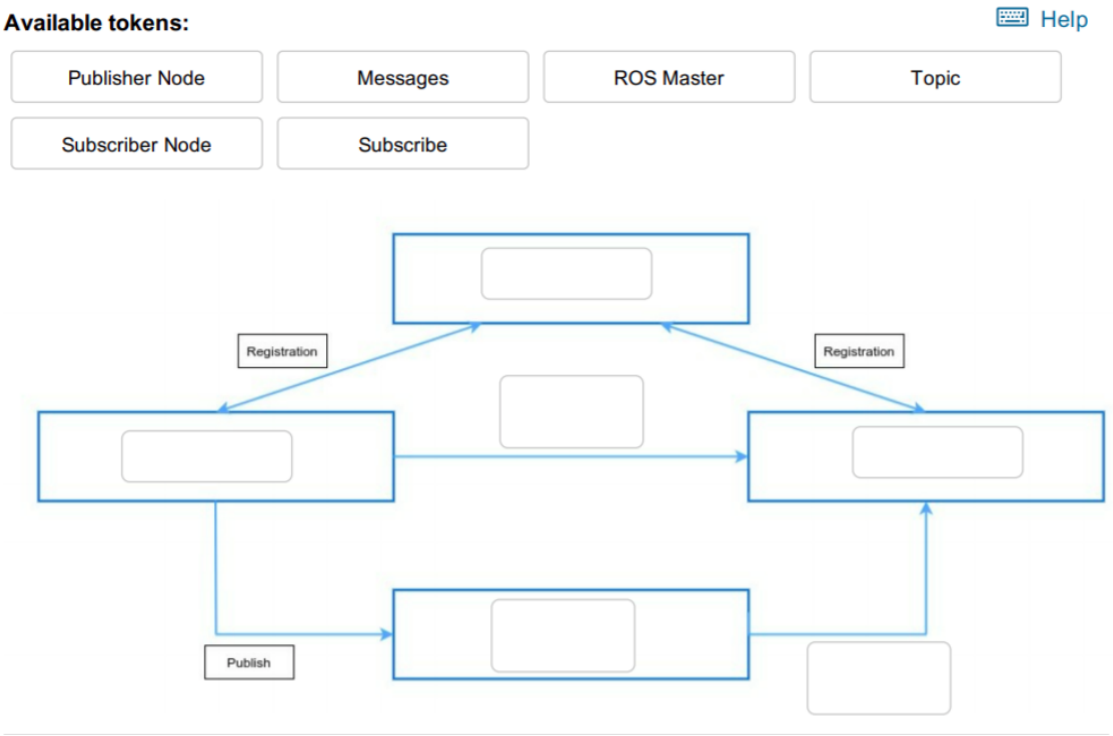
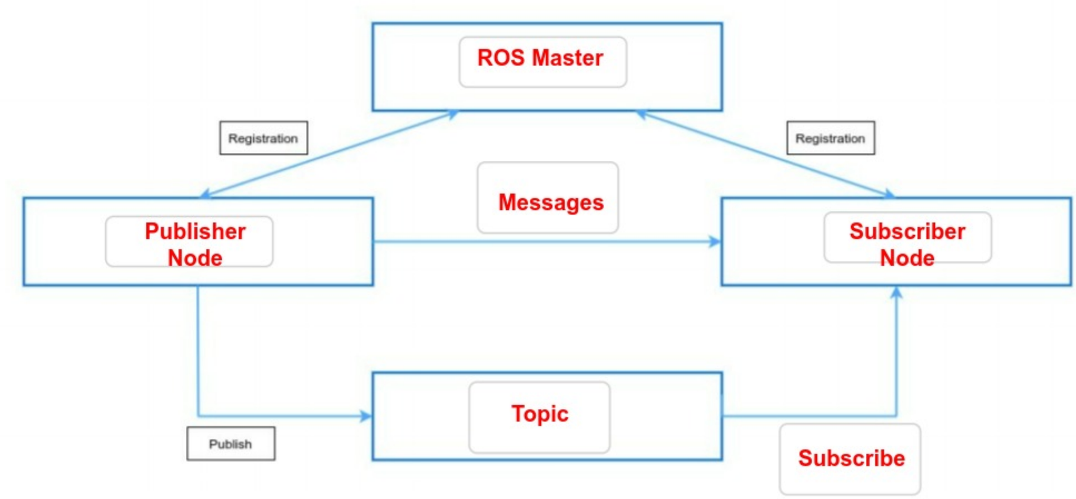
IN3140/IN4140 Introduction to Robotics Final Examination Spring 2021 (with Solutions)

Task 1.1 (7%)

Match the tokens below with the empty fields in the ROS architecture diagram. You can move the tokens by dragging them over to the desired positions. **Each match will grant you one mark, and each mismatch will cost you one mark instead. Perfectly matching all the tokens will award you one additional mark. Unused tokens will provide no reward and incur no penalty.**



Solution:



Note: the worst obtainable score in this task is -6%. The worst score is obtained by using all tokens without getting a single match.

Task 1.2 (3%)

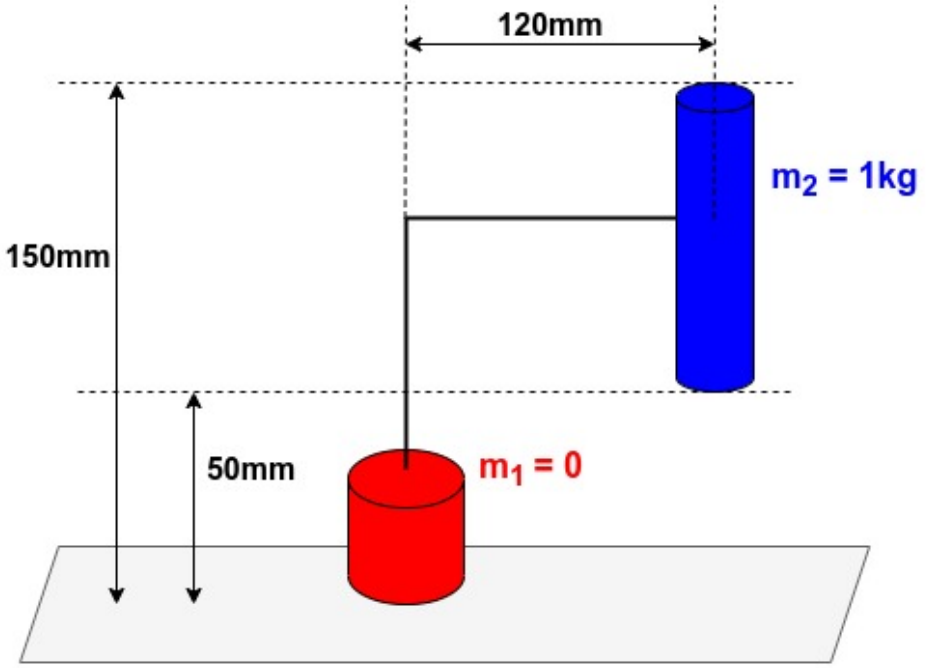
Briefly explain what **RViz**, **Gazebo**, **MoveIt**, and **OpenRave** are and describe their functionality.

Possible Answer:

They are tools/packages that can be used together with ROS due to its modular nature. In particular:

- **RViz** is a visualization package**.** It displays information from the sensors of a robot (such as cameras).
- **Gazebo** is a simulation package. It can display a 3D model of the robot and simulate its behavior.
- **MoveIt** is a robot manipulation package with enabled support for motion planning and kinematics.
- **OpenRave** is also a robot manipulation and motion planning package, similarly to MoveIt.

Task 2.1 (4%)



Consider the system in the image above. Given that the masses $m_1 = 0$ and $m_2 = 1kg$ are both uniformly distributed, and that gravitational acceleration $g = 10ms^{-2}$, what is the total potential energy of the system? **Provide a short numerical answer, including the units of energy. If you are uncertain of your answer, you may show the calculation steps to secure partial rewards.**

Solution:

$P = 1J$

Reasoning: Since $m_1 = 0$, only m_2 contributes to the total potential energy. Further, the body of m_2 occupies the height between 50mm and 150mm over the ground. The midpoint height is 100mm (0.1m). Since m_2 is uniformly distributed, the following holds:

$$P = m_2 g h_2$$

$$P = 1 * 10 * 0.1 = 1(J)$$

Task 2.2 (4%)

Consider a robotic system with the following equation of motion:

$$\tau = \begin{bmatrix} \frac{m_1 b L_1^2 \ddot{\theta}_1}{8} - 2c_1 s_2 \dot{\theta}_1^2 + m_1 g L_1 c_\gamma \\ m_2 \rho L_2 \ddot{\theta}_2 + (c^2 - s^2) s_1 \dot{\theta}_1 \dot{\theta}_2 + \frac{m_2 g L_2 s_{12}}{16} \end{bmatrix}$$

Which expression(s) would correctly describe **Coriolis and centrifugal forces** acting on the system?
Select one or more alternatives. Each wrong selection will cost you two marks.

- $\begin{bmatrix} -2c_1 s_2 \dot{\theta}_1 & 0 \\ 0 & (c^2 - s^2) s_1 \dot{\theta}_2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}$
- $\begin{bmatrix} \frac{m_1 b L_1^2 \dot{\theta}_1}{8} & 0 \\ 0 & m_2 \rho L_2 \dot{\theta}_2 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$
- ■ $\begin{bmatrix} -2c_1 s_2 \dot{\theta}_1 & 0 \\ (c^2 - s^2) s_1 \dot{\theta}_2 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$ ← the only correct answer
- $\begin{bmatrix} \frac{m_1 b L_1^2}{8} & 0 \\ 0 & m_2 \rho L_2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}$

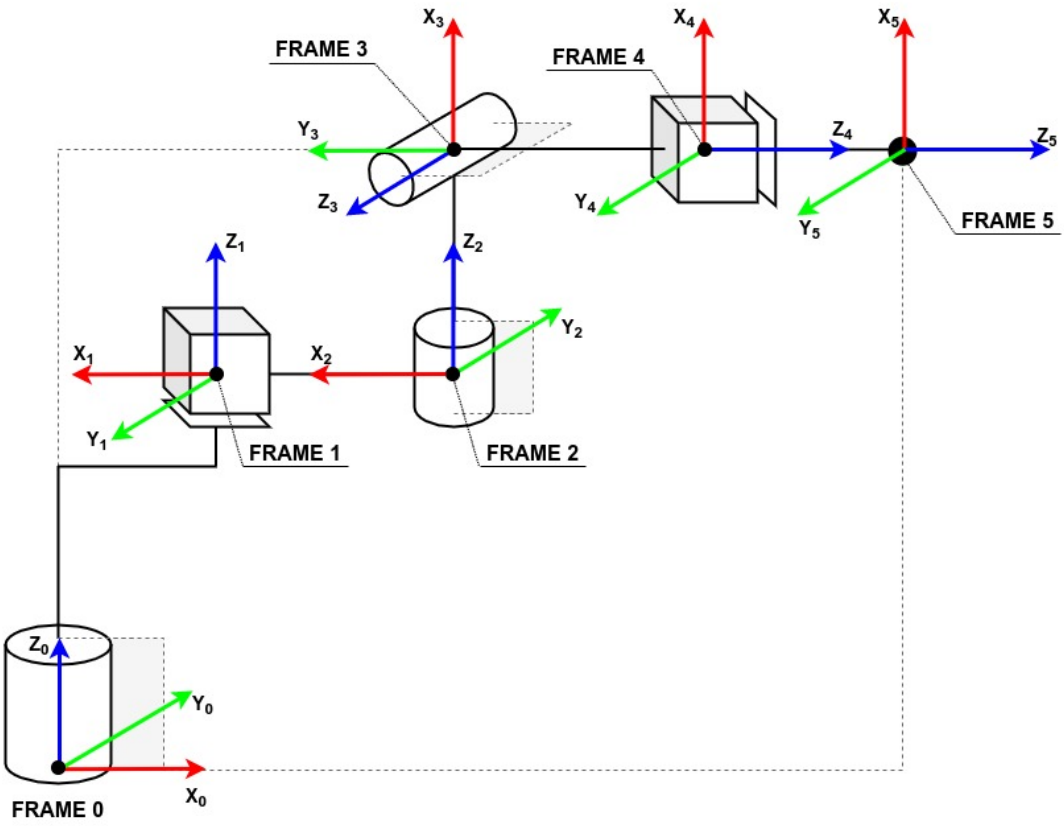
Reasoning: the dynamical model can be decomposed to match the following form:

$$\tau = D(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q)$$

In this form, the term $C(q, \dot{q})\dot{q}$ is the one that describes Coriolis and centrifugal forces. Therefore, one should look for the alternatives that both match this pattern and correctly represent the actual terms of τ. Only one such alternative is offered.

Notes: the options are presented in a random order during the examination. Selecting the correct option provides a reward of 4%. The worst obtainable score in this task is -6%. The worst score is obtained by selecting all options but the correct one.

Task 3.1 (9%)



Observe the manipulator in the image above. Six coordinate frames (FRAME 0 through FRAME 5) have been assigned to the manipulator. However, some of the frames do not completely follow the Denavit-Hartenberg convention. The dashed lines in the image should help you understand the geometry of the space and the axes of joint rotation.

State which of the coordinate frames break the Denavit-Hartenberg convention rules and VERY briefly explain your reasoning. Note that the frames are to be evaluated separately. For example, if you argue that FRAME 1 is "wrong", it cannot be just because FRAME 0 is "wrong". You will:

- **receive one mark for each correctly identified frame and up to two more marks for explaining your choice;**
- **lose one mark for every incorrectly identified frame.**

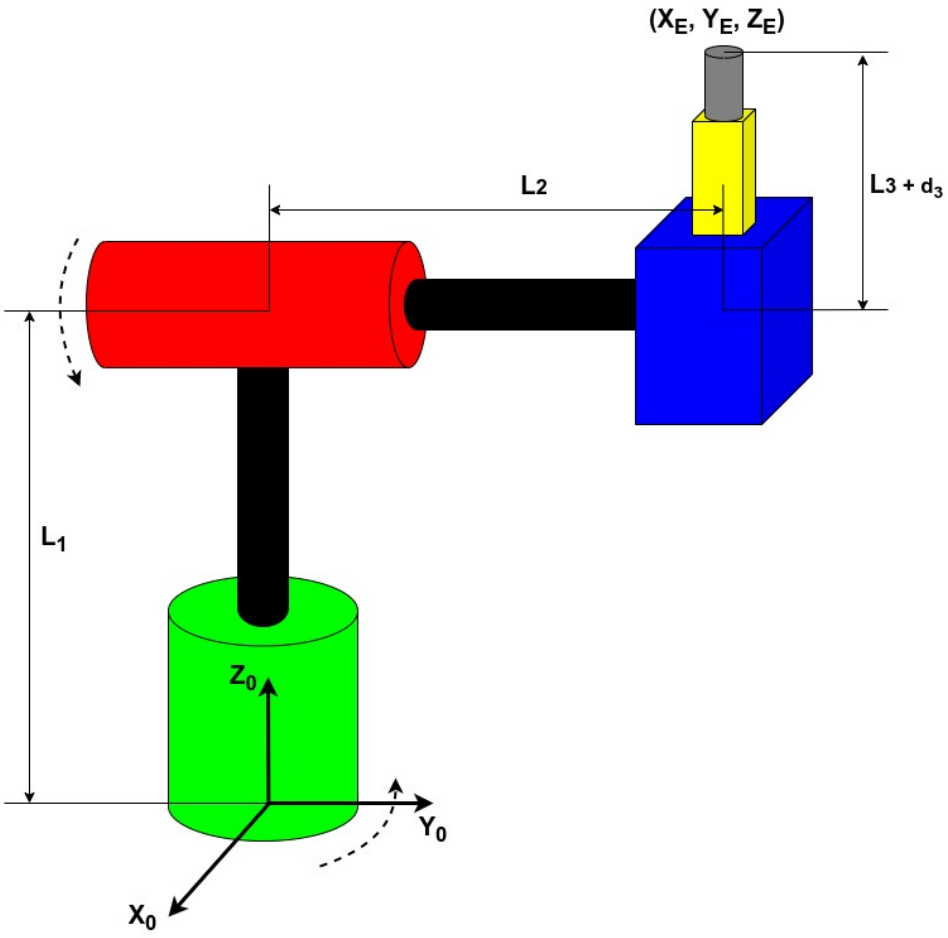
Possible Answer:

The wrongly assigned frames are:

- **FRAME 2**, because Y_2 does not follow the right hand rule;
- **FRAME 3**, because X_3 is not perpendicular to Z_2;
- **FRAME 4**, because X_4 does not intersect Z_3.

Notes: The worst obtainable score in this task is -3%. The worst score is obtained by stating that frames 0, 1, and 5 are incorrectly assigned (without mentioning frames 2, 3, and 4).

Task 4 (File Upload, 45%)

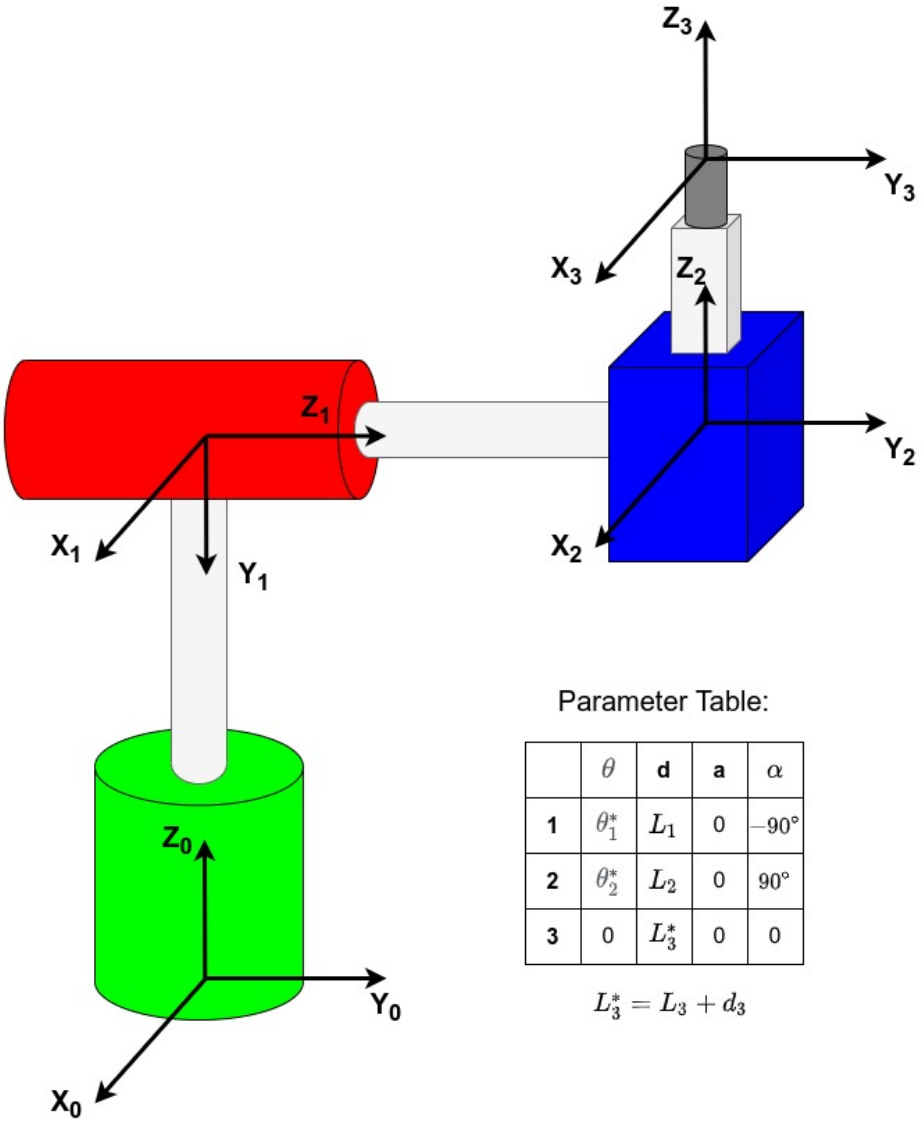


Observe the RRP robot in the image above. The base frame $(\mathbf{X_0}, \mathbf{Y_0}, \mathbf{Z_0})$ has already been assigned to the manipulator, and the axes $\mathbf{X_0}$, $\mathbf{Y_0}$, and $\mathbf{Z_0}$ are all perpendicular to each other.

Task 4.1 (5%)

Assign the remaining coordinate frames (up to and including the end-effector frame) to the robot in accordance with the Denavit-Hartenberg convention. Draw the robot with the assigned frames. Find the Denavit-Hartenberg parameters for the resulting configuration and list them in a table.

Possible Solution:



Parameter Table:

	θ	d	a	α
1	θ_1^*	L_1	0	$-90°$
2	θ_2^*	L_2	0	$90°$
3	0	L_3^*	0	0

$L_3^* = L_3 + d_3$

Task 4.2 (10%)

Derive the forward kinematics for the robot.

Possible Solution:

General transformation matrix:

$$A_i = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

Using (1) with the values from the parameter table:

$$A_1 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_3^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then,

$$H_2^0 = A_1 A_2 = \begin{bmatrix} c_1c_2 & -s_1 & c_1s_2 & -s_1L_2 \\ s_1c_2 & c_1 & s_1s_2 & c_1L_2 \\ -s_2 & 0 & c_2 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Finally,

$$H_3^0 = H_2^0 A_3 = \begin{bmatrix} c_1c_2 & -s_1 & c_1s_2 & -s_1L_2 + c_1s_2L_3^* \\ s_1c_2 & c_1 & s_1s_2 & c_1L_2 + s_1s_2L_3^* \\ -s_2 & 0 & c_2 & L_1 + c_2L_3^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Task 4.3 (10%)

Derive the inverse kinematics for the robot. Amendment made during the examination: $\theta_1 = 0°$ for this task.

Possible Solution:

Given that $\theta_1 = 0°$, the end-effector coordinates are

$$\begin{cases} x = -s_1L_2 + c_1s_2L_3^* = s_2L_3^* \\ y = c_1L_2 + s_1s_2L_3^* = L_2 \\ z = L_1 + c_2L_3^* \end{cases}$$

Calculations for d_3:

$$x^2 + (z - L_1)^2 = (s_2^2 + c_2^2)L_3^{*^2} = L_3^{*^2}$$

$$L_3^* = \sqrt{x^2 + (z - L_1)^2}$$

$$d_3 = L_3^* - L_3$$

Calculations for θ_2:

$$\frac{x}{z - L_1} = \frac{s_2 \cancel{L_3^*}}{c_2 \cancel{L_3^*}}$$

$$\theta_2 = atan2\left(\frac{x}{z - L_1}\right)$$

Task 4.4 (10%)

Derive the **linear** part of the Jacobian matrix (J_v) for the robot.

Possible Solution:

The formula for an RRP robot:

$$J_v = \begin{bmatrix} J_{v_1} & J_{v_2} & J_{v_3} \end{bmatrix} =$$

$$\begin{bmatrix} z_0 \times (o_3 - o_0) & z_1 \times (o_3 - o_1) & z_2 \end{bmatrix}$$

Calculations for J_{v_1}:

$$J_{v_1} = z_0 \times (o_3 - o_0) =$$

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} -s_1 L_2 + c_1 s_2 L_3^* \\ c_1 L_2 + s_1 s_2 L_3^* \\ L_1 + c_2 L_3^* \end{bmatrix} = \begin{bmatrix} -c_1 L_2 - s_1 s_2 L_3^* \\ -s_1 L_2 + c_1 s_2 L_3^* \\ 0 \end{bmatrix}$$

Calculations for J_{v_2}:

$$J_{v_1} = z_1 \times (o_3 - o_1) =$$

$$\begin{bmatrix} -s_1 \\ c_1 \\ 0 \end{bmatrix} \times \begin{bmatrix} -s_1 L_2 + c_1 s_2 L_3^* \\ c_1 L_2 + s_1 s_2 L_3^* \\ c_2 L_3^* \end{bmatrix} = \begin{bmatrix} c_1 c_2 L_3^* \\ s_1 c_2 L_3^* \\ -s_2 L_3^* \end{bmatrix} \leftarrow \text{simplified}$$

Calculations for J_{v_3}:

$$J_{v_3} = z_2 = \begin{bmatrix} c_1 s_2 \\ s_1 s_2 \\ c_2 \end{bmatrix}$$

Answer:

$$J_v = \begin{bmatrix} -c_1 L_2 - s_1 s_2 L_3^* & c_1 c_2 L_3^* & c_1 s_2 \\ -s_1 L_2 + c_1 s_2 L_3^* & s_1 c_2 L_3^* & s_1 s_2 \\ 0 & -s_2 L_3^* & c_2 \end{bmatrix}$$

Task 4.5 (10%)

Briefly explain how you can find singular configurations for this robot. Find the singular configurations and sketch one of them. Briefly explain how the sketched configuration would affect the robot's ability to move in a given direction. **You are allowed to use Python or MATLAB programs to answer this question, but you must write and submit your own code.**

Possible Solution:

Singularities can be found by solving $det(J_v) = 0$. Using Python to simplify $det(J_v)$ for the given robot:

```
import sympy as sp

t1, t2 = sp.symbols("t1 t2") # angles present in Jv
L2, L3 = sp.symbols("L2 L3") # link lengths present in Jv

s1, s2 = sp.sin(t1), sp.sin(t2) # sines
c1, c2 = sp.cos(t1), sp.cos(t2) # cosines

Jv = sp.Matrix([[-c1*L2 -s1*s2*L3, c1*c2*L3, c1*s2],
                [-s1*L2 +c1*s2*L3, s1*c2*L3, s1*s2],
                [0, -s2*L3, c2]])

determinant = sp.simplify(sp.det(Jv))
sp.pprint(determinant)
```

The result is

$$det(J_v) = -s_2 L_3^{*^2}$$

Therefore, singularities are found at

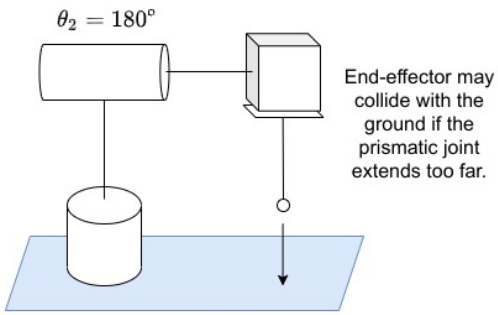
$$-s_2 L_3^{*^2} = 0$$

Since, realistically, $L3^* > 0$, it must be the case that

$$s_2 = 0$$

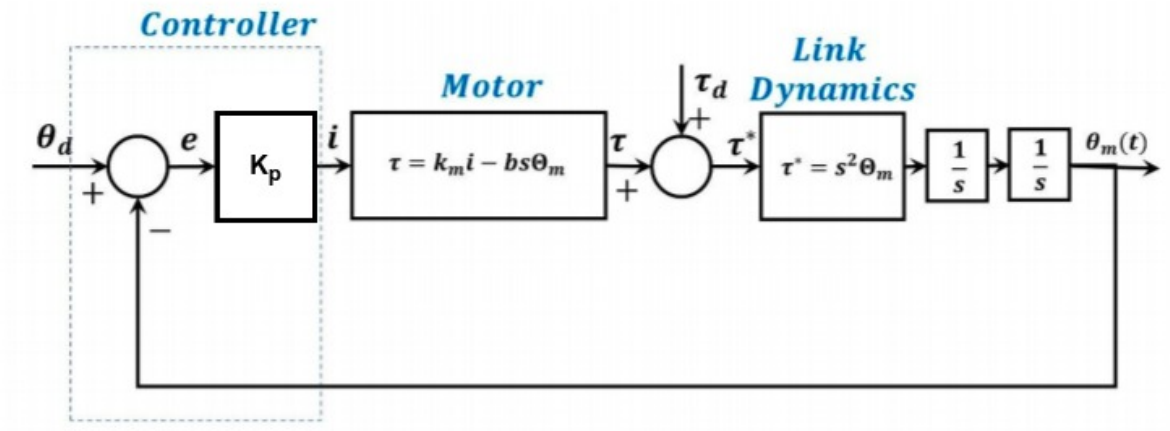
$$\theta_2 = \{0°, 180°\}$$

A simple sketch for the latter configuration (the other configurations and explanations are also accepted):



Task 5 (File Upload, 28%)

A step response test has been performed on a single joint under P-control. A simplified block diagram of the closed-loop system is as follows:



Task 5.1 (10%)

Find the transfer function between the input θ_d and the output θ_m. You may assume that disturbance τ_d is 0 for this question. Note: two solutions (and variations on them) are accepted due to different opinions on how to read the diagram.

Possible Solution 1:

$$s^2\theta_m = K_p k_m(\theta_d - \theta_m) - bs\theta_m + \tau_d$$

$$\theta_m(s^2 + K_p k_m + bs) = K_p k_m \theta_d + \tau_d$$

Note: "$+\tau_d$" may or may not be present.

Answer:

$$\theta_m = \frac{K_p k_m \theta_d + \tau_d}{s^2 + K_p k_m + bs}$$

Possible Solution 2:

$$U_{control}(s) = K_p(\theta_m - \theta_d)$$

$$U_{sys}(s) = k_m i - bs\theta_m + s^2\theta_m + \tau_d$$

Since

$$U_{control}(s) = U_{sys}(s)$$

then

$$K_p(\theta_m - \theta_d) = k_m i - bs\theta_m + s^2\theta_m + \tau_d$$

$$K_p\theta_m + bs\theta_m - s^2\theta_m = k_m i + K_p\theta_d + \tau_d$$

$$\theta_m(K_p + bs - s^2) = k_m i + K_p\theta_d + \tau_d$$

Answer:

$$\theta_m = \frac{k_m i + K_p\theta_d + \tau_d}{K_p + bs - s^2}$$

Note: "$+\tau_d$" may or may not be present.

Task 5.2 (8%)

The system receives a step reference input $\theta_d = \frac{\Omega_d}{s}$ and constant disturbance $\frac{d}{s}$. Use the final value theorem to determine the steady state error. Note: solution depends on the answer provided in 5.1.

Possible Solution:

Using the final value theorem and **the second solution** from *Task 5.1*,

$$\lim_{t\to\infty} \theta_m(t) = \lim_{s\to 0} s\left(\frac{k_m i + K_p\theta_d + \tau_d}{K_p + bs - s^2}\right) =$$

$$\lim_{s\to 0} \frac{sk_m i + sK_p\frac{\Omega_d}{s} + s\frac{d}{s}}{K_p + bs - s^2} =$$

$$\lim_{s\to 0} \frac{sk_m i + K_p\Omega_d + d}{K_p + bs - s^2} =$$

$$\lim_{s\to 0} \frac{\cancel{sk_m i}^{\,0} + K_p\Omega_d + d}{K_p + \cancel{bs}^{\,0} - \cancel{s^2}^{\,0}} =$$

$$\Omega_d + \frac{d}{K_p} \quad \text{(output at } t \to \infty\text{)}$$

Input at $t \to \infty$:

$$\lim_{t\to\infty} \theta_d(t) = \lim_{s\to 0} \cancel{s}\frac{\Omega_d}{\cancel{s}} = \Omega_d$$

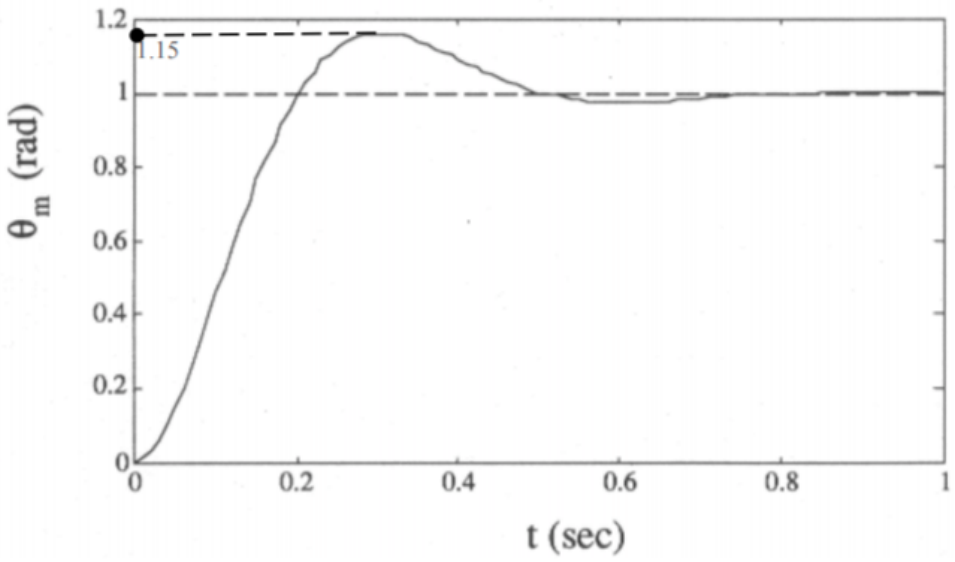
Steady state error:

$$lim_{t\to\infty}(\theta_m(t) - \theta_d(t)) = \Omega_d + \frac{d}{K_p} - \Omega_d = 0 + \frac{d}{K_p}$$

Notes: Using the first solution to 5.1 would yield $\frac{d}{K_p k_m}$ instead. Also, "τ_d" and the terms derived from it may or may not be present due to the assumption made in 5.1.

Task 5.3 (10%)

From the provided response diagram, estimate ζ (damping factor) and ω_n (natural frequency) of the closed-loop system.



Additional information provided for this task during the examination:

Percentage overshoot:

$$Mp\% = \exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right) \times 100\%$$

Settling time (within 5%):

$$t_s = \frac{4}{\zeta\omega_n}$$

Possible Solution:

The peak value of the response is $\theta_m = 1.15rad$, while the steady state value is $1rad$. It can be established that $Mp\% = 15\%$. Using the percentage overshoot formula:

$$\exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right) = 0.15$$

Rearranging:

$$\zeta = \frac{|\ln(0.15)|}{\sqrt{\pi^2 + \ln^2(0.15)}} \approx 0.52$$

Settling time t_s is difficult to determine precisely, and answers in the range of $0.5 - 0.7s$ may be provided. Via the settling time formula,

$$\omega_n = \frac{4}{t_s\zeta} \approx \frac{7.7}{t_s}$$

$$\omega_n \in (11.0, 15.5)$$

Any answer near or within the above range should be acceptable.