

5 Example - Robot modeling

①

Assuming

~~$L_2 \Rightarrow \dot{L}_2$~~ Static $L_2 \Rightarrow \dot{L}_2$

$s_1 \approx \theta_1$, ~~$L_2 \approx$~~ $s_1 + \theta_1 - \theta_1$ Linearization

$$m L_2^2 \ddot{\theta}_1 + 2 m L_2 \dot{L}_2 \dot{\theta}_1 - m g L_2 (s_1 + \theta_1 - \theta_1) = \tau_1$$

$$\underbrace{m L_2^2 \ddot{\theta}_1 - m g L_2 \theta_1}_{\text{Linear model}} + \underbrace{m g L_2 (\theta_1 - s_1)}_{\text{Disturbance}} = \tau_1$$

$$J \ddot{\theta}_1 - G \theta_1 + D = \tau_1$$

(book uses D)

9 Laplace transform

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Definition

$$x(s) = \int_0^{\infty} e^{-st} x(t) dt \quad (1)$$

Example:

Laplace transform of $\frac{dx}{dt}$

$$\mathcal{L}\left\{\frac{dx}{dt}\right\} = \int_0^{\infty} e^{-st} \frac{dx}{dt} dt$$

$$\int u'v = uv - \int uv'$$

$$u' = \frac{dx}{dt}$$

$$u = x(t)$$

$$v = e^{-st}$$

$$v' = -s e^{-st}$$

$$\mathcal{L}\left\{\frac{dx}{dt}\right\} = e^{-st} x(t) \Big|_0^{\infty} - \int_0^{\infty} -s e^{-st} x(t) dt$$

$$= \underbrace{e^{-s\infty} x(\infty) - \frac{e^{-s \cdot 0}}{1} x(0)}_{= x(s) \text{ see (1)}}$$

$$+ s \int_0^{\infty} e^{-st} x(t) dt$$

$= x(s)$ see (1)

$$= \cancel{x(s)} = s x(s) - x(0)$$

$$10 \quad m \ddot{x}(t) + b \dot{x}(t) + k x(t) = f(t) \quad (3)$$

$\downarrow \mathcal{L}$

$$m \mathcal{L}\{\ddot{x}(t)\} + b \mathcal{L}\{\dot{x}(t)\} + k \mathcal{L}\{x(t)\} = \mathcal{L}\{f(t)\}$$

$$m (s^2 X(s) - s x(0) - \dot{x}(0))$$

$$+ b (s X(s) - x(0)) + k X(s) = F(s)$$

$$(m s^2 + b s + k) X(s) = F(s) + m \dot{x}(0) + (m s + b) x(0)$$

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$$J \ddot{\theta} - G \theta_1 + D = \tau_1$$

$$J s^2 \theta_1 - G \theta_1 + D = \tau_1$$

$$J s^2 \theta_1 - G \theta_1 = \tau_1 - D$$

$$\theta_1 = \frac{\tau_1}{J s^2 - 1} \quad (J s^2 - 1) \theta_1 = \tau_1 - D$$

$$\theta_1 = \frac{\tau_1}{J s^2 - 1} - \frac{D}{J s^2 - 1}$$

$$\theta_1 = H \tau_1 - H D$$

$$H = \frac{1}{J s^2 - 1}$$

18 Example mass-spring-damper system (5)

$$\Sigma F = m a = F - b \dot{x} - k x$$

$$m \ddot{x} + b \dot{x} + k x = F$$

$$m s^2 X + b s X + k X = F$$

$$(m s^2 + b s + k) X = F$$

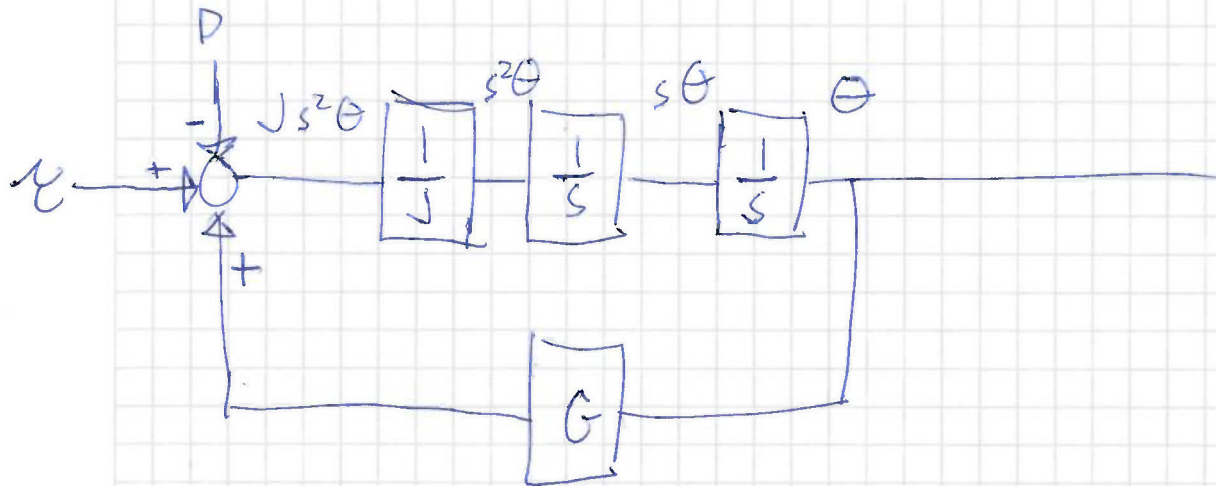
$$\frac{X}{F} = \frac{1}{m s^2 + b s + k}$$

21 Block diagram Robot

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$$J s^2 \Theta - G \Theta + D = \tau$$

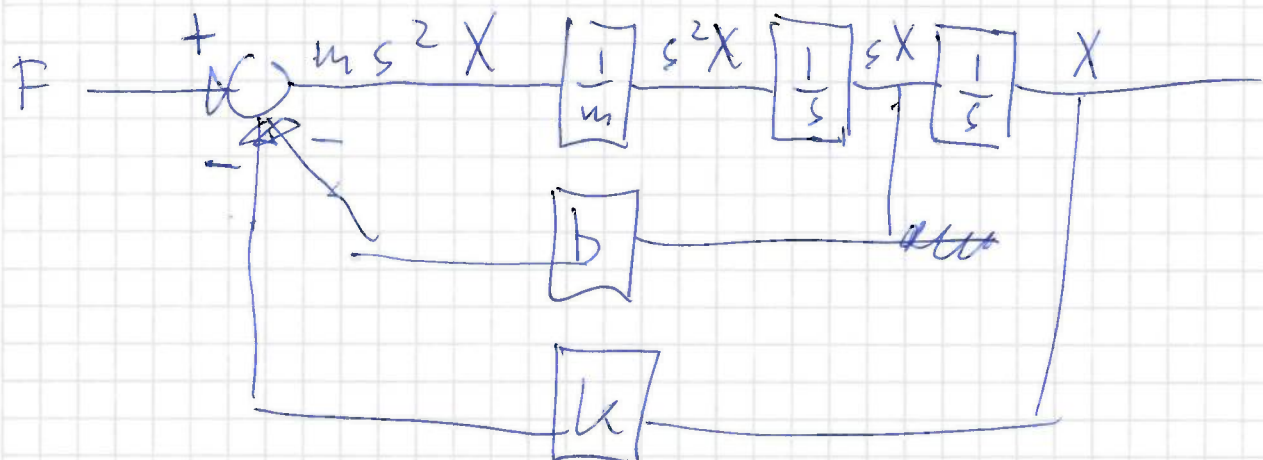
$$J s^2 \Theta = \tau + G \Theta - D$$



22 Block diagram mass-spring-damper

$$m s^2 X + b s X + k X = F$$

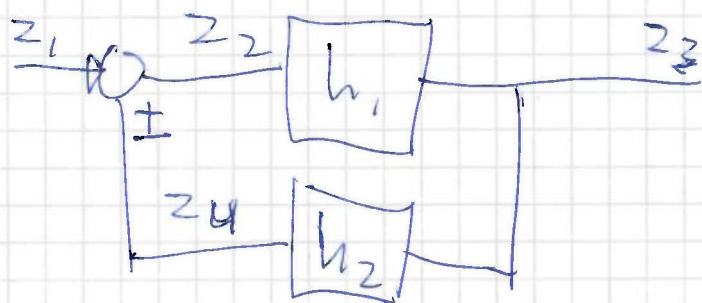
$$m s^2 X = F - b s X - k X$$



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Manipulation of block diagrams

⑦



$$\begin{aligned}
 z_3 &= z_2 h_1 \\
 &= (z_1 \pm z_4) h_1 \\
 &= (z_1 \pm z_3 h_2) h_1
 \end{aligned}$$

$$z_3 \mp z_3 h_2 h_1 = z_1 h_1$$

$$z_3 = \frac{h_1}{1 \mp h_1 h_2} z_1$$

Control theory Intro

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$$m \ddot{x} + f \dot{x} + kx = u$$

$$X (ms^2 + fs + k) = u$$

$$\frac{x}{u} = \frac{1}{ms^2 + fs + k}$$

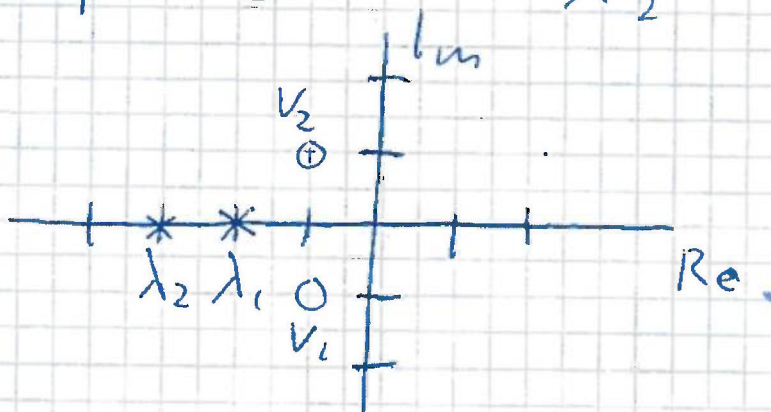
28 Example 1: poles and zeros

$$h(s) = \frac{2s^2 + 4s + 4}{s^2 + 5s + 6}$$

$$= 2 \frac{(s+1+j)(s+1-j)}{(s+2)(s+3)}$$

$$v_1 = -1-j \quad v_2 = -1+j$$

$$\lambda_1 = -2 \quad \lambda_2 = -3$$



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Example 2: Mass-spring-damper system

$$h(s) = \frac{1}{ms^2 + fs + k}$$

$$= \frac{\frac{1}{m}}{s^2 + \frac{f}{m}s + \frac{k}{m}}$$

$$= \frac{\frac{1}{m}}{(s - \lambda_1)(s - \lambda_2)}$$

$$\lambda_{1,2} = -\frac{f}{2m} \left(1 \pm \sqrt{1 - 4\frac{km}{f^2}} \right)$$

Three cases

$\zeta > 1$ I $4\frac{km}{f^2} < 1$

Two distinct real roots

$\zeta = 1$ II $4\frac{km}{f^2} = 1$

Two real roots of same value

$\zeta < 1$ III $4\frac{km}{f^2} > 1$

Two complex conjugate roots

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Root locus plots (Balchen 47.4)

Example:

$$h(s) = \frac{1}{\frac{T_1}{k}(s - \lambda_1)(s - \lambda_2)}$$

$$= \frac{1}{\frac{T_1}{k}(s + \alpha + j\beta)(s + \alpha - j\beta)}$$

$$\lambda_{1,2} = -\frac{1}{2T_1} (1 \pm \sqrt{1 - 4T_1 k}) \quad \forall k \leq \frac{1}{4T_1}$$

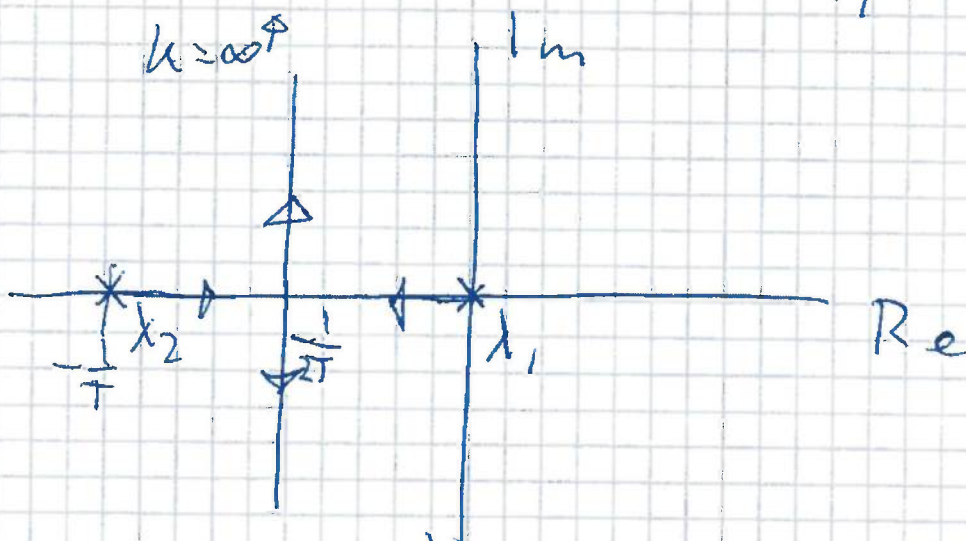
$$\alpha = \frac{1}{2T_1}$$

$$\beta = \frac{1}{2T_1} \sqrt{4T_1 k - 1} \quad \forall k > \frac{1}{4T_1}$$

Starting with $k=0$

$$\lambda_1 = 0$$

$$\lambda_2 = -\frac{1}{T_1}$$

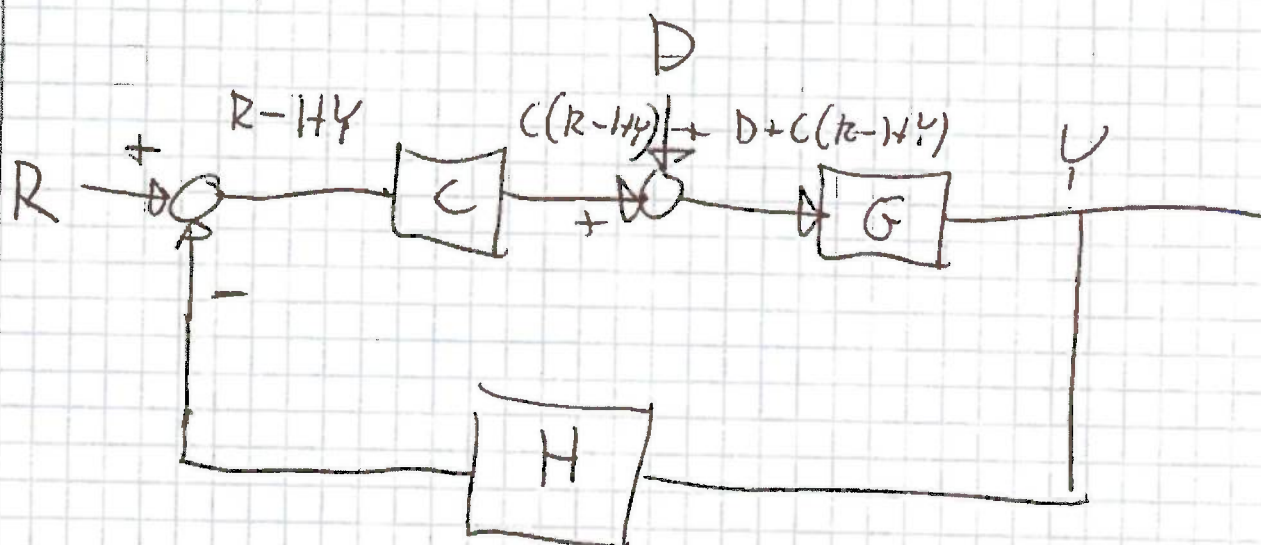


When $k = \frac{1}{4T_1}$ $\lambda_1 = \lambda_2 \Rightarrow \lambda = -\frac{1}{2T_1}$

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Feedback systems

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$$Y = GD + GC R - GC H Y$$

$$(1 + GCH) Y = GD + GCR$$

$$Y = \underbrace{\frac{GC}{1 + GCH}}_U R + \underbrace{\frac{G}{1 + GCH}}_{U_D} D$$