HOMOGENEOUS TRANSFORMS — COMBINE ROTATION AND TRANSLATION

PURPOSE: TO SIMULTANEOUSLY REPRESENT THE POSITION AND ORIENTATION OF ONE COORDINATE FRAME RELATIVE TO ANOTHER AND PERFORM COORDINATE TRANSFORMATIONS.

BASIC ROTATION

$$Rot_{x,\alpha} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C_\alpha & -S_\alpha & 0 \\ 0 & S_\alpha & C_\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Rot_{y,\beta} = \begin{bmatrix} C_\beta & 0 & S_\beta & 0 \\ 0 & 1 & 0 & 0 \\ -S_\beta & 0 & C_\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Rot_{z,\gamma} = \begin{bmatrix} C_\gamma & -S_\gamma & 0 & 0 \\ S_\gamma & C_\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- RELATIVE ANGLES

BASIC TRANSLATION

$$Trans_{x,a} = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Trans_{y,b} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

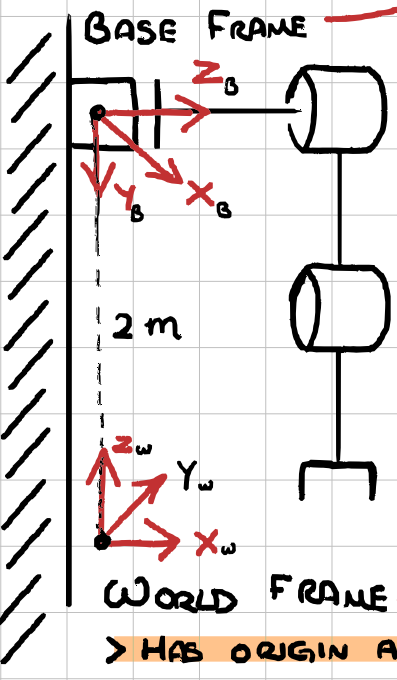
$$Trans_{z,c} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- RELATIVE DISPLACEMENT

☆ THE columns of rotation matrices are mutually orthogonal.
☆ Each column is a unit vector

$$R^{-1} = R^T$$

EXAMPLE (TASK 12, 2023 MODIFIED)



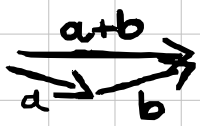
TOTAL VOLUME SWEPT CONSTRAINED BY: · GEOMETRY · MECHANICS

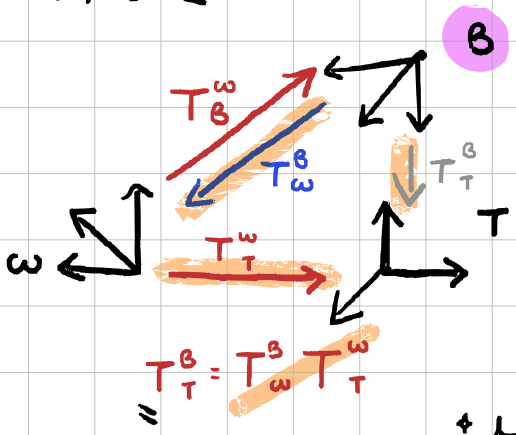
1. DRAW THE WORKSPACE
2. FIND R_B^ω AND d_B^ω TO COMPOSE T_B^ω.
   - ▷ $Z_B \parallel X_\omega$, $Y_B \parallel Z_\omega$, X_B OUT OF PAGE, Y_ω INTO PAGE
   - ▷ IDENTIFY ANGLES
   - ▷ USE THE RIGHT HAND RULE
3. FIND THE INVERSE TRANSFORM
   - ▷ $H^{-1} = \begin{bmatrix} R^T & -R^T d \\ 0 & 1 \end{bmatrix}$

GOING FROM THE BASE FRAME TO THE TASK FRAME

- WHY?

The robot operates in its own frame and needs to understand the task (frame) relative to itself



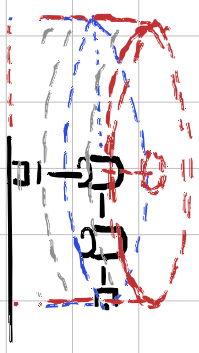


RECOMMENDED READING IN LAST YEAR'S NOTES

- COMPOSITION OF ROTATIONS
- PARAMETERISATION OF ROTATIONS
- HOW TO DO CALCULATIONS WITH HOMOGENEOUS TRANSFORMS

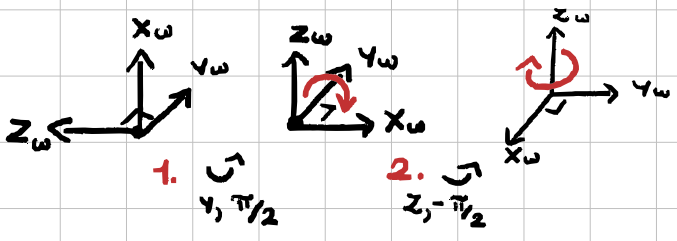
Answers

1.



2.

- How do I rotate the world frame such that it aligns with the base frame?



- Pre-multiplication or post-multiplication?

> fixed-frame
- reversed order

> current frame
- one after another ★

Tasks 2.10-2.13

$$R_B^\omega = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

$$d_B^\omega = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

$$H_B^\omega = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Conclusions:

- When we perform multiple rotations with respect to multiple axes, we must identify the correct matrices and multiply them.
- Matrix multiplication is not commutative, the order of rotations matter.

3.

$$H^{-1} = \begin{bmatrix} R^T & -R^T d \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix} \qquad H_\omega^B = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$