University of Oslo
Faculty of Mathematics and Natural Sciences

Final Exam in: INF4411/3410, fall 2015
Date of exam: 17-Dec-2014
Time: 14:30-18:30
This examination paper consists of ? page(s)
Permitted materials: All printed and written including approved (!) calculator

Make sure that your copy of this examination paper is complete before answering.

Read Carefully:

This is the final exam of INF4410 and INF3410, fall 2015. It will contribute 60% towards the final grade, while the other 40% are contributed by the mandatory labs 2 and 3 of the course. The task under chapter 9 at the end of the exam is *RESERVED for INF4411 ONLY!* Students that are registered as bachelor students in INF3410 shall *NOT* solve this task! No extra points will be given if they should do so!

It is recommended that you read all the tasks before starting with the exam. A good strategy thereafter is to start with the tasks you feel most comfortable with and leave the most difficult ones for last.

It is also recommended to read through all the tasks once more after you have finished, to make sure you have not overlooked anything.

Start a new page for every task you are answering and write your ID-number and task number on the top of all pages you submit. If you use several pages per task also mark the page number next to the task number. Make sure to answer all questions and follow all instructions. Clearly mark your final answers and results (all that are explicitly asked for or instructed to find in a task) with a double underline. Document your deduction of those results and any assumptions you may have had to make.

Note that you may not hand in the original task papers that you are holding now, i.e. do not write any results here but use exclusively the separate sheets of paper that you are going to hand in.

Each question is graded indicating the weight when grading.

Sum of all points:

- INF3410: 22
- INF4411: 27

Chapter 1

Task 1 (3p): Using the device parameters of the 0.18μm process in the table 1.5 in the course book (table 1), and L=0.36μm, select the device width W and

Table 1.5 MOSFET parameters representative of various CMOS technologies and used for rough hand calculations in this text.

	0.8 μm		0.35 μm		0.18 μm		45 nm	
Technology	**NMOS**	**PMOS**	**NMOS**	**PMOS**	**NMOS**	**PMOS**	**NMOS**	**PMOS**
μC_{ox} ($\mu A/V^2$)	92	30	190	55	270	70	280	70
V_{t0} (V)	0.80	–0.90	0.57	–0.71	0.45	–0.45	0.45	–0.45
$\lambda \cdot L$ (μm/V)	0.12	0.08	0.16	0.16	0.08	0.08	0.10	0.15
C_{ox} ($fF/\mu m^2$)	1.8	1.8	4.5	4.5	8.5	8.5	25	25
t_{ox} (nm)	18	18	8	8	5	5	1.2	1.2
n	1.5	1.5	1.8	1.7	1.6	1.7	1.85	1.85
θ (1/V)	0.06	0.135	1.5	1.0	1.7	1.0	2.3	2.0
m	1.0	1.0	1.8	1.8	1.6	2.4	3.0	3.0
$C_{ov}/W = L_{ov}C_{ox}$ (fF/μm)	0.20	0.20	0.20	0.20	0.35	0.35	0.50	0.50
$C_{db}/W \approx C_{sb}/W$ (fF/μm)	0.50	0.80	0.75	1.10	0.50	0.55	0.45	0.50

Table 1.5
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Table 1: Typical process parameters

V_{GS} required to bias the NFET with an intrinsic gain $A_i = 20$ and transconductance $g_m = 50\frac{\mu A}{V}$. What DC current consumption is required?

Fasit:

$$A_i = g_m r_{ds} \tag{1}$$

$$r_{ds} = \frac{1}{\lambda I_D} \tag{2}$$

$$g_m = \mu C_{ox} \frac{W}{L} V_{eff} \tag{3}$$

$$A_i = \mu C_{ox} \frac{W}{L} V_{eff} \frac{2L}{\lambda W \mu C_{ox} V_{eff}^2} \tag{4}$$

$$= \frac{2}{\lambda V_{eff}} = \frac{2L}{L\lambda V_{eff}} \tag{5}$$

$$V_{eff} = \frac{2L}{L\lambda A_i} = \frac{0.36 * 10^{-6}}{0.08 * 10^{-6} * 10} = 0.45\text{V} \tag{6}$$

$$V_{GS} = 0.45 + 0.45 = 0.9V \tag{7}$$

$$g_m = \frac{W}{0.36 * 10^{-6}} 270 * 10^{-6} 0.45 \tag{8}$$

$$W = g_m \frac{0.36 * 10^{-6}}{270 * 10^{-6} 0.45} \tag{9}$$

$$= 50 * 10^{-6} \frac{0.36 * 10^{-6}}{270 * 10^{-6} 0.45^2} \tag{10}$$

$$= 0.148\mu\text{m} \tag{11}$$

$$I_D = \frac{1}{2}\mu C_{ox} \frac{W}{L} V_{eff}^2 = \frac{1}{2} 270 * 10^{-6} \frac{0.148 * 10^{-6}}{0.36 * 10^{-6}} * 0.45^2 \tag{12}$$

$$= 11.24\mu\text{A} \tag{13}$$

$$r_{ds} = \frac{L}{L\lambda I_D} = \frac{0.36 * 10^{-6}}{0.08 * 10^{-6} 11.24 * 10^{-6}} = 400k\Omega \tag{14}$$

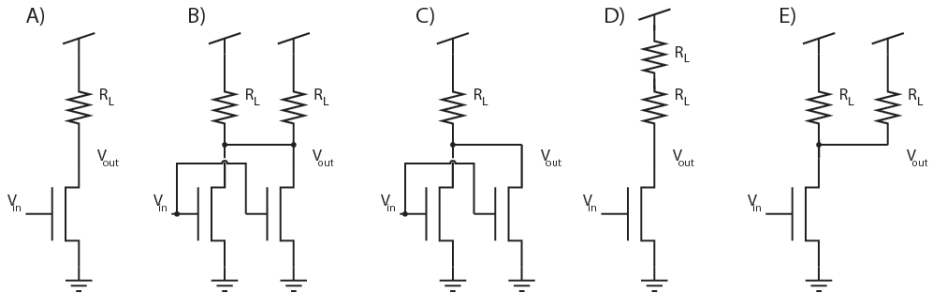


Figure 1: Different common source gain stages.

Chapter 3

Task 2 (4p): Consider the common source gain stages in figure 1. Defining the small signal DC gain of the circuit A) as being -A_A, can you express the DC gains -$A_{B,C,D,E}$ of the other circuits B),C),D), E) relative to this gain -A_A? Assume that all circuits are biassed at the same point of operation in strong inversion and that the resistance R_L is the same as the r_{ds} of the NFET of circuit A). All transistors have the same width to length ratio W/L.

Fasit:

$$A_A = g_m * \frac{1}{2} r_{ds} \tag{15}$$

$$A_B = 2g_m * \frac{1}{4} r_{ds} = A_A \tag{16}$$

$$A_C = 2g_m * \frac{1}{3} r_{ds} = \frac{4}{3} A_A \tag{17}$$

$$A_D = g_m * \frac{2}{3} r_{ds} = \frac{4}{3} A_A \tag{18}$$

$$A_E = g_m * \frac{1}{3} r_{ds} = \frac{2}{3} A_A \tag{19}$$

Chapter 4

Task 3 (4p): For the following three transfer functions, draw a Bode plot with both a magnitude and phase plot. Remember to mark the values of interesting points in the graph clearly and to draw slopes correctly!

$$H_1(s) = 100 \frac{\left(1 + \frac{s}{10^4 \text{rad}}\right)}{\left(1 + \frac{s}{10^4 \text{rad}}\right)\left(1 + \frac{s}{10^5 \text{rad}}\right)} \tag{20}$$

$$H_2(s) = 100 \frac{\left(1 - \frac{s}{10^4 \text{rad}}\right)}{\left(1 + \frac{s}{10^4 \text{rad}}\right)\left(1 + \frac{s}{10^5 \text{rad}}\right)} \tag{21}$$

$$H_3(s) = 100 \frac{\left(1 + \frac{s}{\sqrt{2}10^4 \text{rad}}\right)}{\left(1 + \frac{s}{(1+j)10^4 \text{rad}}\right)\left(1 + \frac{s}{(1-j)10^4 \text{rad}}\right)} \tag{22}$$

Fasit: see figure 3

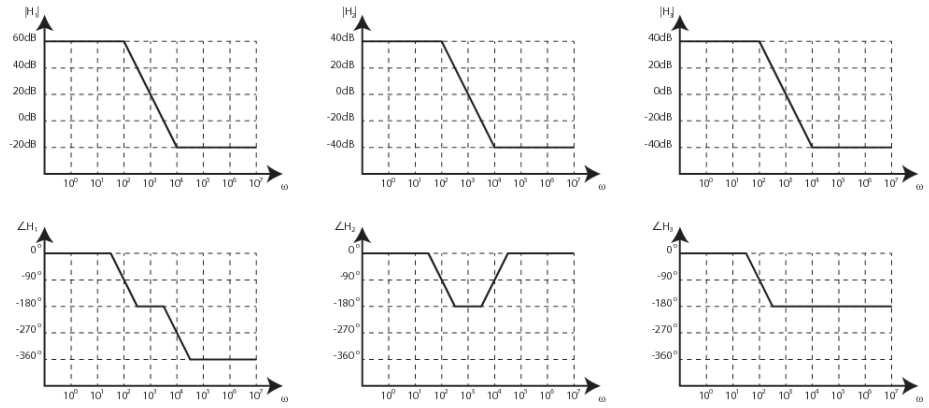


Figure 2: Bodeplots for task 2 of three different transfer functions $H_{1,2,3}$

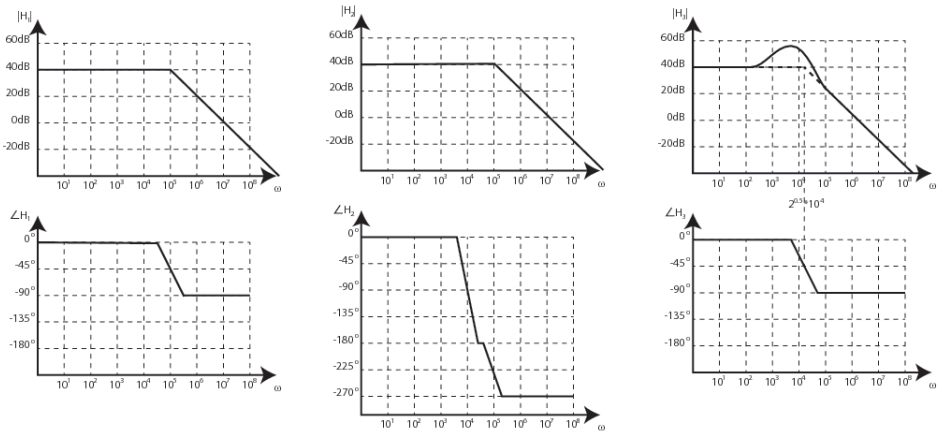


Figure 3: Fasit Task 3

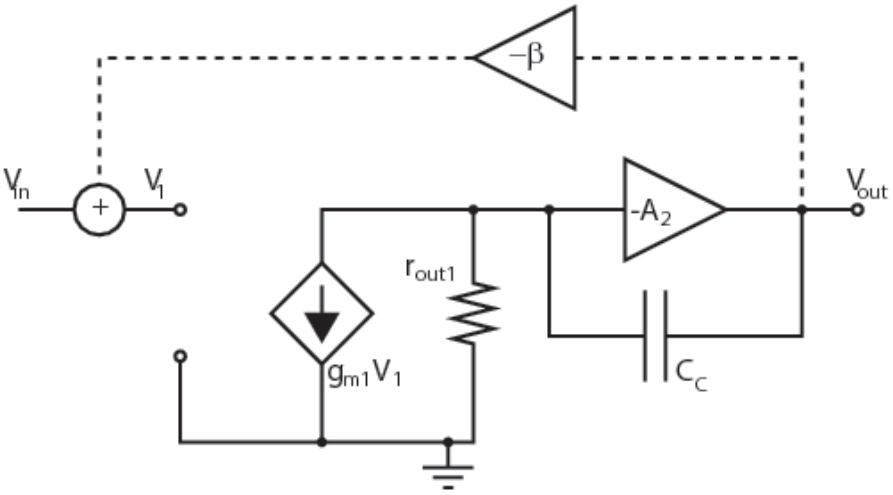


Figure 4: Two stage amplifier with capacitive feedback. The first stage is a transconduction amplifier while the second stage is an idealized voltage amplifier with infinite input impedance and zero output impedance and frequency independent gain.

Task 4 (3p): For the three Bode plots in figure 2, write down the corresponding transfer function in root-form. If there are several solutions, write down all of them.

Fasit:

$$H_1 = 1000\frac{(1-\frac{s}{10^4})(1-\frac{s}{10^4})}{(1+\frac{s}{10^2})(1+\frac{s}{10^2})} \tag{23}$$

$$H_2 = 100\frac{(1+\frac{s}{10^4})(1+\frac{s}{10^4})}{(1+\frac{s}{10^2})(1+\frac{s}{10^2})} \tag{24}$$

$$H_3 = 100\frac{(1+\frac{s}{10^4})(1-\frac{s}{10^4})}{(1+\frac{s}{10^2})(1+\frac{s}{10^2})} \tag{25}$$

$$\tag{26}$$

Chapter 5

Task 5 (3p) Consider the compensated two stage amplifier in figure 4 without (!) the feedback loop (dashed lines), i.e. in open loop and $V_1 = V_{in}$. Note that the second amplifier stage is an ideal amplifier that has infinite input impedance and zero output impedance and a gain -A_2 that is independent of the frequency. Compute the open loop transfer function of the circuit and sketch a Bode plot. What is the order of the transfer function? $A_2 = 25$, $g_{m1} = 5\mu\frac{\mathrm{A}}{\mathrm{V}}$, $r_{out1} = 300\mathrm{k}\Omega$. (Hint: use the Miller theorem)

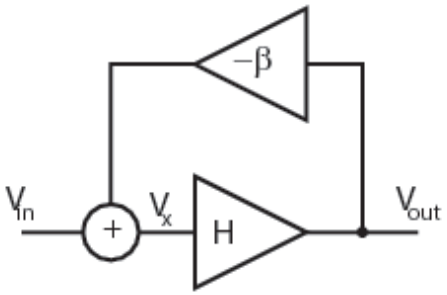


Figure 5: General negative feedback loop

Fasit:

$$g_{m1}v_{in} + v_2(\frac{1}{r_{ds}} + sC_C) = v_{out}sC_C \tag{27}$$

$$v_{out} = -A_2v_2 \tag{28}$$

$$\tag{29}$$

$$g_{m1}v_{in} - v_{out}\frac{1}{A_2}(\frac{1}{r_{ds}} + sC_C) = v_{out}sC_C \tag{30}$$

$$\frac{v_{out}}{v_{in}} = \frac{g_{m1}}{\frac{1}{A_2r_{out1}} + sC_C(1 + \frac{1}{A_2})} \tag{31}$$

$$= \frac{A_2g_{m1}}{\frac{1}{r_{out1}} + sC_C(A_2 + 1)} \tag{32}$$

$$= \frac{A_2g_{m1}r_{out1}}{1 + sr_{out1}C_C(A_2 + 1)} \tag{33}$$

$$\tag{34}$$

That's a first order transfer function.
With numbers (note that C_c was not given so one can only do this symbolically for the pole!):

$$A_{DC} = A_2g_{m1}r_{out1} = 25 * 300000 * 5e - 6 = 37.5 \tag{35}$$

$$\omega_{p1} = \frac{1}{r_{out1}C_C(A_2 + 1)} = \frac{1}{7.8 * 10^6C_C} \tag{36}$$

Some partial results by students to verify. Impedance in node at input of A2:

$$z_2 = \frac{v_2}{i_2} = \frac{r_{out}}{1 + sr_{out}C_C} \tag{37}$$

Task 6 (3p) Given the open loop 2nd order low pass transfer function of equation (38), if you connect it in a feedback loop where the input $v_x = v_{in} - \beta v_{out}$ as depicted in figure 5, what is the biggest β you can choose for the phase margin to be at least 60^o?

$$H(s) = 10\frac{1}{(1 + \frac{s}{100})(1 + \frac{s}{500})} \tag{38}$$

Fasit:

$$\phi = -\arctan\frac{\omega}{\omega_1} - \arctan\frac{\omega}{\omega_2} \quad (39)$$

$$\phi \approx -90^o - \arctan\frac{\omega}{\omega_2} \text{ for } \omega > \omega_1 \quad (40)$$

$$\text{PM} = 60 \Rightarrow \phi = -120^o \quad (41)$$

$$30^o = \arctan\frac{\omega_t}{\omega_2} \quad (42)$$

$$\tan 30^o = \frac{\omega_t}{\omega_2} \quad (43)$$

$$\frac{1}{\sqrt{3}} = \frac{\omega_t}{500} \quad (44)$$

$$\omega_t \approx 189\text{rad/s} \quad (45)$$

A shortcut in the book in table 5.1:$PM = 60^o \Rightarrow \omega_t/\omega_{eq} = 0.580$

$$|H(j\omega)| = 1000\frac{1}{|1+\frac{j289}{100}|\,|1+\frac{j289}{500}|} \quad (46)$$

$$= 10\frac{1}{\sqrt{1+\frac{289^2}{100^2}}\sqrt{1+\frac{289^2}{500^2}}} \quad (47)$$

$$= 10\frac{1}{3.0581 * 1.1550} \quad (48)$$

$$= 2.8312 \text{ gain for } \omega = 289\text{rad/s} \quad (49)$$

$$\beta_{max} = 1/2.8312 = 0.3532 \quad (50)$$

Check: (51)

$$\phi = -\arctan\frac{289}{100} - \arctan\frac{289}{500} = 1.7618\text{rad} = 101^o \quad (52)$$

slightly too short $\Rightarrow$ trial and error: (53)

$$\phi = -\arctan\frac{456}{100} - \arctan\frac{456}{500} = 2.094\text{rad} = 120.00^o \quad (54)$$

$$|H(j\omega)| = 10\frac{1}{\sqrt{1+\frac{354^2}{100^2}}\sqrt{1+\frac{354^2}{500^2}}} \quad (55)$$

$$= 10\frac{1}{4.668 * 1.353} \quad (56)$$

$$\beta_{max} = 1/1.583 = 0.632 \quad (57)$$

Some students used the approximate relationship:

$$\omega_{ta} \approx A_0\omega_{p1} = 1000\text{rad/s} \quad (58)$$

which is a shaky assumption here as ω_{ta} is at lower frequencies if ω_{eq} is close enough to ω_{p1} to matter. I wonder at this assumption in the book, actually! But even so, if they use this and then use$\omega_t = 289$rad/s in

$$\beta\frac{\omega_{ta}}{\omega_{eq}} = \frac{\omega_t}{\omega_{eq}}\sqrt{1+\frac{\omega_t}{\omega_{eq}}} \quad (59)$$

they should get

$$\beta = \frac{\omega_t}{\omega_{ta}}\sqrt{1+\frac{\omega_t}{\omega_{eq}}} \quad (60)$$

$$= \frac{289}{1000}\sqrt{1+\frac{289}{500}} = 0.363 \quad (61)$$

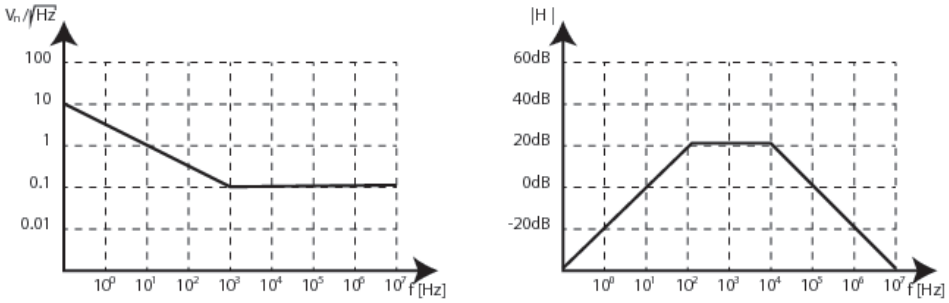


Figure 6: Input noise root spectral density and transfer function of a bandpass amplifier

Chapter 6

Task 7 (2p): An OTA has a G_{MA} (transconductane of the entire circuit when measuring change of output current at a fixed output voltage vs. change of input voltage) of 2mA/V and a DC gain of 60dB. What is its output resistance r_{out}?

Fasit:

$$A = G_{\mathrm{MA}} r_{out} \tag{62}$$

$$r_{out} = 1000/2e - 3 \tag{63}$$

$$= 500\mathrm{k}\Omega \tag{64}$$

Chapter 9

the task of chapter 9 are mandatory for students having registered as master students to course INF4411, and are NOT(!) to be solved for bachelor students registered to course INF3410. No extra point are to be gained for bachelor students registered to course INF3410 by solving the task under chapter 9!

Task 8 (5p): Consider a band pass amplifier that has equivalent input noise root spectral density and transfer function magnitude as shown in figure 7. Sketch the root spectral density for the output signal. Compute the total output root mean square ($V_{n(rms)}$) noise in the depicted noise bandwith from 0.1Hz to 10MHz by piecewise integration. (Hint: Remember that $\int \frac{1}{x} dx = |\ln x|$ and $\int x^a dx = \frac{1}{1+a} x^{a+1}$ for $a \neq -1$)

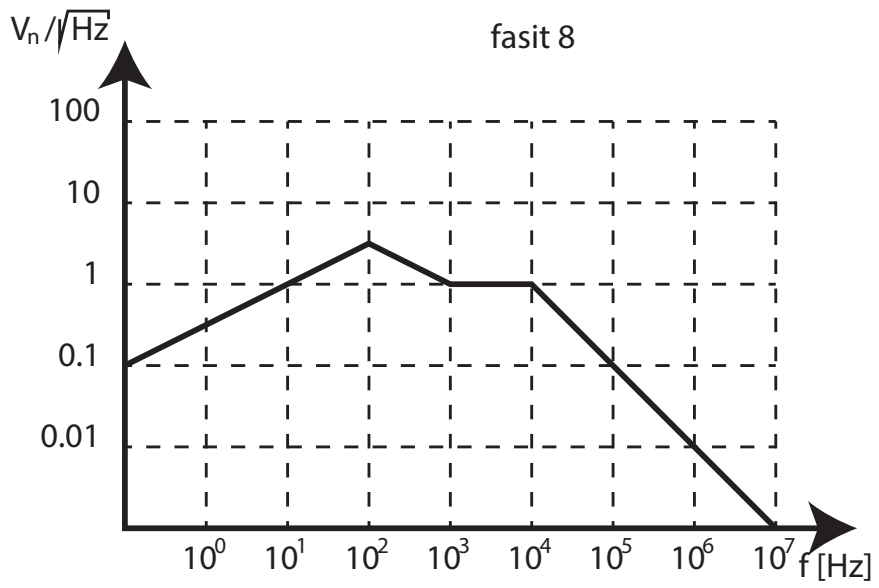


Figure 7: Solution graph for task 8: output noise root spectral density

Fasit:

$$v_{no}(f) = f * 0.1 \ * \ \sqrt{10}/\sqrt{f} \text{ from 0.1Hz to 100Hz} \quad (65)$$

$$\int_{0.1}^{100} v_{no}^2 df = \int_{0.1}^{100} 0.1 f df = 10 - 0.01 = 9.9V^2 \quad (66)$$

$$v_{no}(f) = 10 \ * \ \sqrt{10}/\sqrt{f} \text{ from 100Hz to 1kHz} \quad (67)$$

$$\int_{100}^{1000} v_{no}^2 df = \int_{100}^{1000} \frac{1000}{f} df = 1000(\ln 1000 - ln(100)) = 2303V^2 \quad (68)$$

$$v_{no}(f) = 10 * 0.1 \text{ from 1kHz to 10kHz} \quad (69)$$

$$\int_{1000}^{10000} v_{no}^2 df = 9000V^2 \quad (70)$$

$$v_{no}/\sqrt{Hz} = \frac{100000}{f} * 0.1 \text{ from 10kHz to 10MHz} \quad (71)$$

$$\int_{10000}^{10000000} v_{no}^2 df = \int_{10000}^{10000000} 10^8 f^{-2} df = 10^8(-1 * 10^-7 - 1 * 10^-4) = 9990V^2 \quad (72)$$

$$V_{no(rms)} = \sqrt{9.9 + 2303 + 9000 + 9990} = 145V \quad (73)$$