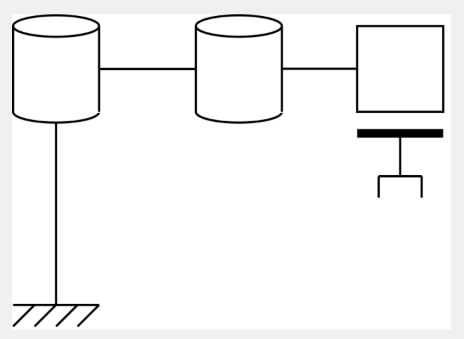
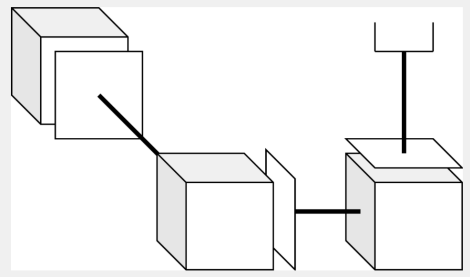
IN3140 exam 2024

Task 1



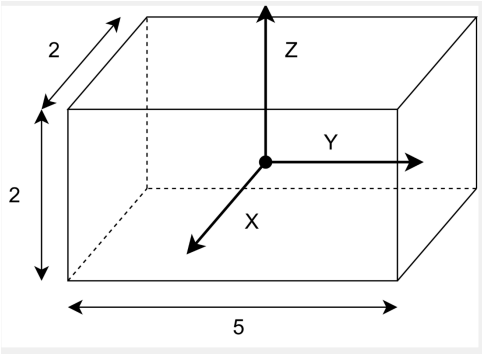
SCARA manipulator

Task 2



Cartesian manipulator

Task 3



$$I_{xy} = -\int_{-1}^{1}\int_{-2.5}^{2.5}\int_{-1}^{1} xy\ \rho(x,y,z)\ dx\,dy\,dz$$

Task 4

$$\tau = \begin{bmatrix} m_1 L_1^2 \ddot{q}_2 - m_2 L_2^2 S_2 \ddot{q}_1 - m_2 L_1 C_1 S_2 \dot{q}_1 + \frac{1}{2} m_1 g L_1 S_2 \\ m_2 L_1 C_2 \ddot{q}_2 + m_2 L_1^2 L_2^2 C_2 \dot{q}_1 \dot{q}_2 \end{bmatrix}$$

Coriolis and centrifugal:

$$C = \begin{bmatrix} -m_2 L_1 C_1 S_2 & 0 \\ m_2 L_1^2 L_2^2 C_2 \dot{q}_2 & \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

Task 5

Inertia:

$$D = \begin{bmatrix} -m_2 L_2^2 S_2 & m_1 L_1^2 \\ 0 & m_2 L_1 C_2 \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix}$$

Task 6 — 1p

PI-controller: $u(t) = K_p e(t) + K_i \int e(t)\,dt$

$u(t) = K_p (q_{desired} - q_{current}) + K_i \int (q_{desired} - q_{current})\,dt$

$K_p = 200$, $K_i = 20$

$q_{desired} = 2$

$q_{current} = 1.25$

$\Delta t = 0.25$

$\dot{q} = 0.05$

integral = 30 + (20 − 1.25) · 0.25 = 30.1875

(30: previous; 20: error; 0.25: time step)

$u(t + 0.25) = 200\,(2.0 - 1.25) + 20 \cdot 30.1875 = 753{,}75$

(200: K_d; $(2.0 - 1.25)$: error; 20: K_i; 30.1875: integral)

Task 7 — 3p

a) Why reality gap is a problem:

- real-world environments are complex difficult to model accurately in simulations and discrepancies between the real-world and the simulation can cause the robot to behave differently in a real world environment than in simulation.
- errors/discrepancies in simulation can be exploited by the AI and thus cause unexpected behavior in reality

b) Why we simulate robots:

- to avoid damaging physical robots
- safety - can avoid dangerous situations
- cheaper because we don't have to develop physical prototypes or spend a lot of time setting up physical experiments
- more effective in terms of ability to test out different algorithms/designs quickly.

↳ something along these lines should be mentioned, but it's not necessary to include all points

Task 8 3p

ROS node:

- Code with a specific purpose in your system
- Can for example be a camera that publishes images or path planning or something like this
- Each node serves a purpose and in a larger system there are typically multiple nodes that address different tasks in the system and can communicate with each other through topics or services.

→ something related to this

Task 9 10p

Why do we have different ROS topics and messages?

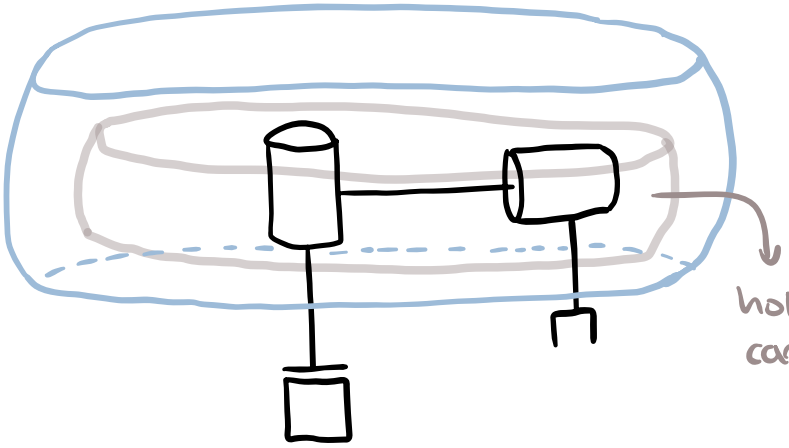
- The topics work as communication channels and we need different ones based on the information that should be communicated on the channel. The point is that each topic is dedicated to a specific task and thus it is known by the nodes that publish and subscribe to the topics what kind of information is shared.

- We need different messages because the messages contain different data that can be structured differently. Naturally, data such as text, numbers or pictures aren't represented in the same way and we need different message types with unique structure and composition for different data.

→ something related to this - all this is not needed for full marks

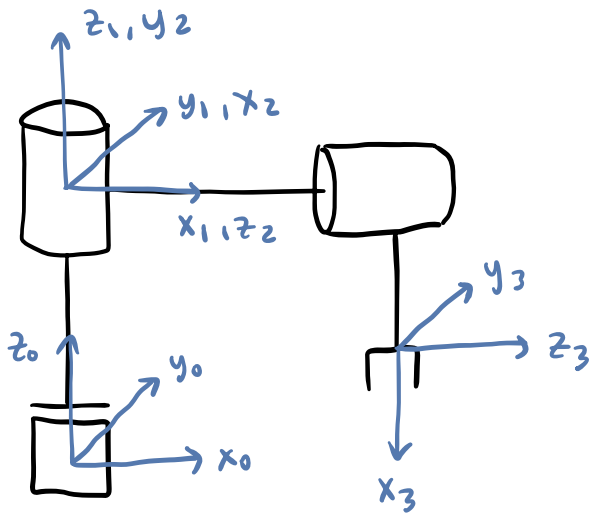
Task 10 2p

a) Workspace:



→ important to include:
- overall donut shape
- inner boundary: show the hole in the donut where the tooltip can't reach
- rounded edges

b) DH-convention 7p



i	θ_i	d_i	α_i	a_i
1	0	L_1^*	0	0
2	$\theta_2^* + 90°$	0	90°	0
3	$\theta_3^* - 90°$	L_2	0	L_3

c) Forward Kinematics 7p

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_1^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -S_2 & 0 & C_2 & 0 \\ C_2 & 0 & S_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\cos(\theta_2^* + 90) = \cos\theta_2^* \cdot \cos 90° - \sin\theta_2^* \cdot \sin 90° = -\sin\theta_2^* = -S_2$

$\sin(\theta_2^* + 90) = \sin\theta_2^* \cdot \cos 90° + \cos\theta_2^* \sin 90° = \cos\theta_2^* = C_2$

$$A_3 = \begin{bmatrix} s_3 & c_3 & 0 & L_3 s_3 \\ -c_3 & s_3 & 0 & -L_3 c_3 \\ 0 & 0 & 1 & L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\cos(\theta_3^* - 90) = \cos\theta_3^* \cos 90^\circ + \sin\theta_3^* \sin 90^\circ = \sin\theta_3^* = s_3$$
$$\sin(\theta_3^* - 90) = \sin\theta_3^* \cos 90^\circ - \cos\theta_3^* \sin 90^\circ = -\cos\theta_3^* = -c_3$$

$$A_1 A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_1^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -s_2 & 0 & c_2 & 0 \\ c_2 & 0 & s_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -s_2 & 0 & c_2 & 0 \\ c_2 & 0 & s_2 & 0 \\ 0 & 1 & 0 & L_1^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 A_2 A_3 = \begin{bmatrix} -s_2 & 0 & c_2 & 0 \\ c_2 & 0 & s_2 & 0 \\ 0 & 1 & 0 & L_1^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_3 & c_3 & 0 & L_3 s_3 \\ -c_3 & s_3 & 0 & -L_3 c_3 \\ 0 & 0 & 1 & L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -s_2 s_3 & -s_2 c_3 & c_2 & -L_3 s_2 s_3 + L_2 c_2 \\ c_2 s_3 & c_2 c_3 & s_2 & L_3 c_2 s_3 + L_2 s_2 \\ -c_3 & s_3 & 0 & -L_3 c_3 + L_1^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

d) Inverse Kinematics (analytical method) 10p

$$x = -L_3 s_2 s_3 + L_2 c_2$$
$$y = L_3 c_2 s_3 + L_2 s_2$$
$$z = -L_3 c_3 + L_1^*$$

$$x^2 = (-L_3 s_2 s_3 + L_2 c_2)^2 = L_3^2 s_2^2 s_3^2 - 2L_2 L_3 s_2 c_2 s_3 + L_2^2 c_2^2$$
$$y^2 = (L_3 c_2 s_3 + L_2 s_2)^2 = L_3^2 c_2^2 s_3^2 + 2L_2 L_3 s_2 c_2 s_3 + L_2^2 s_2^2$$

$$x^2 + y^2 = L_3^2 s_3^2 \underbrace{(s_2^2 + c_2^2)}_{1} + L_2^2 \underbrace{(c_2^2 + s_2^2)}_{1}$$

$$x^2 + y^2 = L_3^2 s_3^2 + L_2^2$$

$$L_3^2 s_3^2 = x^2 + y^2 - L_2^2$$

$$s_3^2 = \frac{x^2 + y^2 - L_2^2}{L_3^2} \Rightarrow s_3 = \frac{\sqrt{x^2 + y^2 - L_2^2}}{L_3} := D$$

$\cos^2\theta + \sin^2\theta = 1 \Rightarrow \cos\theta = \sqrt{1 - \sin^2\theta}$

$$c_3 = \sqrt{1 - s_3^2} = \sqrt{1 - D^2}$$

$$\underline{\theta_3^* = \operatorname{atan2}(\sqrt{1 - D^2}, D)}$$

$$z = -L_3 c_3 + L_1^* \Rightarrow \underline{L_1^* = z + L_3 c_3}$$

$a\cos\theta + b\sin\theta = c \Leftrightarrow \theta = \operatorname{atan2}\left(\frac{b}{a}\right) \pm \operatorname{atan2}\left(\frac{\sqrt{a^2 + b^2 + c^2}}{c}\right)$

$$x = -L_3 s_2 s_3 + L_2 c_2$$

$$\underbrace{L_2}_{a} c_2 \underbrace{- L_3 s_3}_{b} s_2 = \underbrace{x}_{c}$$

$$\theta_2 = \operatorname{atan2}(-L_3 s_3, L_2) \pm \operatorname{atan2}(L_2^2 + L_3^2 s_3^2 + x^2, x)$$

e) Jacobian

$$J_{vi} = \begin{cases} z_0 \times (o_3 - o_0) & \text{rev.} \\ z_0 & \text{prism.} \end{cases} \qquad J_{wi} = \begin{cases} z_0 & \text{rev.} \\ 0 & \text{prism.} \end{cases}$$

$$J = \begin{bmatrix} z_0 & z_1 \times (o_3 - o_1) & z_2 \times (o_3 - o_2) \\ 0 & z_1 & z_2 \end{bmatrix}$$

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad z_2 = \begin{bmatrix} c_2 \\ s_2 \\ 0 \end{bmatrix}$$

$$o_1 = \begin{bmatrix} 0 \\ 0 \\ L_1 \end{bmatrix} \qquad o_2 = \begin{bmatrix} 0 \\ 0 \\ L_1 \end{bmatrix}, \quad o_3 = \begin{bmatrix} -L_3 s_2 s_3 + L_2 c_2 \\ L_3 c_2 s_3 + L_2 s_2 \\ -L_3 c_3 + L_1 \end{bmatrix}$$

$$o_3 - o_1 = \begin{bmatrix} -L_3 s_2 s_3 + L_2 c_2 \\ L_3 c_2 s_3 + L_2 s_2 \\ -L_3 c_3 \end{bmatrix} \begin{matrix} a \\ b \\ c \end{matrix} \qquad o_3 - o_2 = \begin{bmatrix} -L_3 s_2 s_3 + L_2 c_2 \\ L_3 c_2 s_3 + L_2 s_2 \\ -L_3 c_3 \end{bmatrix} \begin{matrix} a \\ b \\ c \end{matrix}$$

$$z_1 \times (o_3 - o_1) = \begin{bmatrix} -b \\ a \\ 0 \end{bmatrix} = \begin{bmatrix} -L_3 c_2 s_3 - L_2 s_2 \\ -L_3 s_2 s_3 + L_2 c_2 \\ 0 \end{bmatrix}$$

x	y	z	x	y
0	0	1	0	1
a	b	c	a	b

$$z_2 \times (o_3 - o_1) = \begin{bmatrix} s_2 c \\ -c_2 c \\ c_2 b - s_2 a \end{bmatrix} = \begin{bmatrix} -L_3 s_2 c_3 \\ L_3 c_2 c_3 \\ L_3 s_3 \end{bmatrix}$$

x	y	z	x	y
c_2	s_2	0	c_2	s_2
a	b	c	a	b

$$c_2(L_3 c_2 s_3 + L_2 s_2) - s_2(-L_3 s_2 s_3 + L_2 c_2) =$$
$$L_3 c_2^2 s_3 + L_2 c_2 s_2 + L_3 s_2^2 s_3 - L_2 s_2 c_2 =$$
$$L_3 s_3 (c_2^2 + s_2^2) =$$
$$L_3 s_3$$

$$J = \begin{bmatrix} 0 & -L_3 C_2 S_3 - L_2 S_2 & -L_3 S_2 C_3 \\ 0 & -L_3 S_2 S_3 + L_2 C_2 & L_3 C_2 C_3 \\ 1 & 0 & L_3 S_3 \\ 0 & 0 & C_2 \\ 0 & 0 & S_2 \\ 0 & 1 & 0 \end{bmatrix}$$

Task II 4p

a)

$$t^w = T_b^w t^b$$

→ Rot y_1 90°

$$R_b^w = \begin{bmatrix} \cos 90° & 0 & \sin 90° \\ 0 & 1 & 0 \\ -\sin 90° & 0 & \cos 90° \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$T_b^w = \begin{bmatrix} 0 & 0 & 1 & 2.7 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$t^b = \begin{bmatrix} -L_3 S_2 S_3 + L_2 C_2 \\ L_3 C_2 S_3 + L_2 S_2 \\ -L_3 C_3 + L_1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.071 \\ 0.35 \\ 0.35 \\ 1 \end{bmatrix}$$?

$$t^w = \begin{bmatrix} 0 & 0 & 1 & 2.7 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.071 \\ 0.35 \\ 0.35 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3.05 \\ 0.35 \\ -0.071 \\ 1 \end{bmatrix}$$

$x = 3.05$ m
$y = 0.35$ m
$z = -0.071$ m

b) Since the parts come with a random orientation, the robot will not be able to grasp the objects since it does not have a wrist. Now it can only grab objects if they come on a specific orientation, so to complete the task, a spherical wrist (3 DOF) or some extra degrees of freedom needs to be added so that it is able to grab objects in any orientation.

Task 12 — 18,5p

a) Kinetic and potential energy

initial J

$$J_v = \begin{bmatrix} 0 & -L_3 C_2 S_3 - L_2 S_2 & -L_3 S_2 C_3 \\ 0 & -L_3 S_2 S_3 + L_2 C_2 & L_3 C_2 L_3 \\ 1 & 0 & L_3 S_3 \end{bmatrix}$$

$$J_w = \begin{bmatrix} 0 & 0 & C_2 \\ 0 & 0 & S_2 \\ 0 & 1 & 0 \end{bmatrix}$$

$$I = \begin{bmatrix} Ixx & 0 & 0 \\ 0 & Iyy & 0 \\ 0 & 0 & Izz \end{bmatrix}$$

$\theta_3 = 0$
$L_3 = 0$
$L_1 = 0$
columns 1 and 3 = 0

$$K = \underbrace{\frac{1}{2} m v^T v}_{①} + \underbrace{\frac{1}{2} w^T I w}_{②}$$

① $$J_v = \begin{bmatrix} 0 & -\frac{2}{3} L_2 S_2 & 0 \\ 0 & \frac{2}{3} L_2 C_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$v = J_v \dot{q} = \begin{bmatrix} 0 & -\frac{2}{3} L_2 S_2 & 0 \\ 0 & \frac{2}{3} L_2 C_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} L_2 S_2 \dot{q}_2 \\ \frac{2}{3} L_2 C_2 \dot{q}_2 \\ 0 \end{bmatrix}$$

$$v^T v = \begin{bmatrix} -\frac{2}{3} L_2 S_2 \dot{q}_2 & \frac{2}{3} L_2 C_2 \dot{q}_2 & 0 \end{bmatrix} \begin{bmatrix} -\frac{2}{3} L_2 S_2 \dot{q}_2 \\ \frac{2}{3} L_2 C_2 \dot{q}_2 \\ 0 \end{bmatrix}$$

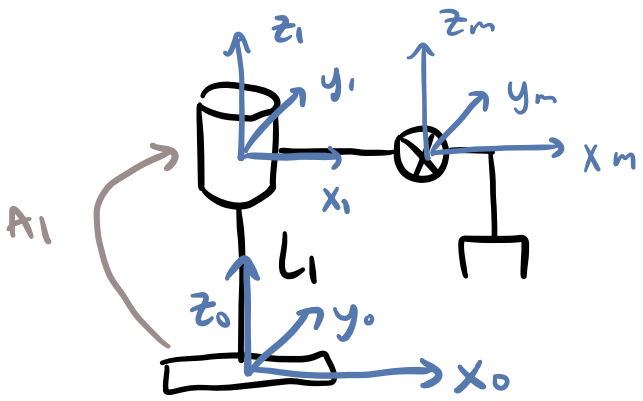
$$= \frac{4}{9} L_2^2 S_2^2 \dot{q}_2^2 + \frac{4}{9} L_2^2 C_2^2 \dot{q}_2^2$$

$$= \frac{4}{9} L_2^2 \dot{q}_2^2 (C_2^2 + C_2^2) = \underline{\frac{4}{9} L_2^2 \dot{q}_2^2}$$

② $$J_w = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\omega = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{q}_2 \end{bmatrix}$$

$$I = R\, I\, R^T$$



$$R = A_1 \cdot Rot_{z,\theta_2}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$I = \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \begin{bmatrix} c_2 & s_2 & 0 \\ -s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} I_{xx}c_2 & -I_{yy}s_2 & 0 \\ I_{xx}s_2 & I_{yy}c_2 & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \begin{bmatrix} c_2 & s_2 & 0 \\ -s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} I_{xx}c_2^2 + I_{yy}s_2^2 & I_{xx}c_2s_2 - I_{yy}s_2c_2 & 0 \\ I_{xx}s_2c_2 - I_{yy}s_2c_2 & I_{xx}s_2^2 + I_{yy}c_2^2 & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}$$

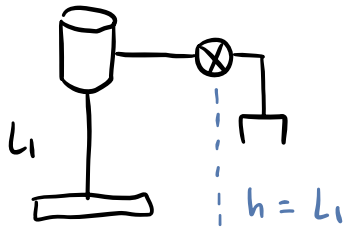
$$\omega^T I \omega = \begin{bmatrix} 0 & 0 & \dot{q}_2 \end{bmatrix} \begin{bmatrix} I_{xx}c_2^2 + I_{yy}s_2^2 & I_{xx}c_2s_2 - I_{yy}s_2c_2 & 0 \\ I_{xx}s_2c_2 - I_{yy}s_2c_2 & I_{xx}s_2^2 + I_{yy}c_2^2 & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{q}_2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & I_{zz}\dot{q}_2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{q}_2 \end{bmatrix} = I_{zz}\dot{q}_2^2$$

$$K = \frac{1}{2} m \cdot \frac{4}{9} L_2^2 \dot{q}_2^2 + \frac{1}{2} I_{zz} \dot{q}_2^2 = \frac{1}{2}\dot{q}_2^2 \left(\frac{4}{9} L_2^2 + I_{zz} \right)$$

$P = mgh$

$$P = mgL_1$$



b) $$L = K - P = \frac{1}{2}\dot{q}_2^2\left(\frac{4}{9}mL_2^2 + I_{zz}\right) - mgL_1$$

$$\tau = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_2} - \frac{\partial L}{\partial q_2}$$

$$\frac{\partial L}{\partial \dot{q}_2} = \frac{\frac{1}{2}\dot{q}_2^2\left(\frac{4}{9}mL_2^2 + I_{zz}\right)}{\partial \dot{q}_2}$$

$$= \dot{q}_2\left(\frac{4}{9}mL_2^2 + I_{zz}\right)$$

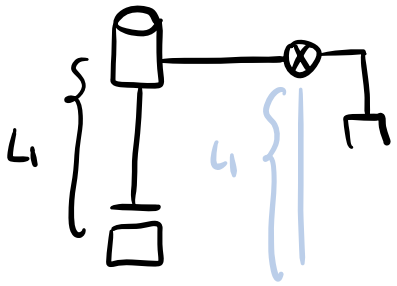
$$\frac{d}{dt}\left(\dot{q}_2\left(\frac{4}{9}mL_2^2 + I_{zz}\right)\right) = \ddot{q}_2\left(\frac{4}{9}mL_2^2 + I_{zz}\right)$$

$$\frac{\partial L}{\partial q_2} = \frac{\frac{1}{2}\dot{q}_2^2\left(\frac{4}{9}mL_2^2 + I_{zz}\right)}{\partial q_2} = 0$$

$$\tau = \ddot{q}_2\left(\frac{4}{9}mL_2^2 + I_{zz}\right)$$

$$\tau = D\ddot{q}_2 \,, \quad D = \frac{4}{9}mL_2^2 + I_{zz}$$

c)



$P = mgh \quad \Rightarrow P = mgL_1$

L_1 in this case, the height depends on the first prismatic joint and thus the potential energy does too.

Since we add the prismatic joint back, when calculating the kinetic and potential energy, the first column of the jacobian is not 0 anymore and $L_1 \neq 0$.

For J_W, this column is 0 regardless. But J_V will instead be

$$J_V = \begin{bmatrix} 0 & -2/3 L_2 S_2 & 0 \\ 0 & 2/3 L_2 C_2 & 0 \\ 1 & 0 & 0 \end{bmatrix} \qquad J_W = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

This will lead to the addition of the contribution of $\dot{q}_1$ to the kinetic energy of the system.

$$V = \begin{bmatrix} -2/3 L_2 S_2 \dot{q}_2 \\ 2/3 L_2 C_2 \dot{q}_2 \\ \dot{q}_1 \end{bmatrix}$$

Since there is only 1 mass point still, the rest of the energy calculation is similar as in 6. The new lagrange should thus be:

$$V^T V = 4/9 L_2^2 \dot{q}_2^2 + \dot{q}_1^2$$

Since J_W is the same as before, the angular part of the kinetic energy is the same, so:

$$K = \frac{1}{2} m \left(4/9 L_2^2 \dot{q}_2^2 + \dot{q}_1^2 \right) + I_{zz} \dot{q}_2^2$$

And the lagrangian is:

$$L = \frac{1}{2} m \left(4/9 L_2^2 \dot{q}_2^2 + \dot{q}_1^2 \right) + I_{zz} \dot{q}_2^2 - mgL_1$$

However, when using the euler-lagrange equation to find the dynamics of the system, we now need to account for both joints, so the lagrangian is both differentiated with respect to the first and second joint.

This means that we need to find

$$\tau_1 = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_1} - \frac{\partial L}{\partial q_1} \quad \text{and} \quad \tau_2 = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_2} - \frac{\partial L}{\partial q_2}$$

and set it up on the form

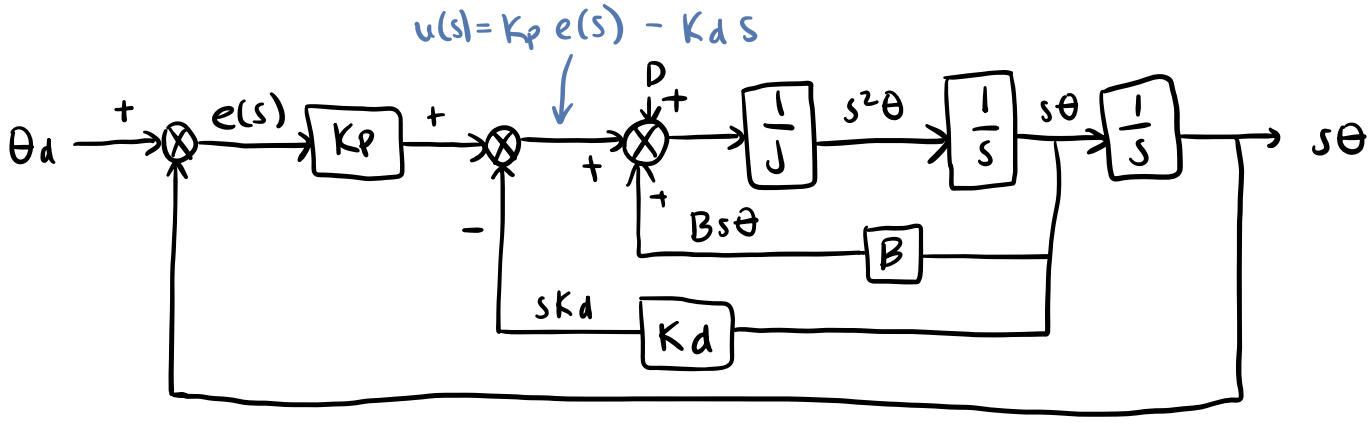
$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

to find the total dynamics of the system.

→ here it is important to show the overall approach and not accurate calculations

Task 13 18

a)



b) PD - controller

c) $u_{controller} = u_{system}$ (laplace domain of $\tau = J\ddot{\theta}(t) + B\dot{\theta}$)

$$K_p(\theta_d - \theta) - K_d s\theta = Js^2\theta + Bs\theta + D$$

$$K_p\theta_d - K_p\theta - K_d s\theta = Js^2\theta + Bs\theta + D$$

$$K_p\theta_d - D = \theta(Js^2 + Bs + K_p + K_d s) \quad *$$

$$H(s) = \frac{K_p - D}{Js^2 + Bs + K_p + K_d s} =$$

$$\underbrace{\frac{K_p}{Js^2 + Bs + K_p + K_d s}}_{\theta / \theta_d} - \frac{D}{Js^2 + Bs + K_p + K_d s}$$

$$\frac{\theta}{\theta_d} = \frac{K_p}{Js^2 + Bs + K_p + K_d s}$$

d) $\theta_d = \Psi/s$, $D = d/s$, $e_{ss} = \lim_{s \to 0} sE(s)$

$$\theta = \frac{K_p\theta_d - D}{Js^2 + Bs + K_p + K_d s} = X(s)$$

$\lim_{t \to \infty} x(t) = \lim_{s \to 0} sX(s)$

$$E(s) = (\theta_d - \theta) = \left(\theta_d - \frac{K_p\theta_d - D}{Js^2 + Bs + K_p + K_d s}\right)$$

$$= \frac{\theta_d(Js^2 + Bs + K_p + K_d s)}{Js^2 + Bs + K_p + K_d s} - \frac{K_p\theta_d - D}{Js^2 + Bs + K_p + K_d s}$$

$$= \frac{\theta_d Js^2 + \theta_d Bs + \theta_d K_p + \theta_d K_d s - K_p\theta_d - D}{Js^2 + Bs + K_p + K_d s}$$

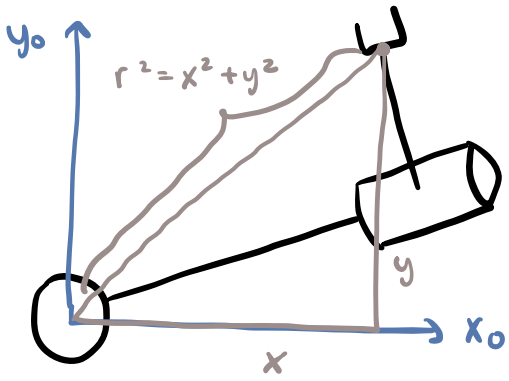
$$= \frac{\theta_d(Js^2 + Bs + \theta_d K_d s - D}{Js^2 + Bs + K_p + K_d s}$$

$$e_{ss} = \lim_{s \to 0} sE(s) = \frac{s(\Psi/s(Js^2 + Bs + \theta_d K_d s) - d/s)}{Js^2 + Bs + K_p + K_d s}$$

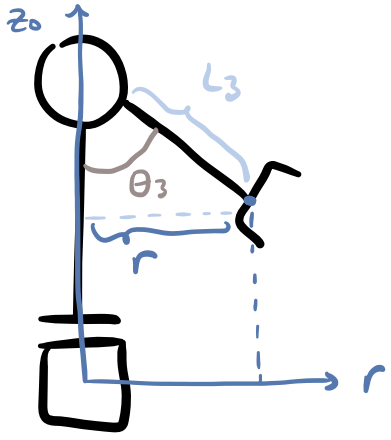
$$= \lim_{s \to 0} \frac{\Psi(Js^2 + Bs + \theta_d K_d s) - d}{Js^2 + Bs + K_p + K_d s} = -\frac{d}{K_p}$$

Geometrical approach:

Top view:

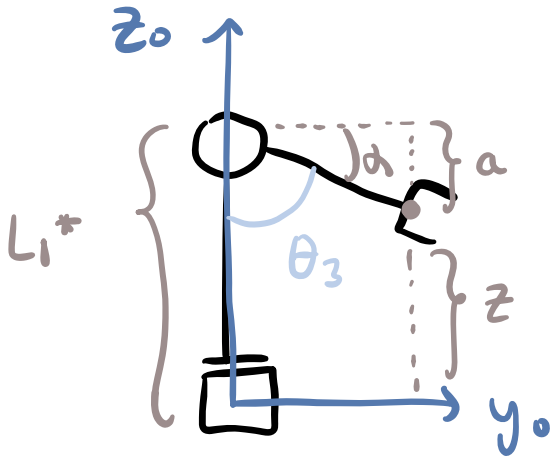


Front view:



$$\sin\theta_3 = \frac{r}{L_3}$$

Front view:



$$L_1^* = z + a$$

$$\sin\alpha = \frac{a}{L_3} \Rightarrow a = L_3 \sin\alpha$$

$$\alpha = 90 - \theta_3$$

$$\sin\alpha = \sin(90 - \theta_3) = \cos\theta_3$$

$$a = L_3 C_3$$

$$\underline{\underline{L_1^* = z + L_3 C_3}}$$