General Information (IN3140)

Examination in IN3140 - Introduction to Robotics (Spring 2023)

Examination date: June 9th, 2023
Examination time: 15:00 - 19:00 (4 hours)
Resources: Relevant formulas and calculators (see bottom of screen)
Permitted materials:

- Spong, Hutchinson & Vidyasagar, *Robot Modelling and Control*
- Karl Rottman, *Matematisk formelsamling*

A simple and an advanced calculator are available in Inspera.

The examination consists of seventeen tasks that may be solved in arbitrary order:

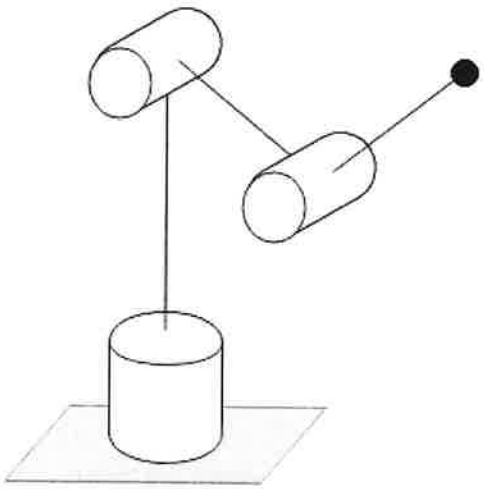
Tasks 1-9, **12 and 14** are either multiple choice questions or short essay tasks - **they are to be answered on the computer**. There is no penalty for guessing on multiple choice questions.

Tasks 10, 11, 13 and **15** are conventional tasks that will test your knowledge of Ros, kinematics, dynamics and control theory - **these tasks are to answered by submitting hand drawings.** You use the handed out sketch sheets. It is possible to use several sheets per assignment. See instructions on how to fill in the sketch sheets in the link below the assignment overview.

You are not supposed to submit hand drawings for any other tasks than **Tasks 10, 11, 13** and **15**. You will NOT be given extra time to fill in the information boxes on the sketch sheets (assignment codes, candidate number, etc.)

1 Manipulator 1

A manipulator of which type is displayed in the following image?



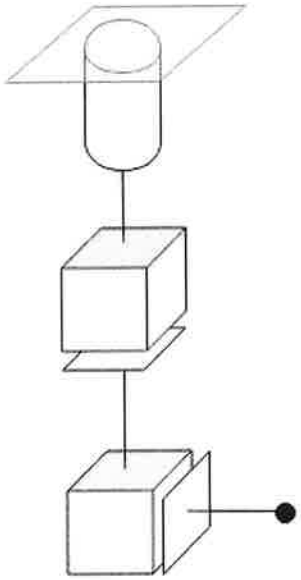
Select one alternative:

- ○ Elbow-down manipulator
- ○ PPP manipulator
- ✓ Articulated manipulator
- ○ Cartesian manipulator

Maximum marks: 1

2 Manipulator 2

A manipulator of which type is displayed in the following image?



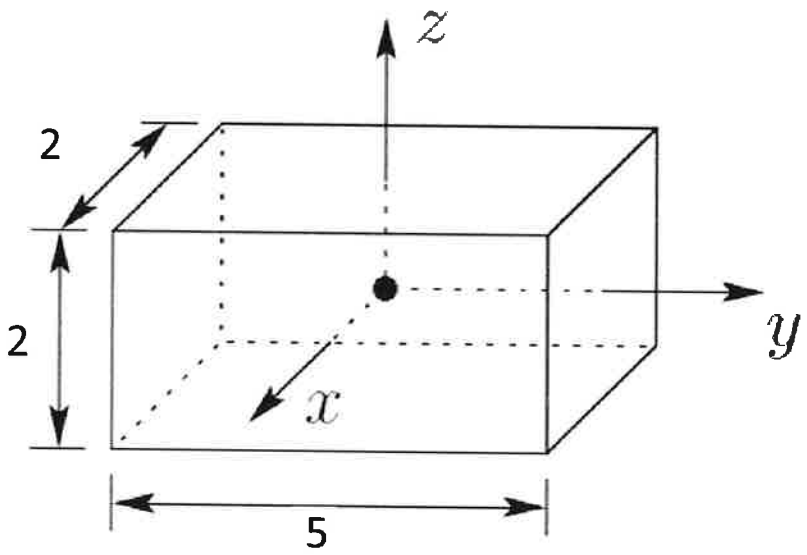
Select one alternative:

- PPR manipulator
- Articulated manipulator
- Cylindrical manipulator
- Spherical manipulator

Maximum marks: 1

3 Inertia calculation

In Dynamics the Inertia (I) of an object can be expressed as a matrix. Given the box and the coordinate frame below:



What is the formula for calculating I_{xx}?

Select one alternative:

- $I_{xx} = \int_{-1}^{1} \int_{0}^{2\pi} \int_{0}^{r} r^2 p(x,y,z) dr d\theta dz$
- $I_{xx} = \int_{-1}^{1} \int_{-2.5}^{2.5} \int_{-1}^{1} (x^2(y+z)) p(x,y,z) dx dy dz$
- ✓ $I_{xx} = \int_{-1}^{1} \int_{-2.5}^{2.5} \int_{-1}^{1} (y^2+z^2) p(x,y,z) dx dy dz$
- $I_{xx} = \int_{-1}^{1} \int_{-2.5}^{2.5} \int_{-1}^{1} (xy+xz) p(x,y,z) dx dy dz$

Maximum marks: 1.5

4 Controller choise

Given a system where there are no friction or dampening, and there are no external forces working on the manipulator.

What type of controller(s) would make the manipulator converge towards a setpoint?
Select one or more alternatives:

- [ ] P-controller
- [x] PD-controller
- [x] PID-controller
- [ ] PI-controller
- [ ] D-controller
- [ ] I-controller

Maximum marks: 2

5 Ki parameter

What happends if K_i has a too high value in a PID controller?

Select one alternative:

- ○ We stop before we reach the desired position
- ○ We get a slower response
- ○ Nothing
- [x] We get oscillations

Maximum marks: 1.5

6 Dynamics_Coriolis

The general dynamics of a system can be expressed by the following equation:
$\tau = D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q)$

Given the following Dynamics:

$$\tau = \begin{bmatrix} m_1 m_2 L_1^2 \ddot{\theta}_1 + m_2 L_2^2 \ddot{\theta}_2 - m_1 m_2 L_1 c_1 s_2 \dot{\theta}_1^2 + m_1 g L_1 c_1 \\ m_2 L_1^2 L_2 \ddot{\theta}_1 + c_1 s_1 \dot{\theta}_2 + \frac{m_2 g L_1 s_1}{4} \end{bmatrix}$$

Which expression(s) would describe the **Coriolis and Centrefugal forces** acting on the system?
Select one alternative:

- $\begin{bmatrix} -m_1 m_2 L_1 c_1 s_2 & -m_1 m_2 L_1 c_1 s_2 \\ 0 & c_1 s_1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$
- $\begin{bmatrix} m_1 m_2 L_1^2 & m_2 L_2^2 \\ m_2 L_1^2 L_2 & 0 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}$
- $\begin{bmatrix} -m_1 m_2 L_1 c_1 s_2 \dot{\theta}_1 & 0 \\ 0 & c_1 s_1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$
- $\begin{bmatrix} m_2 L_2^2 & m_1 m_2 L_1^2 \\ 0 & m_2 L_1^2 L_2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}$

Maximum marks: 1.5

7 Dynamics_Inertial

The general dynamics of a system can be expressed by the following equation:

$\tau = D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q)$

Given the following Dynamics:

$$\tau = \begin{bmatrix} m_1 m_2 L_1^2 \ddot{\theta}_1 + m_2 L_2^2 \ddot{\theta}_2 - m_1 m_2 L_1 c_1 s_2 \dot{\theta}_1^2 + m_1 g L_1 c_1 \\ m_2 L_1^2 L_2 \ddot{\theta}_1 + c_1 s_1 \dot{\theta}_2 + \frac{m_2 g L_1 s_1}{4} \end{bmatrix}$$

Which expression(s) would describe the **Inertia forces** acting on the system?

Select one alternative:

- $\begin{bmatrix} -m_1 m_2 L_1 c_1 s_2 & -m_1 m_2 L_1 c_1 s_2 \\ 0 & c_1 s_1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$
- $\begin{bmatrix} m_2 L_2^2 & m_1 m_2 L_1^2 \\ 0 & m_2 L_1^2 L_2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}$
- $\begin{bmatrix} -m_1 m_2 L_1 c_1 s_2 \dot{\theta}_1 & 0 \\ 0 & c_1 s_1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$
- $\begin{bmatrix} m_1 m_2 L_1^2 & m_2 L_2^2 \\ m_2 L_1^2 L_2 & 0 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}$

Maximum marks: 1.5

8 Reality Gap

Why do we have reality gaps in our simulations, and what can we do to minimalize its impact?

Fill in your answer here

Format

We have a reality gap since the simulation cannot capture everything from the real world.

To minimalize the impact we can discourage the Ai to exploit the Reality gap by giving it negative rewards for the exploitation.
We can also increase the simulation fidelity. (Reduce the gap)

We can encurage slow, static movements. add noise

Learn after deployment, so it continues to learn in the real world.

Words: 0

Maximum marks: 3

9 Ros 1

What is the difference between Rviz and Gazebo?

Fill in your answer here

Gazebo is a simulaton. It shows you what is happening.

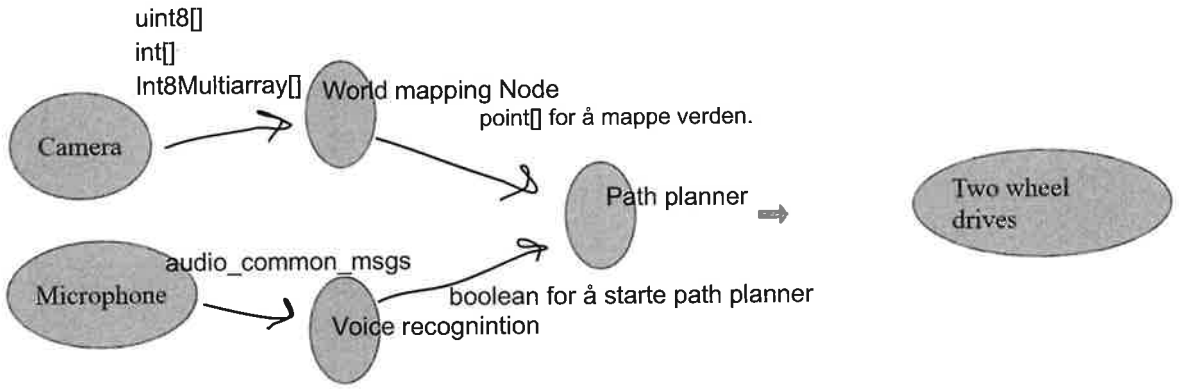
RVIZ is short for ROS visualization. it visualizes what the robot is seeing and doing.
It shows you what the robot is thinking it is doing.

Maximum marks: 3

10 Ros robot assistant

In this exercise you can answer with digital hand drawing. Use the handed out sketch sheets. See instructions on filling out the sketch sheets in the link below the task bar.

Assume we have a robot assistant equipped with a microphone and a camera. Using its microphone, the robot listens to the user giving it different commands. As soon as the robot detects the user saying "come", it starts finding the user's position and moving towards them. We want you to develop a ROS program with at least three nodes to perform this task. Complete the below figure by adding the relevant nodes and a few explanations about each one. Use arrows to specify the topics that each node publishes or subscribes to. Also, above the arrows, mention what type of message is transferred on each topic. You can add more nodes if you think it is needed.



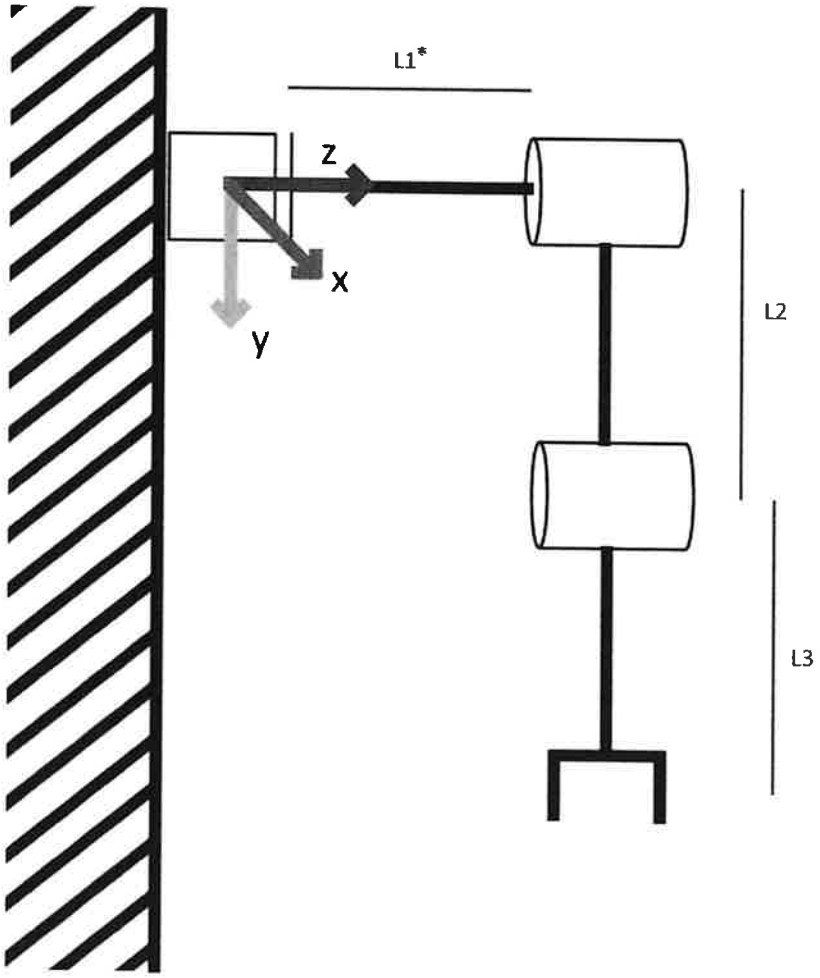
Maximum marks: 4

Noe slikt typ:

Funker med node for å mappe verden som får data fra kamera.
Du har også node som finner ut når noen sier "come"

Du kombinerer så mapping og voice rec slik at roboten kan
reise til personen som sa "come" via en path planner.

11 Kinematics



In this exercise you can answer with digital hand drawing. Use the handed out sketch sheets. See instructions on filling out the sketch sheets in the link below the task bar.

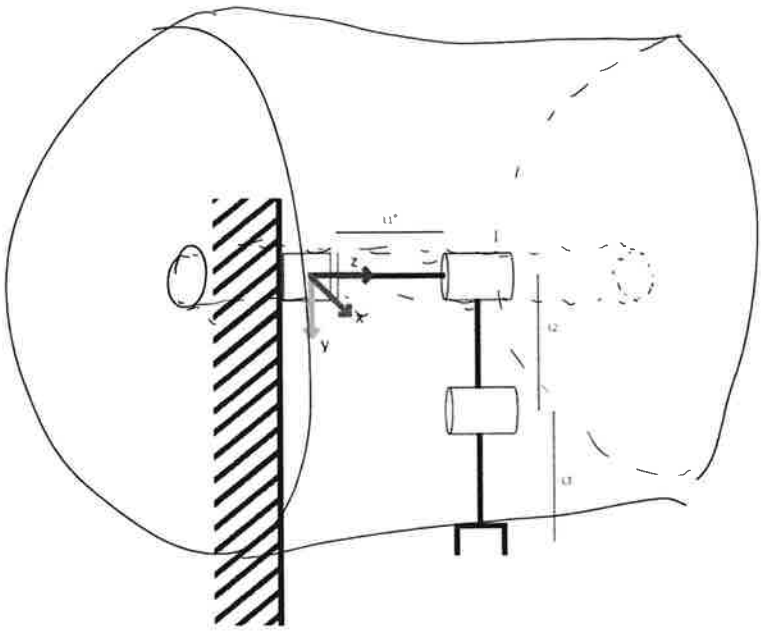
The figure above shows the manipulator used in this task. The first prismatic joint in the base of the manipulator moves along the **Z0** axis. The total length between the base and first revolute joint is noted as $L_1{}^*$. θ_2 and θ_3 are the joint angles of the second and third joint. Their zero-position is as shown in the drawing. L2 is the length between the first revolute joint and the second revolute joint, while L3 is the length from the second revolute joint to the tool tip. The base coordinate frame to be used is given in the figure above.

a) 2% Draw the manipulator workspace (include the outer and the inner boundaries).

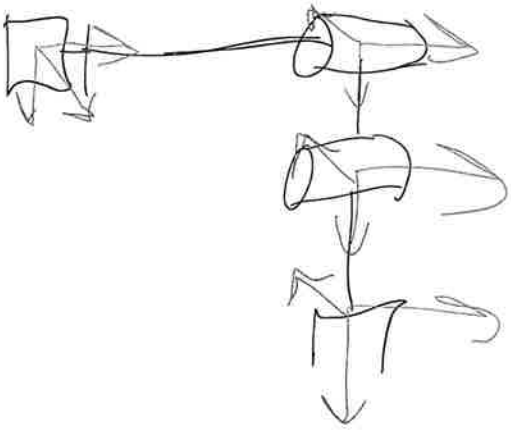
b) 7% Assign coordinate frames in accordance with the Denavit-Hartenberg convention to the manipulator in the figure in a separate drawing . Find the Denavit-Hartenberg parameters and place them into a DH-table.

c) 7% Derive the forward kinematics of the manipulator from the base coordinate system to the end-effector.

d) 10% Derive the inverse kinematics of the manipulator.



→noe slikt trenger ikke å ha hulleder i midten. (da er $L_3 > L_2$)



PH	θ	α	d	a
1	$\pi/2$	0	L_1^*	0
2	θ_2	0	0	L_2
3	θ_3	0	0	L_3

```
forwardkinematics =

[-sin(th_2 + th_3), -cos(th_2 + th_3), 0, - L3*sin(th_2 + th_3) - L2*sin(th_2)]
[ cos(th_2 + th_3), -sin(th_2 + th_3), 0,   L3*cos(th_2 + th_3) + L2*cos(th_2)]
[                0,                 0, 1,                                  L1]
[                0,                 0, 0,                                   1]
```

Inv:

$z = L_1$ *

$r^2 = x^2 + y^2$

$r^2 = (-L_3 s_{23} - L_2 s_2)^2 + (L_3 c_{23} + L_2 c_2)^2$

$r^2 = L_3^2 s_{23}^2 + L_3 s_{23} L_2 s_2 + L_2^2 s_2^2 + L_3^2 c_{23}^2 + L_3 c_{23} L_2 c_2 + L_2^2 c_2^2$

$r^2 = L_3^2 + L_2^2 + L_3 s_{23} L_2 s_2 + L_3 c_{23} L_2 c_2$

$c_{23} = c_2 c_3 - s_2 s_3$

$s_{23} = s_2 c_3 + s_3 c_2$

$r^2 - L_3^2 - L_2^2 = L_3 L_2 s_2 (s_2 c_3 + s_3 c_2) + L_3 L_2 c_2 (c_2 c_3 - s_2 s_3)$

$r^2 - L_3^2 - L_2^2 = L_3 L_2 s_2^2 c_3 + \cancel{L_3 L_2 s_2 c_2 s_3} + L_3 L_2 c_2^2 c_3 - \cancel{L_3 L_2 s_2 c_2 s_3}$

$r^2 - L_3^2 - L_2^2 = L_3 L_2 c_3$

$\frac{r^2 - L_3^2 - L_2^2}{L_3 L_2} = c_3$

$1 = s^2 + c^2 \rightarrow s^2 = 1 - c^2 \Rightarrow s = \sqrt{1 - c^2}$

$r^2 = x^2 + y^2$

$$\theta_3 = \operatorname{atan2}\left(\sqrt{1 - \left(\frac{x^2+y^2-L_3^2-L_2^2}{L_3 L_2}\right)^2},\ \frac{x^2+y^2-L_3^2-L_2^2}{L_3 L_2}\right)$$

θ_3 regnes nå som gitt variabel

har:

har:

$a\cos\theta + b\sin\theta = c \Leftrightarrow \theta = \operatorname{atan2}\left(\frac{b}{a}\right) \pm \operatorname{atan2}\left(\frac{\sqrt{a^2+b^2+c^2}}{c}\right)$

$Y = L_3 C_{23} + L_2 C_2$

$C_{23} = C_2 C_3 - S_2 S_3$

↳ $Y = L_3 C_2 C_3 - L_3 S_2 S_3 + L_2 C_2$

$Y = (L_3 C_3 + L_2) C_2 + (-L_3 S_3) S_2$

$c \quad a \quad b$

↳ setter så inn i formel

$\theta_2 = \operatorname{atan2}\left(\frac{-L_3 S_3}{L_3 C_3 + L_2}\right) \pm \operatorname{atan2}\left(\frac{\sqrt{Y^2 + (L_3 S_3 + L_2)^2 + (L_3 S_3)^2}}{Y}\right)$

Skriver opp DH for Worldframe til

Base

Link	θ	α	d	a
0	$-\pi/2$	$-\pi/2$	L_0	0

ny forrover:

```
[               0,                0, 1,                                                  L1]
[ sin(th_2 + th_3), cos(th_2 + th_3), 0,       L3*sin(th_2 + th_3) + L2*sin(th_2)]
[-cos(th_2 + th_3), sin(th_2 + th_3), 0, L0 - L3*cos(th_2 + th_3) - L2*cos(th_2)]
[               0,                0, 0,                                                   1]
```

Posisjonen e:

$X = 0,7\,m$

$Y = 0,4 \cdot 0,965\,2583 + 0,15 \approx \underline{0,54}$ (0,536 37...)

$Z = 2 - 0,4 \cdot 0,2588 - 0,3 \cdot 0,866 \approx \underline{1,64}$ (1,636 64...)

```
[0, - L3*cos(th_2 + th_3) - L2*cos(th_2), -L3*cos(th_2 + th_3)]
[0, - L3*sin(th_2 + th_3) - L2*sin(th_2), -L3*sin(th_2 + th_3)]
[1,                                    0,                    0]
[0,                                    0,                    0]
[0,                                    0,                    0]
[0,                                    1,                    1]
```

Potensiell energi: $\left(-L_2 L_2/2 + h_0\right) m$ ← ikke nødvendig

$-\frac{0,3_m}{2} \cdot \frac{\sqrt{2}}{2} \cdot 0,5_{kg} \cdot 10_{m/s^2} = \frac{3}{4} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{3^3}}{2^3} = 0,6495\,j \rightarrow Joul$

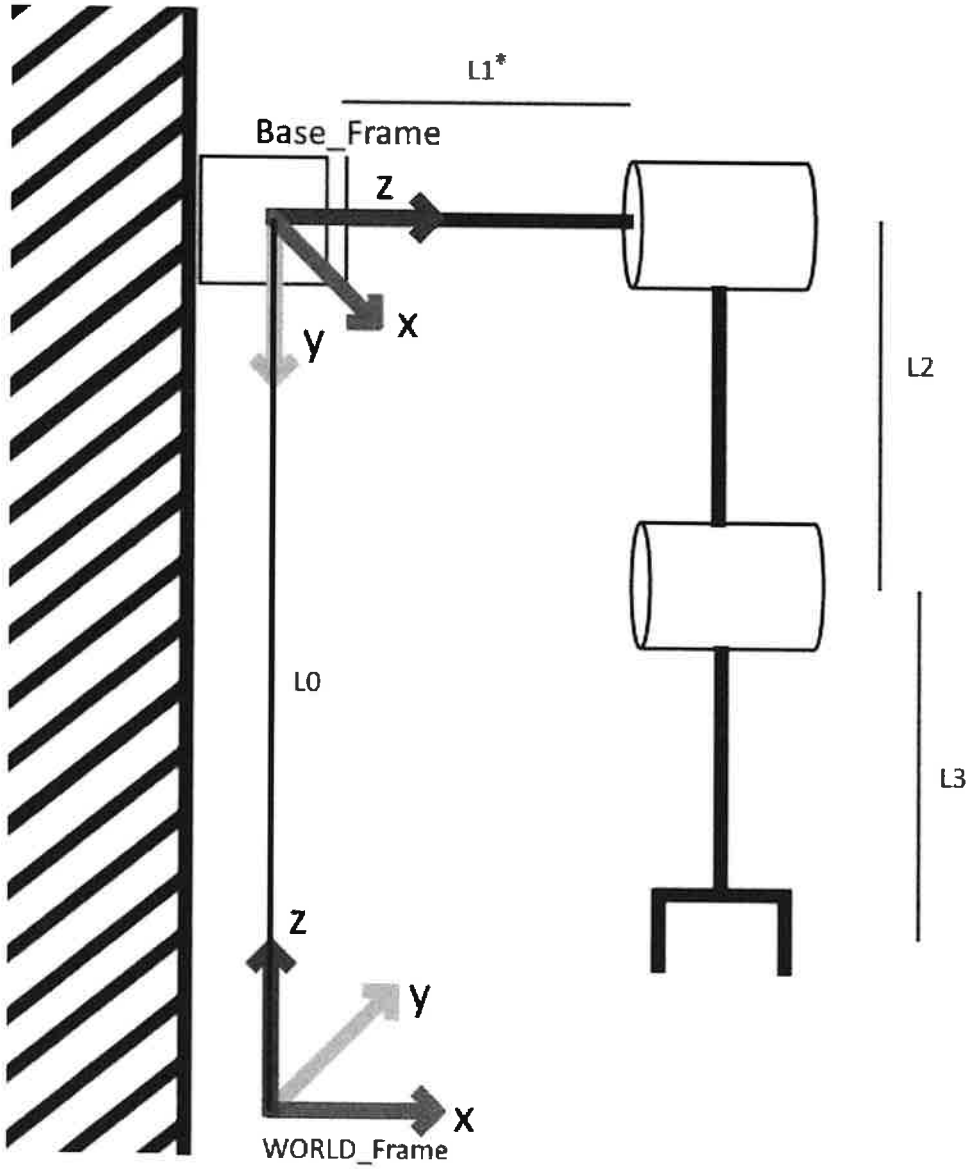
e) 10% Derive the Jacobian matrix of the manipulator.

Maximum marks: 36

a) WS ser litt ut som ett sylinder. om L3 er lenger enn L2 så er den hul i midten

12 Position World frame

The manipulator is the same as in the previous tasks, but a world coordinate frame has been established vertically below the base of the manipulator with a distance of L0 between them.



Given the parameters:
$L0 = 2.0m$,
$L1^* = 0.7m$,
$L2 = 0.3m$,
$L3 = 0.4m$,
$\theta_2 = 30°$,
$\theta_3 = 45°$

a) 1% What is the position of the tool-tip (end of manipulator) given in the WORLD_Frame?
Fill in your answer here

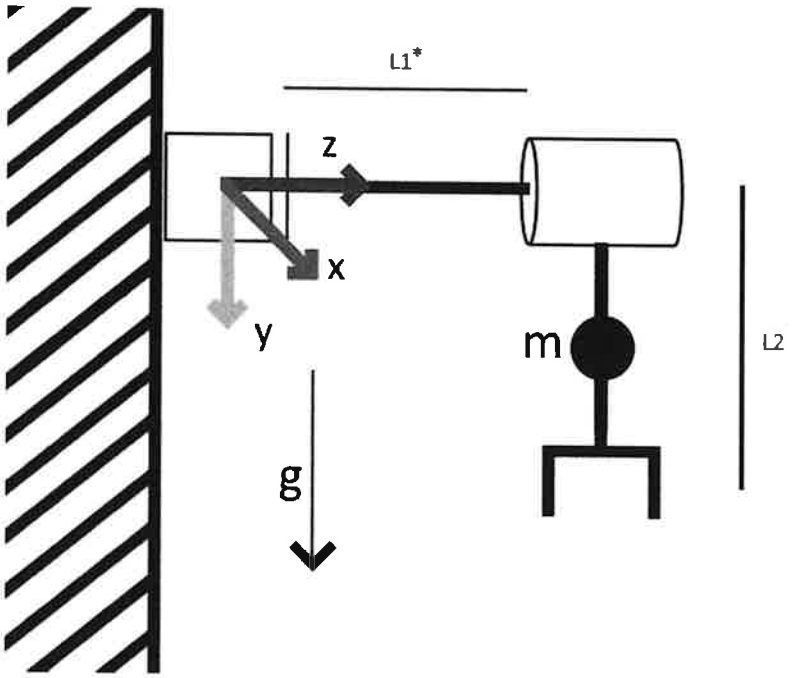
b) 3% The manipulator above has been tasked to work in a assembly-cell. The parts to assemble will come on a conveyor belt below the robot. The parts will have a **random orientation**, and the task is to mount the parts in a given **position** and **orientation**.
What challenges would the robot meet, and what types of modifications would we need to do to the robot so it can complete its task?

Fill in your answer here

The main challange would be to rotate the object as we need atleast a 6dof to reach any position and have any type of rotation.
To fix this you could add a wrist to the robot.

Maximum marks: 4

13 Dynamics



In this exercise you can answer with digital hand drawing. Use the handed out sketch sheets. See instructions on filling out the sketch sheets in the link below the task bar.

The figure above shows a simplified version of the manipulator from the previous tasks. This manipulator has two degrees of freedom. Assume that the system has **only one** mass **m** - the mass of the second link. Assume further that the mass is a point mass located at the middle of the second link (L2 is the total length of the second link).

a) 2% Produce the **symbolic** expression of the system's potential energy.

b) 1% Assuming that **L1** = 700mm, **L2** = 300m, **M** = 500g, θ_2 = 30° and **g** = 10 m/s^2, **numerically** calculate the system's potential energy. State the units.

c) 6% Find the **symbolic** expression of the system's kinetic energy.

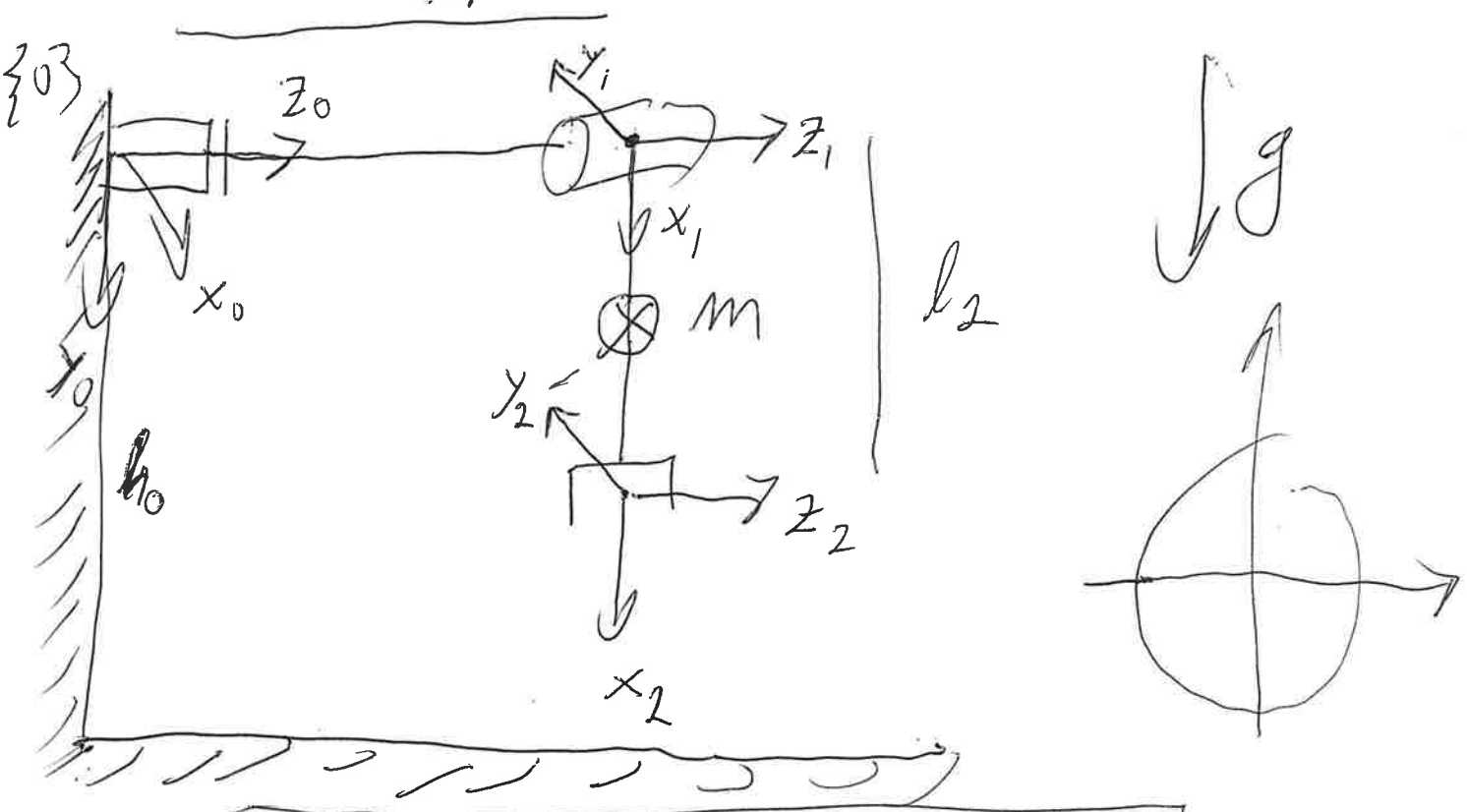
d) 7% Assemble the Lagrangian $\mathcal{L}$ accordingly. Then, derive the dynamics of the manipulator using the Euler-Lagrange formulation. Write the equations of motion in the following form:

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau$$

e) 4% The mass in L2 are now changed to a uniformly distributed mass. In general, what changes to the calculation of the Lagrangian $\mathcal{L}$ would need to be done, and why?

Maximum marks: 20

Dynamics



i	θ_i	α_i	d_i	a_i
1	$\frac{\pi}{2}$	0	l_1^*	0
2	θ_2^*	0	0	l_2

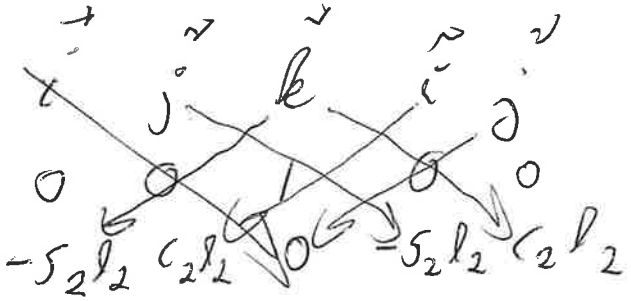
$$A_1 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & l_1^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} C_2 & -S_2 & 0 & C_2 l_2 \\ S_2 & C_2 & 0 & S_2 l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A_1 \cdot A_2 = \begin{bmatrix} -S_2 & -C_2 & 0 & -S_2 l_2 \\ C_2 & -S_2 & 0 & C_2 l_2 \\ 0 & 0 & 1 & l_1^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Jacobian for Dynamics:

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \begin{bmatrix} Z_0 & Z_1 \times (O_2 - O_1) \\ 0 & Z_1 \end{bmatrix}$$

$$= \left[\begin{array}{c|c} 0 & -C_2 l_2 \\ 0 & -S_2 l_2 \\ 1 & 0 \\ \hline 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{array}\right]$$



$$\Rightarrow \begin{matrix} (0 - C_2 l_2)\vec{i} \\ +(-S_2 l_2 - 0)\vec{j} \\ +(0-0)\vec{k} \end{matrix} \Bigg\} \begin{bmatrix} -C_2 l_2 \\ -S_2 l_2 \\ 0 \end{bmatrix}$$

c)

Kinetic Energy:

$$J_{v_{c_1}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} ; \quad N_{c_1} = J_{N_{c_1}} \cdot \dot{q} , \quad \dot{q} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$J_{v_{c_2}} = \begin{bmatrix} 0 & -C_2 l_2 \\ 0 & -S_2 l_2 \\ 1 & 0 \end{bmatrix} ; \quad N_{c_2} = J_{N_{c_2}} \cdot \dot{q}$$

$$K = \frac{1}{2} \dot{q}^T \left\{ m J_{N_{c_1}}^T J_{N_{c_1}} + m \cdot J_{N_{c_2}}^T \cdot J_{N_{c_2}} \right\} \dot{q}$$

$$* \quad J_{N_{C_1}}^T \cdot J_{N_{C_1}} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\frac{1}{2} \begin{bmatrix} \dot{q}_1 & \dot{q}_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \dot{q}_1 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$= \frac{1}{2} \dot{q}_1^2$$

$$* \quad J_{N_{C_2}}^T \cdot J_{N_{C_2}} = \begin{bmatrix} 0 & 0 & 1 \\ -c_2 l_2 & -s_2 l_2 & 0 \end{bmatrix} \begin{bmatrix} 0 & -c_2 l_2 \\ 0 & -s_2 l_2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & c_2^2 l_2^2 + s_2^2 l_2^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & l_2^2 \end{bmatrix}$$

$$\frac{1}{2} \begin{bmatrix} \dot{q}_1 & \dot{q}_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & l_2^2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \dot{q}_1 & \dot{q}_2 l_2^2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$= \frac{1}{2} \left(\dot{q}_1^2 + l_2^2 \dot{q}_2^2 \right)$$

$$\Rightarrow K = \frac{1}{2} m \left(\dot{q}_1^2 + l_2^2 \dot{q}_2^2 \right), \text{ massen ligger midt på } l_2' \Rightarrow l_2' = \frac{l_2}{2}$$

Potensiellenergi: Nullnivå som i figuren.

a) $P = mgh = mg\left(h_0 - c_2 \cdot \frac{l_2}{2}\right)$

b) $P = (-)\frac{0{,}3\,m}{2} \cdot \frac{\sqrt{3}}{2} \cdot 0{,}5\,kg \cdot 10\,m/s^2 = 0{,}6495$ Joule $\left(0{,}6372\ ^{m}/_{g=9.81}\right)$

d) $\mathcal{L} = K - P = \frac{1}{2} m \left(\dot{q}_1^2 + \left(\frac{l_2}{2}\right)^2 \dot{q}_2^2\right) - mg\left(h_0 - c_2 \frac{l_2}{2}\right)$

Euler Langrange

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_j} - \frac{\partial \mathcal{L}}{\partial q_0} = \tau_0$$

$$\frac{\partial \mathcal{L}}{\partial q_1} = [\text{illegible}] = 0 \; ; \; \frac{\partial \mathcal{L}}{\partial q_2} = -mg s_2 \frac{l_2}{2}$$

$$\frac{\partial \mathcal{L}}{\partial \dot{q}_1} = \frac{1}{2} m \cdot 2 \cdot \dot{q}_1 \; ; \; \frac{\partial \mathcal{L}}{\partial \dot{q}_2} = \frac{1}{2} m \cdot 2 \cdot \dot{q}_2 \cdot \frac{l_2^2}{4}$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_1} = m\ddot{q}_1 \; ; \; \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_2} = \frac{m\ddot{q}_2 l_2^2}{4}$$

$$F_1 = m\ddot{q}_1 - 0 = m\ddot{q}_1$$

$$\tau_2 = \frac{1}{4} m l_2^2 \ddot{q}_2 + mg \frac{l_2}{2} s_2 \Rightarrow [\text{illegible}]$$

$D(q)\cdot\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau$

$$D = \begin{bmatrix} m & 0 \\ 0 & \frac{m l_2^2}{4} \end{bmatrix} \; ; \; g = \begin{bmatrix} 0 \\ \frac{mgl_2}{2} \end{bmatrix}$$

c) $m \rightarrow$ Fra punktmasse til kjevnt fordelt (distribuert) masse over L_2.

Forenkling

- En punktmasse har ikke treghetsmoment. Når vi nå har med å gjøre en distribuert masse, vil denne ha et treghetsmoment avhengig de ulike akse $\neq 0$. $I \neq 0$.
- Vi må da ta hvem det generelle uttrykket for kinetisk energi som også tar med seg bidragene fra rotasjon med treghetsmomentet som matriker rotasjonsbevegelser.

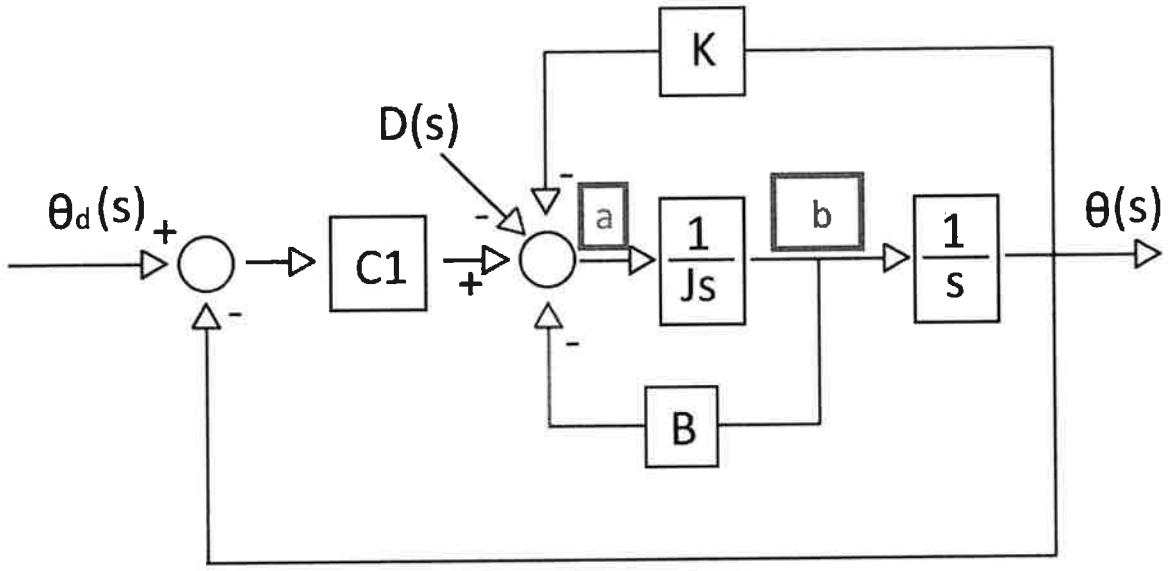
$$K = \frac{1}{2} m v^T v + \frac{1}{2} w^T \mathcal{I} w$$

- Ser vi på Jacobian til manipulatoren består denne av $J = \begin{bmatrix} J_v \\ J_w \end{bmatrix}$
- Treghetsmomentet vil også transformeres fra opprinnelig I, forhold til KS armleddet, til et treghetsmoment gitt i forhold til KS basesystemet.

$$\Rightarrow \mathcal{I} = R I R^T \; ; \; v_i = J_{v_i} \cdot \dot{q} \; ; \; w_i = J_{w_i} \cdot \dot{q}$$

$$\Rightarrow K = \frac{1}{2} \dot{q}^T \left[\sum_{i=1}^{n} \left\{ m_i J_{v_i}(q)^T \cdot J_{v_i}(q) + J_{w_i}(q)^T R_i(q) \cdot I_i \cdot R_i(q)^T \cdot J_{w_i}(q) \right\} \right]$$

14 Control_theory1

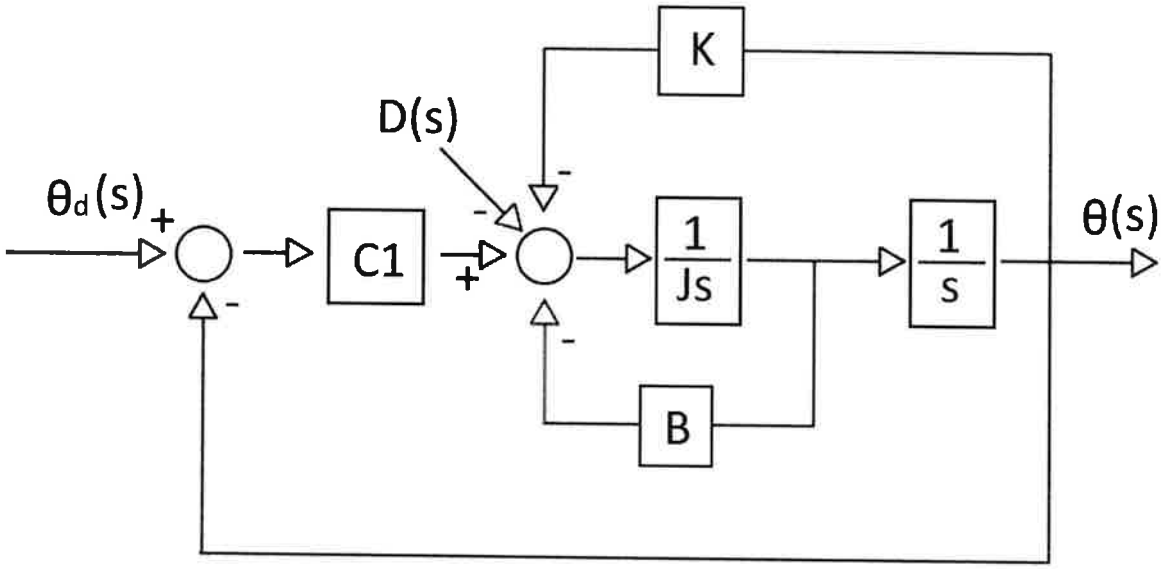


Given the block diagram of the closed-loop system above. Find the symbolic value of the signal in a and b.

a= J*s*s*th
b= s*th

Maximum marks: 4

15 Control_theory2



In this exercise you can answer with digital hand drawing. Use the handed out sketch sheets. See instructions on filling out the sketch sheets in the link below the task bar.

The equation for a system in the time domain follows the equation
$\tau = J\ddot{\theta}(t) + B\dot{\theta} + K\theta(t)$

The above block diagram shows the system with a controller in the frequency domain.

a) 2% What type of controller does the system contain? P

b) 6% Find the Transfer function H(s) between the desired angle (θ_d) and the output angle (θ) of the system.

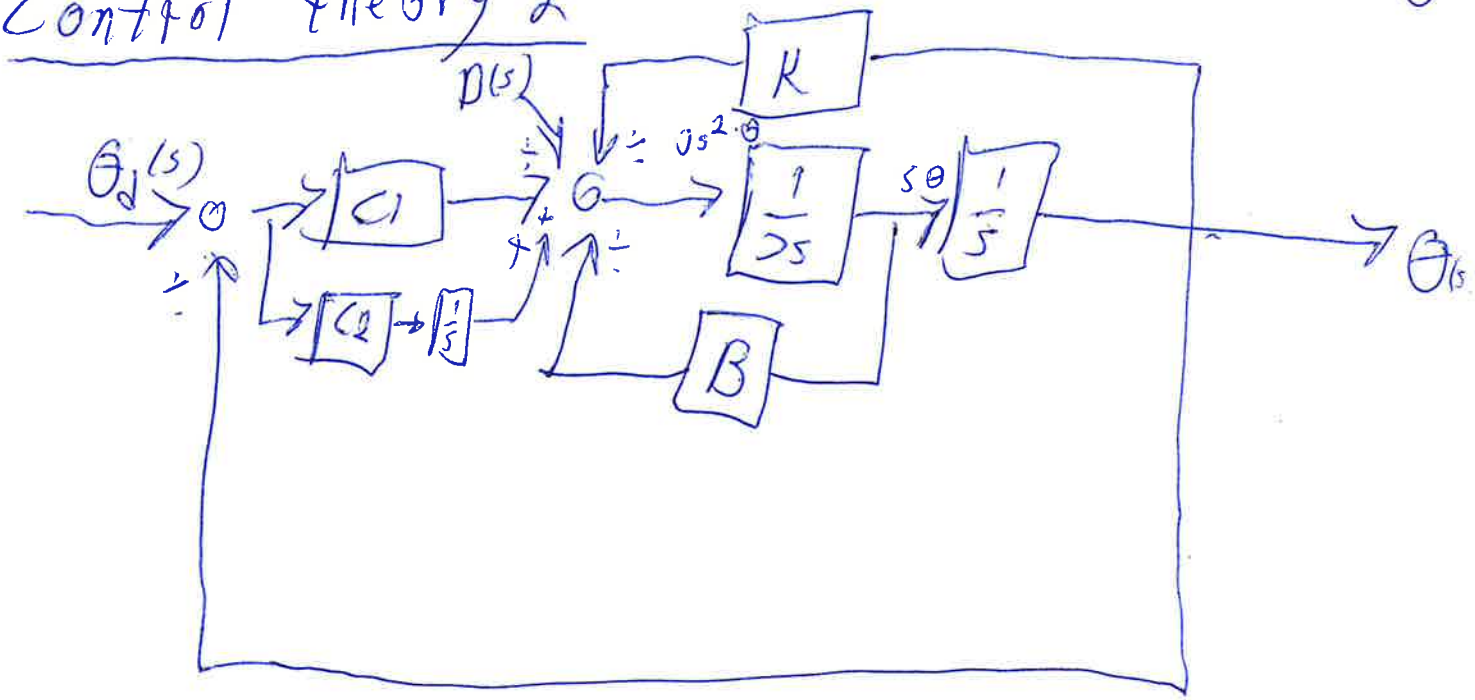
c) 6% Assume that the system receives a step reference input $\theta_d(s) = \frac{\psi}{s}$ and constant disturbance $D = \frac{d}{s}$. Use the final value theorem below to determine the steady-state error:

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$$

d) 2% Discuss the results from task c. What changes to the controller would be needed to obtain zero steady-state error? **And** show that you obtain zero steady state error.

Maximum marks: 16

control theory 2



$$\hat{C} = J\ddot{\theta}(t) + B\dot{\theta}(t) + K\theta(t)$$

a) Type of controller: P-Proportional controller

b) ~~$(\theta_d - \theta) \cdot C_1 \cdot \frac{1}{Js^2}$~~

$$(\theta_d(s) - \theta(s)) \cdot C_1 - D(s) - B \cdot s\,\theta(s) - K\,\theta(s) = Js^2\theta(s)$$

$$J\theta s^2 = C_1\theta_d - C_1\theta - D - Bs\theta - K\theta$$

$$J\theta s^2 + C_1\theta + Bs\theta + K\theta = C_1\theta_d - D$$

$$\theta(Js^2 + Bs + K + C_1) = C_1\theta_d - D$$

$$H(s) = \frac{\theta}{\theta_d} = \frac{C_1}{Js^2 + Bs + (K + C_1)}$$

c) $\Theta_d(s) = \frac{\Psi}{s}$; $D(s) = \frac{d}{s}$

Steady-state error

final value theorem : $\lim_{t\to\infty} \theta(t) = \lim_{s\to 0} s\cdot\theta(s)$

$\Rightarrow \lim_{s\to 0} s \dfrac{C_1\Theta_d - D}{Js^2 + Bs + (K + C_1)}$ solve in for Θ_d of D

$\Rightarrow \lim_{s\to 0} s \cdot \dfrac{C_1\cdot\frac{\Psi}{s} - \frac{d}{s}}{Js^2 + Bs + (K + C_1)} = \dfrac{C_1\Psi - d}{Js^2 + Bs + (K + C_1)}$

$\Rightarrow$ [illegible] $= \dfrac{C_1\Psi - d}{K + C_1} \neq 0$

d) Introduce an Integral part in the controller to ensure steady-state error = 0.

$(\Theta_d - \Theta)\left(C_1 + \frac{C_2}{s}\right) - D - Bs\Theta - K\Theta = Js^2\Theta$

$Js^2\Theta \neq C_1\Theta_d + C_2\cdot\frac{\Theta_d}{s} - C_1\Theta - C_2\frac{\Theta}{s} - D - Bs\Theta - K\Theta$

$Js^2\Theta + C_1\Theta + \frac{C_2}{s}\Theta + Bs\Theta + K\Theta = C_1\Theta_d + \frac{C_2}{s}\Theta_d - D$ /s

$Js^3\Theta + Bs^2\Theta + (C_1 + K)s\cdot\Theta + C_2\Theta = C_1\Theta_d\cdot s + C_2\Theta_d - Ds$

$\Rightarrow \Theta(Js^3 + Bs^2 + (C_1 + K)s + C_2) = (C_1 s + C_2)\Theta_d - Ds$

$\Rightarrow \dfrac{\Theta}{\Theta_d} = H(s) = \dfrac{C_1 s + C_2}{Js^3 + Bs^2 + (C_1 + K)s + C_2}$

Final value theorem:

$$\lim_{t\to\infty} \theta(t) = \lim_{s\to 0} s\cdot\theta(s)$$

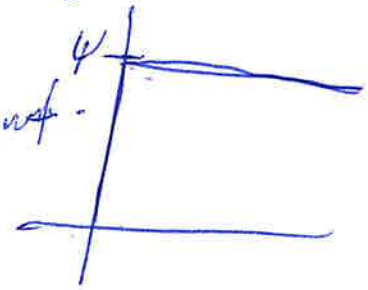
$$= \lim_{s\to 0} -s\,\frac{(c_1 s + c_2)\,\theta_d - Ds}{Js^3 + Bs^2 + (c_1+k)s + c_2}$$

setting $\theta_d = \frac{\psi}{s}$, $D = \frac{d}{s}$

$$\Rightarrow = \lim_{s\to 0} \frac{(c_1 s + c_2)\frac{\psi}{s} - d}{Js^3 + Bs^2 + (c_1+k)s + c_2}$$

$$= \frac{c_2\,\psi}{c_2} = \psi$$

→ step input.



$$\Rightarrow \text{Error } (\theta - \theta_d) = \psi - \psi = 0$$

($\theta = \psi$, $\theta_d = \psi$)