^2}\right)$$

Now its time to find θ_1:

$$a\cos\theta + b\sin\theta = c$$

$$\Updownarrow$$

$$\theta = \operatorname{atan2}(b, a) \pm \operatorname{atan2}\left(\frac{\sqrt{a^2+b^2+c^2}}{c}\right)$$

Since we found θ_2 we can use it as a known value for calculating θ_1.

Using the forwards kinematics for Y

$$y = a_1 c_1 + a_2 s_1 c_2 + a_2 c_1 s_2$$

↳ rewriting to the formula above

$$y = c_1(a_1 + a_2 s_2) + a_2 s_1 c_2$$

$$a = a_1 + a_2 s_2$$

$$b = a_2 c_2$$

$$c = y$$

$$\theta_1 = \operatorname{atan2}(a_2 c_2, a_1 + a_2 s_2) \pm \operatorname{atan2}\left(\sqrt{a_1^2 + a_2^2 + y^2 + 2 s_2 a_1 a_2}, y\right)$$