University of Oslo
Faculty of Mathematics and Natural Sciences

Final Exam in: INF4411/3410, fall 2016
Date of exam: 14-Dec-2015
Time: 09:00-13:00
This examination paper consists of * page(s)
Permitted materials: All printed and written including approved (!) calculator

Make sure that your copy of this examination paper is complete before answering.

Read Carefully:

This is the final exam of INF4410 and INF3410, fall 2016. It will contribute 60% towards the final grade, while the other 40% are contributed by the mandatory labs 2 and 3 of the course. The task * under chapter * is *RESERVED for INF4411 ONLY!* Students that are registered as bachelor students in INF3410 shall *NOT* solve this task! No extra points will be given if they should solve the task!

It is recommended that you read all the tasks before starting with the exam. A good strategy thereafter is to start with the tasks you feel most comfortable with and leave the most difficult ones for last.

It is also recommended to read through all the tasks once more after you have finished, to make sure you have not overlooked anything.

Start a new page for every task you are answering and write your ID-number and task number on the top of all pages you submit. If you use several pages per task also mark the page number next to the task number. Make sure to answer all questions and follow all instructions. Clearly mark your final answers and results (all that are explicitly asked for or instructed to find in a task) with a double underline. Document your deduction of those results and any assumptions you may have had to make.

Note that you may not hand in the original task papers that you are holding now, i.e. do not write any results here but use exclusively the separate sheets of paper that you are going to hand in.

Each question is graded indicating the weight when grading.

Chapter 1

Task 1 (3p): Through repeated application of the Thvenin's Theorem, find the Thvenin equivalent of the circuit in Fig 1 between node 4 and Gnd and hence find the current that would flow through a 4kΩ load resistor connected between node 4 and Gnd.

Table 1.5 MOSFET parameters representative of various CMOS technologies and used for rough hand calculations in this text.

	0.8 μm		0.35 μm		0.18 μm		45 nm	
Technology	**NMOS**	**PMOS**	**NMOS**	**PMOS**	**NMOS**	**PMOS**	**NMOS**	**PMOS**
μC_{ox} ($\mu A/V^2$)	92	30	190	55	270	70	280	70
V_{t0} (V)	0.80	–0.90	0.57	–0.71	0.45	–0.45	0.45	–0.45
$\lambda \cdot L$ (μm/V)	0.12	0.08	0.16	0.16	0.08	0.08	0.10	0.15
C_{ox} ($fF/\mu m^2$)	1.8	1.8	4.5	4.5	8.5	8.5	25	25
t_{ox} (nm)	18	18	8	8	5	5	1.2	1.2
n	1.5	1.5	1.8	1.7	1.6	1.7	1.85	1.85
θ (1/V)	0.06	0.135	1.5	1.0	1.7	1.0	2.3	2.0
m	1.0	1.0	1.8	1.8	1.6	2.4	3.0	3.0
$C_{ov}/W = L_{ov}C_{ox}$ (fF/μm)	0.20	0.20	0.20	0.20	0.35	0.35	0.50	0.50
$C_{db}/W \approx C_{sb}/W$ (fF/μm)	0.50	0.80	0.75	1.10	0.50	0.55	0.45	0.50

Table 1.5
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Table 1: Typical process parameters

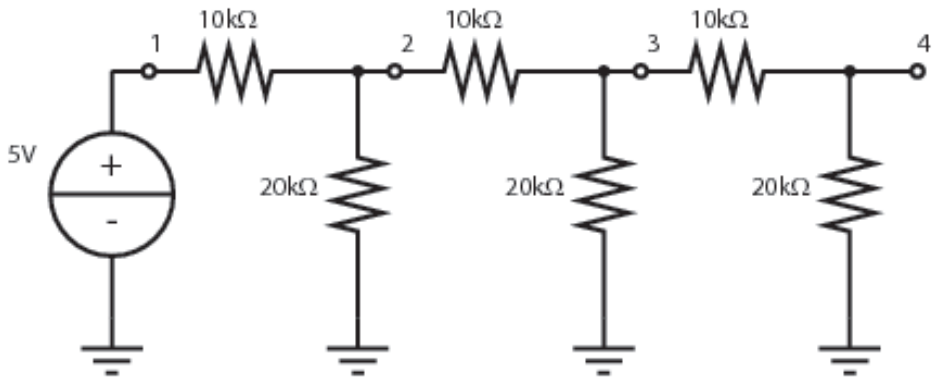


Figure 1:

Note that V_i is the Thvenin equivalent source voltage for node i and NOT the actual voltage at node i.

$$\begin{aligned} V_2 &= 5\text{V} * \frac{20\text{k}\Omega}{10\text{k}\Omega + 20\text{k}\Omega} = 5 * 2/3 = 3.33 && (1)\\ R_2 &= 10\text{k}\Omega||20\text{k}\Omega = 10 * 20/(10+20) = 200/30 = 20/3 = 6.67\text{k}\Omega && (2)\\ & && (3) \end{aligned}$$

$$\begin{aligned} V_3 &= 3.33\text{V} * \frac{20\text{k}\Omega}{16.67\text{k}\Omega + 20\text{k}\Omega} = 3.33 * 20/36.67 = 1.82\text{V} && (4)\\ R_3 &= 16.67\text{k}\Omega||20\text{k}\Omega = 16.67 * 20/(16.67+20) = 9.09\text{k}\Omega && (5)\\ & && (6) \end{aligned}$$

$$\begin{aligned} V_4 &= 1.82\text{V} * \frac{20\text{k}\Omega}{19.09\text{k}\Omega + 20\text{k}\Omega} = 1.82 * 20/(39.09) = 0.93\text{V} && (7)\\ R_4 &= 19.09\text{k}\Omega||20\text{k}\Omega = 19.09 * 20/(19.09+20) = 9.76\text{k}\Omega && (8)\\ I_L &= V_4/(R_4 + R_L) = 0.93/(4e3 + 9.76e3) = 67.6\mu\text{A} && (9) \end{aligned}$$

Some student has written all in one as

Chapter 2

Task 2 (4p): An OpAmp has a CMRR of 40dB and a differential gain of 25dB. If the negative input terminal is kept at Gnd and an sinusoid input signal of 100mV peak to peak amplitude is applied to the positive input terminal, what is the peak to peak amplitude of the output signal? Write the result with at least 3 digits accuracy! (Ask yourself: what is the peak-to-peak amplitude of v_{id} and what is the peak-to-peak amplitude of v_{cm} and how do they combine to generate the output?)

$$\begin{aligned} A_d &= 10^{25/20} = 17.78 && (10)\\ A_{cm} &= 10^{(25-40)/20}0.1778 && (11)\\ v_{o,max} &= A_d * v_{id,max} + A_{cm} * v_{cm,max} && (12)\\ &= A_d * (v_{+,max}) + A_{cm} * (v_{+,max})/2 && (13)\\ &= A_d * 50mV + A_{cm} * 25mV && (14)\\ &= 893.584mV && (15)\\ v_{o,min} &= A_d * v_{id,min} + A_{cm} * v_{cm,min} && (16)\\ &= A_d * (v_{+,min}) + A_{cm} * (v_{+,min})/2 && (17)\\ &= A_d * -50mV + A_{cm} * -25mV && (18)\\ &= -893.584mV && (19)\\ v_{o,pp} &= 1.787V && (20)\\ & && (21) \end{aligned}$$

Note v_{id} has only half the peak to peak swing of v_+

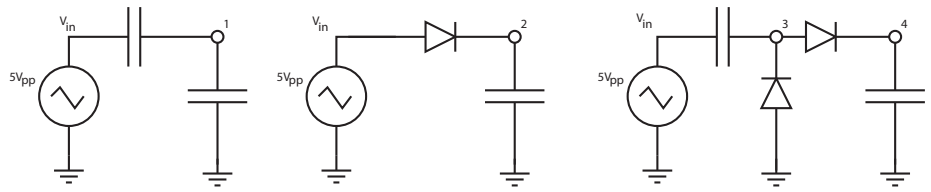


Figure 2:

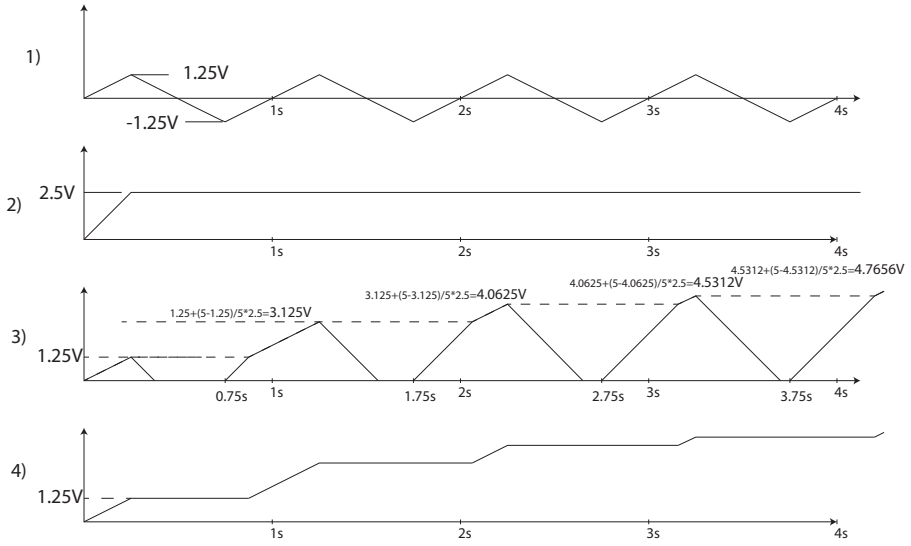


Figure 3: Solution for task 3

Chapter 3 (only for INF4411!)

The task 3 of chapter 3 is mandatory for students having registered as master students to course INF4411, and are NOT(!) to be solved for bachelor students registered to course INF3410. No extra point are to be gained for bachelor students registered to course INF3410 by solving this task under chapter 3!

Task 3 (only INF4411, *p): Assume an ideal diode model, i.e. where the diode does not conduct for voltages equl or lower than zero accross it and does conduct with no resistance for positive voltages in figure 2. The source is an ideal triangular voltage source with 5Vpp, i.e. the maxima are ± 2.5V and the voltage rises and falls linearly between them. Assume that at time $t = 0$ all voltages are at 0V, i.e. Gnd and that the voltage source is $5 * \sin(t * 2\pi)$, i.e. one cycle is 1s. All capacitors do have the same capacitance C. Draw the waveforms resulting at all 7 nodes marked in the figure with the numbers 1 to 7. for 4 full cycles. Clearly mark all maxima occuring in the waveforms and write their values.

Chapter 5

Task 4 (*p): Using the parameters of the 180nm process in table 1, write down the drain-source resistance r_{DS} in the triode region for very small V_{DS} and the drain-source resistance r_o in the active region for an nFET with a gate voltage $V_{GS} = 0.6V$ and $W = 720$nm and $L = 360$nm.

$$r_{ds} = \frac{1}{k_n v_{OV}} = 1/(270e-6 * 720/360 * (0.6 - 0.45)) = 12.35\text{k}\Omega \quad (22)$$

$$r_o = \frac{1}{\lambda I'_D} = \frac{L}{L\lambda 0.5 k_n V_{ov}^2} = 360e-9/(0.08e-6 * 0.5 * 720/360 * 270e-6 * 0.15^2) = 740.74\Omega \quad (23)$$

Chapter 6

Task 5 (*p): While we did not explicitly give an equation for g_m in *weak inversion* in the lecture, can you derive an expression for g_m in dependence of I_d (i.e. $g_m(I_d)$) in weak inversion if you use the weak inversion expression ofI_d in the Sedra/Smith book $I_d = I_S e^{\frac{V_{gs}}{nV_T}}$? Do not just write down the result but do explain briefly how to derive that result!

$$g_m = \frac{\partial I_d}{\partial V_{gs}} \quad (24)$$

$$\frac{\partial f(g(x)}{\partial x} = g' f'(g) \quad (25)$$

$$\Rightarrow \frac{\partial c_1 * e^{c_2 x}}{\partial x} = c_1 c_2 e^{c_2 x} \quad (26)$$

$$\Rightarrow \frac{\partial I_d}{\partial V_{gs}} = \frac{1}{nV_T} I_S e^{\frac{V_{gs}}{nV_T}} \quad (27)$$

$$= \frac{I_d}{nV_T} \quad (28)$$

Chapter 7

Task 6 (*p): Given the current steering circuit in figure 4 (7.4 in the book) and using the parameters of the 0.35μm process in table 1 (at the start of the exam sheets), but neglecting the slope factor n (i.e. $n = 1$) with a supply of Vdd=3.3V and -Vss=Gnd=0, you shall design the circuit such that I_{ref} is 10μA. The width and length of transistor Q_1 shall be $W = 1\mu$m and $L = 1\mu$m. What resistance R do you need?

Task * (*p): In continuation of the previous task all transistor lengths shall be the same $L = 1\mu$m. Give the widths $W_{2..5} =?$ of all transistors $Q_{2..5}$ if you want to achieve $I_2 = 0.5 * I_{REF}$, $I_3 = 3 * I_{REF}$, and $I_5 = 9 * I_{REF}$. Try to find two (simple) solutions! (There are no explicit requirements with respect to the range of the drain voltages.)

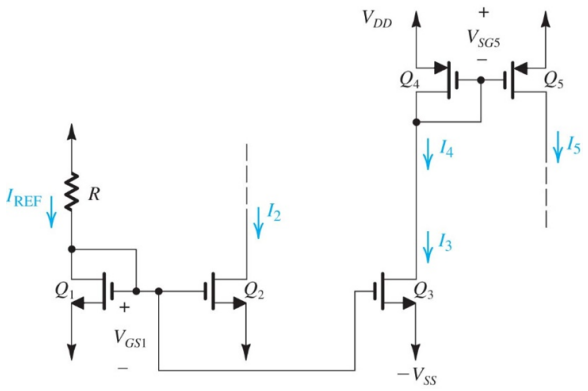


Figure 4:

$$V_{ov} = \sqrt{\frac{I_d}{\frac{1}{2}k_n}} = sqrt(10e-6/(0.5*190e-6*1/1)) = 0.32\text{V} \quad (29)$$

$$V_{gs} = 0.57 + 0.32 = 0.89V \quad (30)$$

$$V_R = 3.30 - 0.89 = 2.41V \quad (31)$$

$$R = 2.41/10e-6 = 241\text{k}\Omega \quad (32)$$

$$W_{Q2} = 0.5\mu\text{m} \quad (33)$$

$$W_{Q3} = 3\mu\text{m} \quad (34)$$

$$\frac{W_{Q5}}{W_{Q4}} = 3 \quad (35)$$

so for example (36)

$$W_{Q4} = 3\mu\text{m} \quad (37)$$

$$W_{Q5} = 9\mu\text{m} \quad (38)$$

or (39)

$$W_{Q4} = 1\mu\text{m} \quad (40)$$

$$W_{Q5} = 3\mu\text{m} \quad (41)$$

(42)

Chapter 8

Task 7 (*p): Using the parameters of the 0.35μm process in table 1 (at the start of the exam sheets) , but neglecting the slope factor n (i.e. $n = 1$), calculate the width W =? of an nFET with length $L = 1\mu$m that shall have an intrinsic gain $A_0 = 80$ at a drain current of 2μA. You may assume that the transistor is in strong inversion. If you use this same transistor with the same drain current as a common source amplifier with a current-source load consisting of a pFET with the same dimensions (i.e. W and L are the same as for the nFET), what will be the gain of that common source amplifier? What will be the gate to source voltage V_{DS} of the pFET?

The nFET width:

$$
\begin{aligned}
A_0 &= g_m r_o && (43)\\
r_o &= \frac{1}{\lambda I_D} = \frac{L}{L\lambda I_D} && (44)\\
&= 1e-6/(0.16e-6*2e-6) = 3.125\mathrm{M}\Omega && (45)\\
\Rightarrow g_m &= \frac{A_0}{r_o} = 80/3.125e6 = 25.6\frac{\mu\mathrm{A}}{\mathrm{V}} && (46)\\
V_{ov} &= \sqrt{\frac{2I_D}{k_n}} && (47)\\
g_m &= k_n V_{ov} = k_n\sqrt{\frac{2I_D}{k_n}} = \sqrt{2k_n I_d} && (48)\\
\Rightarrow k_n &= \frac{g_m^2}{2I_D} = 25.6e-6^2/2/2e-6 = 163.8\frac{\mu\mathrm{A}}{\mathrm{V}} && (49)\\
k_n &= \mu_m C_{ox} W/L = 190e-6*W/1e-6 && (50)\\
\Rightarrow W &= 1e-6/190e-6*163.8e-6 = 0.862\mu\mathrm{m} && (51)\\
\text{check:} & && (52)\\
V_{ov} &= \sqrt{\frac{2I_D}{k_n}} = sqrt(2*2e-6/163.8e-6) = 156.3\mathrm{mV} && (53)\\
I_d &= 0.5k_n V_{ov}^2 = 0.5*163.8e-6*156.3e-3^2 = 2.00\mu\mathrm{A} && (54)\\
g_m &= k_n V_{ov} = 163.8e-6*156.3e-3 = 25.6\frac{\mu\mathrm{A}}{\mathrm{V}} && (55)\\
A_0 &= 25.6e-6*3.125e6 = 80 && (56)\\
& && (57)
\end{aligned}
$$

Alternative approach:

$$
\begin{aligned}
A_0 &= \frac{2}{\lambda V_{ov}} && (58)\\
\Rightarrow V_{ov} &= \frac{2L}{L\lambda A_0} = 2*1e-6/0.16e-6/80 = 0.1562 && (59)\\
I_d &= 2\mu\mathrm{tA} = 0.5k_n V_{ov}^2 = 0.5*190e-6*W/1e-6*156.3e-3^2 && (60)\\
\Rightarrow W &= 2e-6*2/(190e-6/1e-6*156.3e-3^2) = 0.862\mu\mathrm{m} && (61)\\
& && (62)
\end{aligned}
$$

The pFET drain resistance is the same since L and $L\lambda$ is the same so the gain total gain will be:

$$g_m(r_o||r_o) = g_m\frac{r_o}{2} = A_0/2 = 40 \quad (63)$$

And V_{ov}:

$$V_{ov} = \sqrt{\frac{2I_D}{k_n}} = sqrt(2*2e-6/(55e-6*(0.862e-6/1e-6))) = 290.5\mathrm{mV} \quad (64)$$

and

$$V_{GS} = V_{tp} + V_{ov} = 0.71 + 0.2905 = 1.0005\mathrm{V} \quad (65)$$

Chapter 9

Task 8 (*p): Do a open circuit time constant analysis of the circuit in figure 5 and use it to approximate its cut-off frequency ω_H. Use $g_{m1} = 10\mu\frac{\mathrm{A}}{\mathrm{V}}$

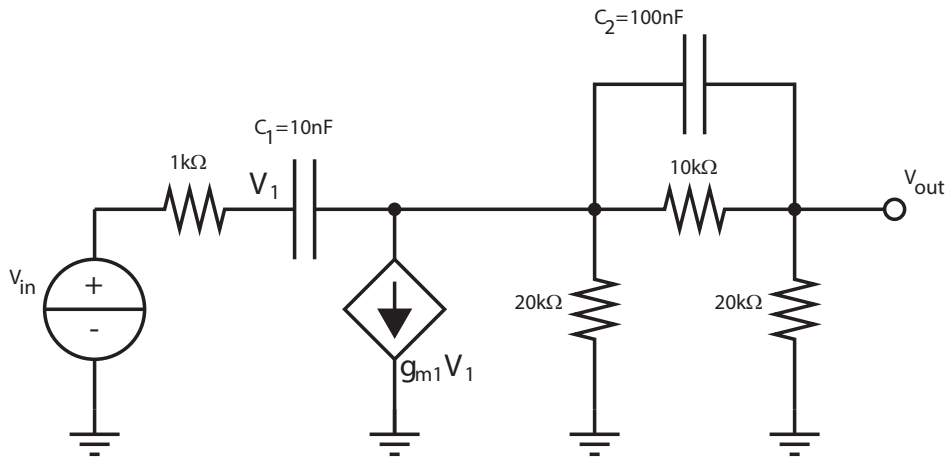


Figure 5:

C1 is tricky but analogous to the CS amplifier on page 735 in the book:

$$I_X = g_{m1}V_1 + V_2/[30e3 * 20e3/(30e3 + 20e3)] \quad (66)$$
$$= g_{m1} * V_1 + (V_1 + V_x)/R_L \quad (67)$$
$$= -g_{m1} * I_X * R_{sig} - I_X * R_{sig}/R_L + V_x/R_L \quad (68)$$
$$= I_X * (-g_{m1} * R_{sig} - R_{sig}/R_L) + V_x/R_L \quad (69)$$
$$\Rightarrow V_x/R_L = I_X(1 + g_{m1} * R_{sig} + R_{sig}/R_L) \quad (70)$$
$$R_X = V_X/I_X = R_L(1 + g_{m1} * R_{sig} + R_{sig}/R_L) \quad (71)$$
$$= R_L + R_L g_{m1} * R_{sig} + R_{sig} \quad (72)$$
$$R_X = 12e3 + 10e - 6 * 12e3 * 1e3 + 1e3 = 13.120\text{k}\Omega \quad (73)$$
$$(74)$$

So:

$$\tau_1 = 10e - 9 * 13.120e3 = 131.2\mu\text{s} \quad (75)$$

For C2:

$$R_x = 10e3 * 40e3/(10e3 + 40e3) = 8\text{k}\Omega \quad (76)$$

So:

$$\tau_2 = 100e - 9 * 8e3 = 800\mu\text{s} \quad (77)$$
$$\omega_H = \frac{1}{\tau_1 + \tau_2} = 1/(800e - 6 + 131.2e - 6) = 1.074\text{k}\frac{\text{rad}}{\text{s}} \quad (78)$$

Transfer Functions and Bode Plots

Task 9 (*p): Consider the Bode plot in figure 7. Note that it is drawn 'piece wise linear', i.e. ignoring the rounding off of knee-points that would not really have such non-differtiable, apprupt transitions. Write down the corresponding transfer function $A_1(s)$.

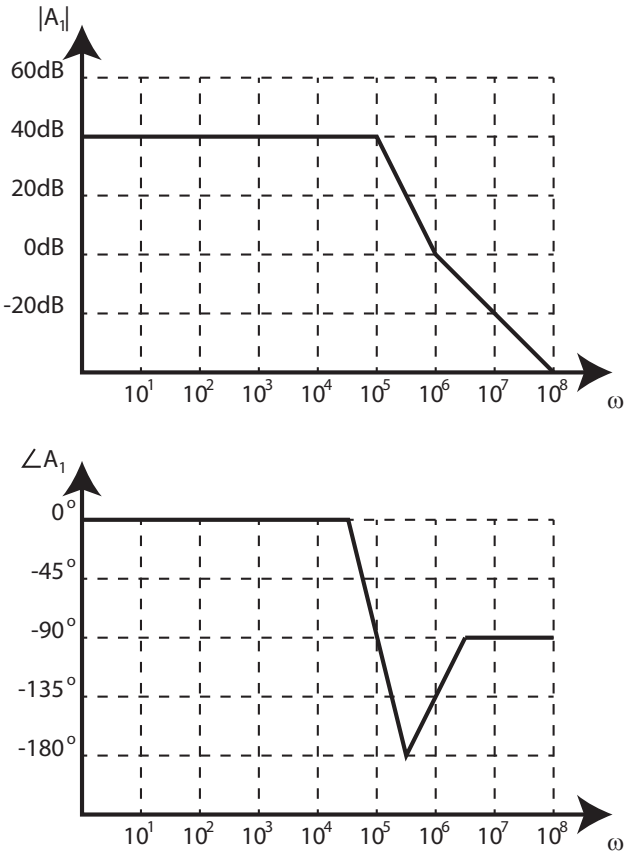

|A1|
60dB
40dB
20dB
0dB
-20dB
10^1 10^2 10^3 10^4 10^5 10^6 10^7 10^8
ω
∠A1
0°
-45°
-90°
-135°
-180°
10^1 10^2 10^3 10^4 10^5 10^6 10^7 10^8
ω

Figure 6:

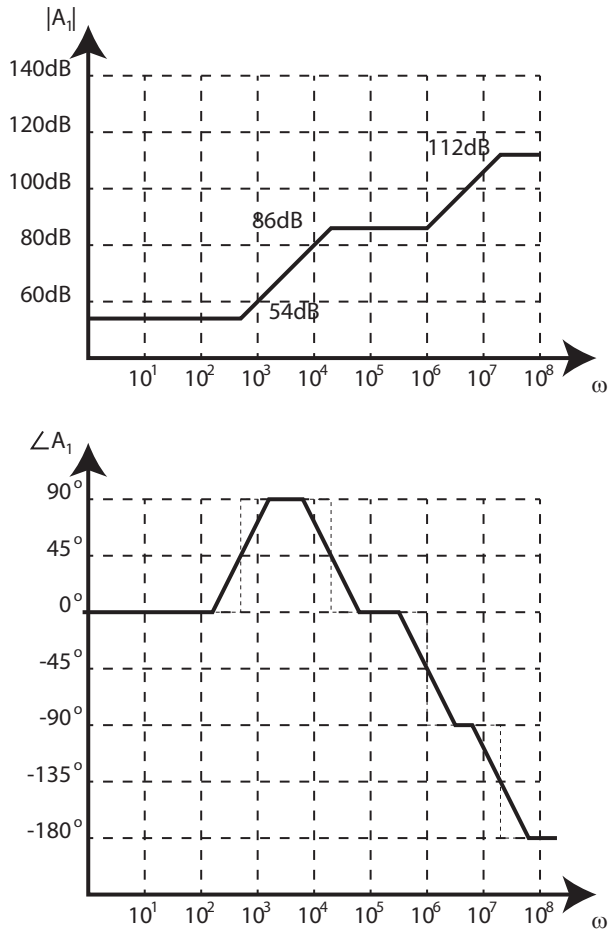


Figure 7: Solution for task 10

Task 10 (*p): Draw a Bode plot with both magnitude- and phase-plot for the following transfer function $A_2(s)$. Make sure to use dB units on the y-axis and $\log_{10}\omega$ on the y-axis. Use piece-wise linear approximations in your plots, i.e. do not 'round' the transitions between linear regions in your log-log plots, just like in figure 7.

$$A_2(s) = \frac{(5 * 10^2 + s)(1 - \frac{s}{10^6})}{(1 + \frac{s}{2*10^7})(1 + \frac{s}{2*10^4})} \tag{79}$$

Solution Task 9:

$$A_1(s) = A_{DC}\frac{1 + \frac{s}{\omega_z}}{(1 + \frac{s}{\omega_{p1}})(1 + \frac{s}{\omega_{p2}})}$$
$$\omega_z = 10^6$$
$$\omega_{p1} = \omega_{p2} = 10^5$$