# Inverse Kinematics - tips

1 Shorten expressions

→ $\cos^2(\theta_i) + \sin^2(\theta_i) = 1$

→ $2S_2S_3L_3 - 2S_2S_3L_3 = 0$

2 Divide the expressions by each other

→ $\frac{y}{x} = \frac{S_2 L_3 C_2 S_3}{C_2 L_3 C_2 S_3} = \frac{S_2}{C_2} = \tan(\theta_2)$

$\theta_2 = \text{atan2}\left(\frac{y}{x}\right) = \text{atan2}(y, x)$

3 Square the expressions and add/subtract

→ $x^2 = L_2^2C_2^2 + L_3^2S_2^2S_3^2 - 2L_2L_3C_2S_2S_3$

$y^2 = L_2S_2^2 + L_3^2C_2^2S_3^2 + 2L_2L_3C_2S_2S_3$

$x^2+y^2 = L_2(C_2^2+S_2^2) + L_3^2S_3^2(S_2^2+C_2^2)$

$x^2+y^2 = L_2 + L_3^2S_3^2$

$S_3 = \sqrt{\frac{x^2+y^2-L_2}{L_3^2}}$

→ Sometimes it's easier to move $L_1$ over:

$z = S_2L_3 + L_1 \Rightarrow (z-L_1)^2 = S_2^2L_3^2$

4 When you have $\sin(\theta)$, find $\cos(\theta)$ to use atan2

→ from 3: $s_3 = \sqrt{\frac{x^2+y^2-L_2}{L_3^2}} := D$ (name it)

$$c_3 = \sqrt{1-s_3^2} = \sqrt{1-D^2}$$

$$\theta_3 = \operatorname{atan2}\left(D, \sqrt{1-D^2}\right)$$

→ the same applies if we have an expression for $\cos(\theta)$ instead of $\sin(\theta)$

5 Use $a\cos(\theta) + b\sin(\theta) = c$ and formula sheet

→ $\underbrace{L_3 s_3 c_2}_{a} + \underbrace{L_2 s_2}_{b} = \underbrace{y}_{c}$

$$\operatorname{atan2}(b,a) \pm \operatorname{atan2}\left(\sqrt{a^2+b^2-c^2}, c\right)$$

$$\theta_2 = \operatorname{atan2}(L_2, L_3 s_3) \pm \operatorname{atan2}\left(\sqrt{L_3^2 s_3^2 - y^2}, y\right)$$

→ a and b cannot consist of any variables unless they are already computed (known) (all of them cannot depend on each other)

6 Multiply an expression with $\sin\theta$ and one with $\cos\theta$

→ $x = L_3 c_1 c_2 + L_2 s_2 c_1 - L_{off} s_1$

$y = L_3 s_1 c_2 + L_2 s_2 s_1 - L_{off} c_1$

$s_1 x = L_3 s_1 c_1 c_2 + L_2 s_2 c_1 s_1 - L_{off} s_1^2$

$c_1 y = L_3 s_1 c_1 c_2 + L_2 s_2 c_1 s_1 + L_{off} c_1^2$

$c_1 y - s_1 x = L_{off} (s_1^2 + c_1^2)$

$y\, c_1 - x\, s_1 = L_{off}$

$a \quad b \quad c$

$\theta_1 = \operatorname{atan2}(-x, y) \pm \operatorname{atan2}(\sqrt{x^2+y^2-c^2}, L_{off})$

7 Only one variable left, rearrange

$\rightarrow z = L_1^* + s_2 \Rightarrow L_1^* = s_2 - z$

($\theta_2$ has to be found first)