University of Oslo
Faculty of Mathematics and Natural Sciences

Final Exam in: INF4411/3410, fall 2018
Date of exam: 30-Nov-2018
Time: 09:00-13:00
This examination paper consists of * page(s)
Permitted materials: All printed and written including approved (!) calculator

Make sure that your copy of this examination paper is complete before answering.

Read Carefully:

This is the final exam of INF4410 and INF3410, fall 2018. It will contribute 60% towards the final grade, while the other 40% are contributed by the mandatory labs 2 and 3 of the course. The task 3 under chapter 3 is *RESERVED for INF4411 ONLY!* Students that are registered as bachelor students in INF3410 shall *NOT* solve this task! No extra points will be given if they should solve the task!

It is recommended that you read all the tasks before starting with the exam. A good strategy thereafter is to start with the tasks you feel most comfortable with and leave the most difficult ones for last.

It is also recommended to read through all the tasks once more after you have finished, to make sure you have not overlooked anything.

Start a new page for every task you are answering and write your ID-number and task number on the top of all pages you submit. If you use several pages per task also mark the page number next to the task number. Make sure to answer all questions and follow all instructions. Clearly mark your final answers and results (all that are explicitly asked for or instructed to find in a task) with a double underline. Document your deduction of those results and any assumptions you may have had to make.

Note that you may not hand in the original task papers that you are holding now, i.e. do not write any results here but use exclusively the separate sheets of paper that you are going to hand in.

Each question is graded indicating the weight when grading.

Chapter 1

Task 1 (4p): Write down the Kirchoff equations (sum of currents into a node is 0A) for nodes v_{O1}, v_{x2}, v_{O2} and the 2 transfer functions of the two circuits in figure 1 A) and B).

Task 1 Solution: A)

Kirchoff:

Table 1.5 MOSFET parameters representative of various CMOS technologies and used for rough hand calculations in this text.

	0.8 μm		0.35 μm		0.18 μm		45 nm	
Technology	**NMOS**	**PMOS**	**NMOS**	**PMOS**	**NMOS**	**PMOS**	**NMOS**	**PMOS**
μC_{ox} ($\mu A/V^2$)	92	30	190	55	270	70	280	70
V_{t0} (V)	0.80	–0.90	0.57	–0.71	0.45	–0.45	0.45	–0.45
$\lambda \cdot L$ (μm/V)	0.12	0.08	0.16	0.16	0.08	0.08	0.10	0.15
C_{ox} ($fF/\mu m^2$)	1.8	1.8	4.5	4.5	8.5	8.5	25	25
t_{ox} (nm)	18	18	8	8	5	5	1.2	1.2
n	1.5	1.5	1.8	1.7	1.6	1.7	1.85	1.85
θ (1/V)	0.06	0.135	1.5	1.0	1.7	1.0	2.3	2.0
m	1.0	1.0	1.8	1.8	1.6	2.4	3.0	3.0
$C_{ov}/W = L_{ov}C_{ox}$ (fF/μm)	0.20	0.20	0.20	0.20	0.35	0.35	0.50	0.50
$C_{db}/W \approx C_{sb}/W$ (fF/μm)	0.50	0.80	0.75	1.10	0.50	0.55	0.45	0.50

Table 1.5
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Table 1: Typical process parameters

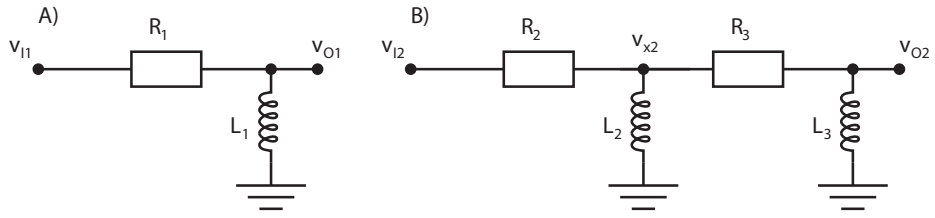


Figure 1: Two RL filters

$$v_{o1}(\frac{1}{sL_1}+\frac{1}{R_1}) = v_{i1}\frac{1}{R_1}$$

Transfer function:

$$\begin{aligned}\frac{v_{o1}}{v_{i1}} &= \frac{\frac{1}{R_1}}{\frac{1}{sL_1}+\frac{1}{R_1}} \\ &= \frac{s\frac{L_1}{R_1}}{s\frac{L_1}{R_1}+1}\end{aligned}$$

B)
Kirchoff:

$$\begin{aligned}v_{o2}(\frac{1}{sL_3}+\frac{1}{R_3}) &= v_{x2}\frac{1}{R_3} \\ v_{x2}(\frac{1}{sL_2}+\frac{1}{R_2}+\frac{1}{R_3}) &= v_{i2}\frac{1}{R_2}+v_{o2}\frac{1}{R_3}\end{aligned}$$

eliminating v_{x2}:

$$\begin{aligned}v_{o2}(\frac{R_3}{sL_3}+1) &= v_{x2} \\ v_{o2}(\frac{R_3}{sL_3}+1)(\frac{1}{sL_2}+\frac{1}{R_2}+\frac{1}{R_3}) &= v_{i2}\frac{1}{R_2}+v_{o2}\frac{1}{R_3}\end{aligned}$$

transfer function

$$\begin{aligned}\frac{v_{o2}}{v_{i2}} &= \frac{\frac{1}{R_2}}{(\frac{R_3}{sL_3}+1)(\frac{1}{sL_2}+\frac{1}{R_2}+\frac{1}{R_3})-\frac{1}{R_3}} \\ &= \frac{s^2\frac{L_2L_3}{R_2R_3}}{(1+s\frac{L_3}{R_3})(1+s\frac{L_2}{R_2}+s\frac{L_2}{R_3})-s^2\frac{L_2L_3}{R_3^2}} \\ &= \frac{s^2\frac{L_2L_3}{R_2R_3}}{1+s\frac{L_2}{R_2}+s\frac{L_2}{R_3}+s\frac{L_3}{R_3}+s^2\frac{L_2L_3}{R_2R_3}+s^2\frac{L_2L_3}{R_3^2}-s^2\frac{L_2L_3}{R_3^2}} \\ &= \frac{s^2\frac{L_2L_3}{R_2R_3}}{1+s(\frac{L_2}{R_2}+\frac{L_2}{R_3}+\frac{L_3}{R_3})+s^2\frac{L_2L_3}{R_2R_3}}\end{aligned}$$

Points: 1 for each Kirchoff equation and transfer function. 0.5 deduction for A not in root form.

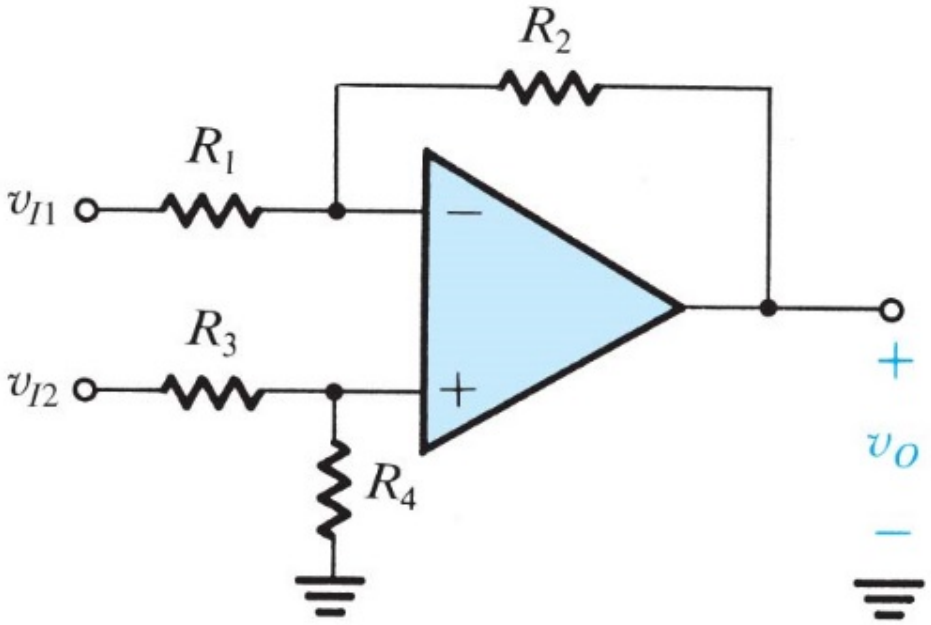


Figure 2: Differential amplifier

Chapter 2

Task 2 (3p): Consider that the differential amplifier in figure 2 uses the following resistor values: $R_1 = 1\text{k}\Omega, R_2 = 2\text{k}\Omega, R_3 = 3\text{k}\Omega, R_4 = 4\text{k}\Omega$. What function does it implement exactly? Write out the function $v_O = f(v_{I2}, v_{I1})$.

Task 2 Solution: Keeping v_{I2} constant you get:

$$\frac{\Delta v_{I1}}{R_1} = \frac{-\Delta v_{O1}}{R_2}$$
$$\Delta v_{O1} = -\Delta v_{I1}\frac{R_2}{R_1}$$

Keeping v_{I1} constant you get:

$$\Delta v_x = \Delta v_{I2}\frac{R_4}{R_3 + R_4}$$

and

$$\Delta v_x = \Delta v_{O2}\frac{R_1}{R_1 + R_2}$$

$\Rightarrow$

$$\Delta v_{O2} = \Delta v_{I2}\frac{R_4}{R_3 + R_4}\frac{R_1 + R_2}{R_1}$$

Where v_X is the voltage at both direct input nodes to the amp.

Superimposing the two:

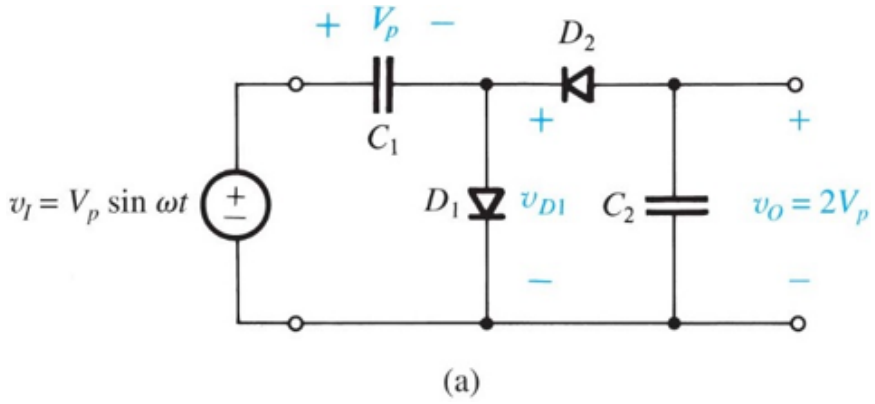


Figure 3: Figure 3.36 from the book: Voltage doubler

$$\begin{aligned}\Delta v_O &= \Delta v_{O2} + \Delta v_{O1} \\ &= \Delta v_{I2} \frac{R_4}{R_3 + R_4} \frac{R_1 + R_2}{R_1} - \Delta v_{I1} \frac{R_2}{R_1}\end{aligned}$$

Starting from zero input and establishing that $v_O = 0$ for zero input:

$$\begin{aligned}v_O &= v_{O2} + v_{O1} \\ &= v_{I2} \frac{R_4}{R_3 + R_4} \frac{R_1 + R_2}{R_1} - v_{I1} \frac{R_2}{R_1}\end{aligned}$$

With numbers

$$\begin{aligned}v_O &= v_{I2} \frac{4}{3+4} \frac{1+2}{1} - v_{I1} \frac{2}{1} \\ &= v_{I2} \frac{12}{7} - 2v_{I1}\end{aligned}$$

Points: 1.5 for each individual amplification with one input constant.

Chapter 3 (only for INF4411!)

The task 3 of chapter 3 is mandatory for students having registered as master students to course INF4411, and are NOT(!) to be solved for bachelor students registered to course INF3410. No extra point are to be gained for bachelor students registered to course INF3410 by solving this task under chapter 3!

Task 3 (only INF4411, 3p): Figure 3 depicts a voltage doubler as described in the book. Consider the diodes to be ideal, i.e. non-conducting for negative voltages and zero resistance for positive voltages. In **steady state** the output

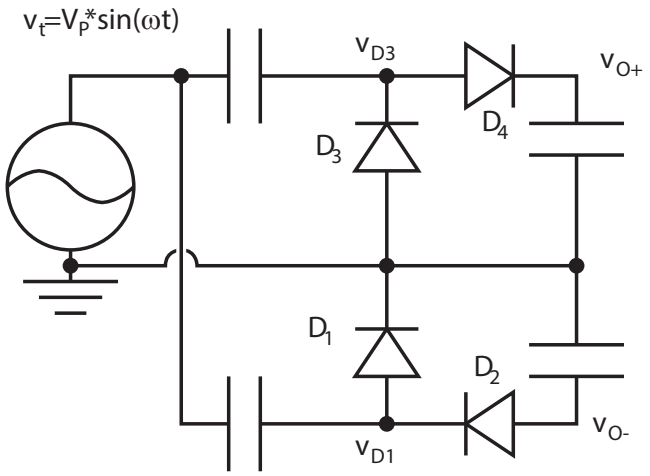


Figure 4: Rectifier derived from voltage doubler

is $-2V_P$ where V_P is the amplitude, i.e. half the peak-to-peak voltage. Now consider the circuit in figure 4 that is an extension of the voltage doubler. Can you sketch the voltages v_{D1}, v_{D3} and v_O together with the input v_t (all referenced to Gnd!) over one cycle of the input signal, again in **steady state** (i.e. not in start-up but after the voltages have become stable from one cycle to the next)?

Task 3 Solution: The lower part of the circuit is simply the previous voltage doubler upside down with output at $-2V_p$. The upper part does the exact same but with inverted sign, i.e. its output is at $+2V_p$. v_{D1} is a sine in parallel with $v_I$but shifted down by V_p, i.e. swinging from $-2V_p$ to 0V. v_{D3} is shifted up by V_P, i.e. swinging from 0 to $+2V_p$.

Points: waveform D1 and D3 correct 1 point each. Total output correct another point

Chapter 5

Task 4 (3p) A particular NMOS transistor is operated in its saturation region at a constant v_{GS}, i_D is found to be 100μA for $v_{DS} = 1V$ and 110μA for $v_{DS} = 2V$. Find the corresponding values for r_o, V_A, and λ.

Task 4 Solution: V_A can be found by extrapolating that line until it crosses the x-axis:

$$100\mu\text{A} + (1\text{V} + V_A)(110\mu\text{A} - 100\mu\text{A})/(2\text{V} - 1\text{V}) = 0$$
$$V_A * 10\mu\text{A/V} = -100\mu\text{A} - 1\text{V} * 10\mu\text{A/V}$$
$$V_A = -10\text{V} - 1\text{V} = 9\text{V}$$

r_0 is also simple actually:

$$r_0 = (2\text{V} - 1\text{V})/(110\mu\text{A} - 100\mu\text{A}) = 1\text{V}/10\mu\text{A} = 100\text{k}\Omega$$

as for λ one needs to be careful to use the correct I'_D when deriving it from r_0 which is the I_D one would get with infinite r_0 or $\lambda = 0$, so that's in fact where the line would cross the y-axis, i.e. at 90μA. So:

$$\lambda = \frac{1}{I'_D r_0} = \frac{1}{90\mu\text{A}100\text{k}\Omega} = \frac{1}{9}1/\text{V}$$

One can also derive it directly from the two sample points though:

$$\begin{aligned}
&I_D = 0.5k_n V_{ov}^2(1 + V_{DS}\lambda)\\
&\frac{1}{0.5k_n V_{ov}^2} = \frac{(1 + V_{DS}\lambda)}{I_D}\\
&\frac{(1 + 1\text{V} * \lambda)}{100\mu\text{A}} = \frac{(1 + 2\text{V} * \lambda)}{110\mu\text{A}}\\
&110\mu\text{A} + 110\mu\text{AV} * \lambda = 100\mu\text{A} + 200\mu\text{AV} * \lambda\\
&10\mu\text{A} = 90\mu\text{AV} * \lambda\\
&\lambda = \frac{10\mu\text{A}}{90\mu\text{AV}} = \frac{1}{9}1/\text{V}
\end{aligned}$$

Points: one for each correct parameter. For $r_o = 90\text{k}\Omega$ or $r_o = 81.1\text{k}\Omega$ or for λ falsely derived from r_0 with the wrong current (i.e. 0.1) 0.5 points.

Chapter 6

Task 5 (2p): Consider the two CS amplifiers in figures 5 A) and B). Both circuits shall be operating in strong inversion. If the DC offset V_I of the input v_I in subfigure A) is V_{IA} (i.e. the input that sets the output voltage nicely in the linear range at $\frac{\text{Vdd}}{2}$) what is the correct DC offset V_{IB} in subfigure B) as a function of V_{IA}?

Task 5 Solution:

$$\begin{aligned}
&I_{DB} = 2I_{DA}\\
&V_{ovB} = \sqrt{2}V_{ovA}\\
&V_{IB} = V_{GSB} = \sqrt{2}(V_{IA} - v_{tn}) + v_{tn}
\end{aligned}$$

Points:

1 for getting sqrt(2) correct. 1 for not making a mistake with vtn subtraction/addition

Task 6 (2p): A NMOS CS amplifier with a total resistive load $R_D = r_o||R_L$ has a small signal gain of -20V/V at the point of operation $V_{GS} = 0.9V$. It is implemented in the 0.35μm process of table 1 and has W/L=1. What is the size of R_D?

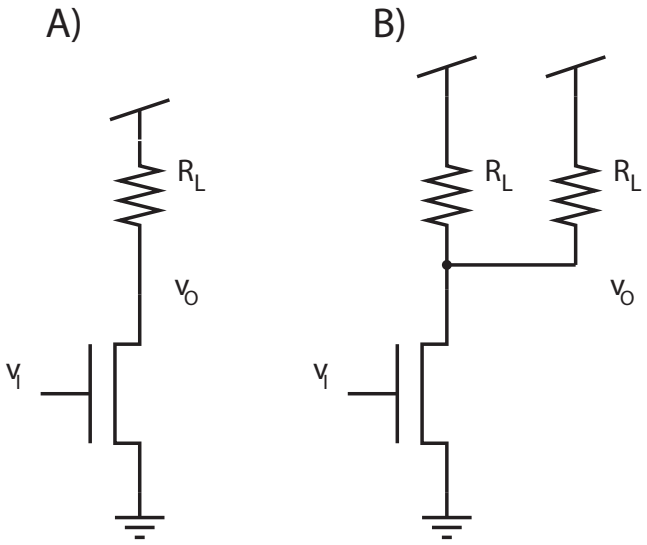


Figure 5: Two CS gain stages with different loads

Task 6 Solution:

$$\begin{aligned} g_m &= k_n V_{ov} \\ &= 190\mu\text{A/V}^2 * 0.33V = 62.7\mu\text{A/V} \\ A &= g_m R_D \\ R_D &= \frac{A}{g_m} = \frac{20}{62.7\mu\text{A/V}} = 319\text{k}\Omega \end{aligned}$$

points:

1 for gm

1 for the rest correct

Chapter 7

Task 7 (3p): A telescopic cascode gain stage with cascode current source load depicted in figure **??** is implemented with the 0.35μ process in table 1 and all transistors have a W/L=0.5. Vdd is 3.3V and the input DC offset is $V_I = 0.8V$. What is the correct bias for V_{G4} such that the output is approximately in the middle of the linear range at Vdd/2? What is the maximum value for V_{G3} and the minimum value for V_{G1} such that all transistors remain in saturation for the output being at Vdd/2?

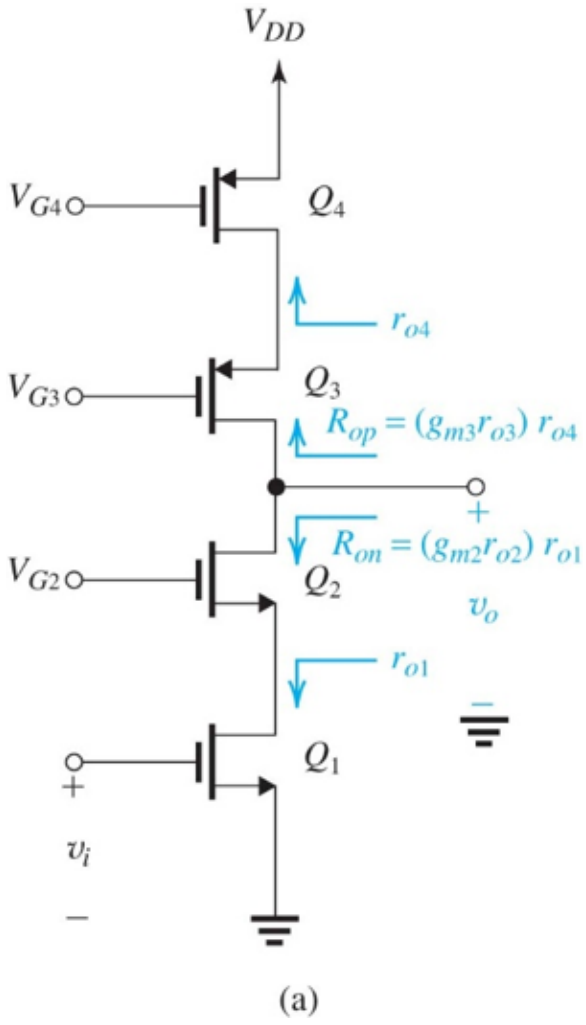


Figure 6: Telescopic cascode gain stage with cascode current source load.

Task 7 Solution:

$$I_{Dn} = I_{Dp} = I_D$$
$$I_D = 0.5 * 190\mu\text{A/V}^2 * 0.5 * 0.23\text{V}^2 = 2.513\mu\text{A}$$
$$V_{ovp} = \sqrt{\frac{2.513\mu\text{A}}{0.25 * 55\mu\text{A/V}^2}} = 0.4275$$
$$V_{G4} = \text{Vdd} - v_{tp} - V_{ovp} = 3.3\text{V} - 0.71\text{V} - 0.4275\text{V} = 2.1625$$
$$V_{G3} = V_{G4} - V_{ovp} = 2.1625\text{V} - 0.4275\text{V} = 1.735\text{V}$$
$$V_{G2} = V_I + Vovn = 0.8\text{V} + 0.23\text{V} = 1.03\text{V}$$

Points:

1 for ID
1 for correct VG4
1 for correct VG3 and VG2

Chapter 8

Task 8 (4p): A currrent loaded MOS differential amplifier (fig **??**) is constructed in 0.35μm technology with the parameters from table 1. The width length of all transistors is 1μm and the width of the transistors in the current mirror and differential pair is also 1μm, while the bias transistor has a width of 2μm. If V_{OV} of the bias transistor is 0.3V, what is the output resistance R_{SS} of the bias transistor, the common mode current gain G_{mcm} of the differential pair transistors, the current gain A_m of the current mirror, and finally the common mode voltage gain A_{cm} of the circuit?

Task 8 Solution: From the book $G_{mcm} \approx \frac{1}{2R_{SS}}$

$$R_{SS} = r_o = \frac{1}{\lambda I_D}$$
$$\lambda(1\mu\text{m}) = 0.161/\text{V}$$
$$I_D = 0.5 k_n V_{ov}^2 = 0.5 * 2 * 190\mu\text{A/V}^2 * 0.3^2\text{V}^2 = 17.1\mu\text{A}$$
$$R_{SS} = 1/17.1\mu\text{A}/0.16\ 1/\text{V} = 366\text{k}\Omega$$
$$G_{mcm} \approx 1/2/366\text{k}\Omega = 1.37\mu\text{A/V}$$

A_m according to the book:

$$A_M = \frac{1}{1 + \frac{1}{g_{m3} r_{o3}}}$$
$$V_{ovp} = \sqrt{\frac{I_{D3}}{k_p}} = \sqrt{8.55\mu\text{A}/0.5/55\mu\text{A/V}^2} = 0.56\text{V}$$
$$g_{m3} = k_n V_{ov3} = 55\mu\text{A/V}^2 * 0.56\text{V} = 30.7\mu\text{A/V}$$
$$r_{o3} = \frac{1}{\lambda I_{D3}} = 1/0.16\ 1/\text{V}/8.55\mu\text{A} = 731\text{k}\Omega$$
$$A_M = 1/(1 + 1/731e - 3/30.7e - 6) = 0.9573$$

A_{cm} from the approximation in the book:

$$A_{cm} \approx -\frac{1}{2g_{m3}R_{SS}} = -1/2/30.7e-6/366e3 = -0.445$$

Someone also used the more precise formula:

$$A_{cm} - (1 - A_m)G_{mcm}(R_{om}||R_{o2})$$
$$R_{om} = r_{o3} = 731\text{k}\Omega$$
$$R_{o2} = 2R_{SS} + r_{o2} + g_{m2}r_{o2}2R_{SS}$$
$$r_{o2} = r_{o3} \text{ same } \lambda \text{ in this process and it's the same } I_d$$
$$V_{ovn} = 0.3\text{V Because the bias transistor is twice as wide, it's the same for all nMOS}$$
$$g_{m2} = 190\mu\text{A} * 0.3\text{V} = 57\mu\text{A/V}$$
$$R_{o2} = 731\text{k}\Omega + 731\text{k}\Omega + 57\mu\text{A/V}731\text{k}\Omega731\text{k}\Omega = 32\text{M}\Omega$$

$A_{cm} = -0.0418$

Points:

1 for ID/RSS/Gmcm
2 for Vovp/gm3/ro3/Am
1 for Acm

Chapter 9

Task 9 (5p): For a NMOS CS-amplifier with a single PMOS transistor current source load using 0.35μm technology with parameters from table 1 the width W and lenght L of both transistors shall be 1μm. V_{ov} for the NMOS point of operation is 0.2V. Compute the DC gain, the -3dB cutoff frequency and the unity gain frequency without any load and with a signal source output resistance R_{sig} that is the same as the total output resistance R_O of the circuit. Use the Miller approach to derive an equivalent capacitance at the input node from the capacitance between input and output. Only take C_{gs} and C_{gd} into account.

Task 9 Solution:

$$I_D = 0.5k'_n\frac{W}{L}V_{OV}^2 = 0.5 * 190e-6 * 1 * 0.2^2 = 3.8\mu\text{A}$$
$$r_{on} = \frac{L}{L\lambda I_D} = 1e-6/0.16e-6/3.8e-6 = 1.644\text{M}\Omega$$
$$r_{op} = \frac{L}{L\lambda I_D} = 1e-6/0.16e-6/7.6e-6 = 1.644\text{M}\Omega$$
$$\Rightarrow R_{sig} = R_O = r_{on}||r_{op} = 822\text{k}\Omega$$

$$g_m = k_n * V_{OV} = 190e-6 * 0.2 = 38\mu\text{S}$$
$$A_0 = g_m R_O = 38e-6 * 822e3 = 31.24$$

$$C_{gd} = W * C_{ov}/W = 1e-6 * 0.2e-9 = 0.2\text{fF}$$
$$C_{eq} = (1 + A_0) * C_{gd} = (31.24 + 1) * 0.2e-15 = 6.45\text{fF}$$
$$C_{gs} = \frac{2}{3} W * L * C_{ox} + W * C_{ov}/W = 2/3 * 1e-12 * 4.5e-3 + 0.2e-15 = 3.2\text{fF}$$
$$C_{tot} = C_{gs} + C_{eq} = 9.65\text{fF}$$
$$\omega_{p1} = \frac{1}{R_{sig} C_{tot}} = 1/822e3/9.65e-15 = 126\text{Mrad/s}$$
$$\omega_t = A_0 * \omega_{p1} = 31.24 * 126e6 = 3.94\text{Grad/s}$$

For ω_t some used an approximation:

$$\begin{aligned}
\omega_t &= \omega_H * A_0 \\
&= \frac{1}{R_{sig} C_{tot}} A_0 \\
&= \frac{1}{R_{sig}(C_{gd}(A_0 + 1) + C_{gs})} A_0 \\
&\approx \frac{1}{R_{sig}(C_{gd} A_0 + C_{gs}} A_0 \\
&= \frac{1}{R_{sig}(C_{gd} + \frac{C_{gs}}{A_0}} \\
&= 1/822e3/(0.2e-15 + 3.2e-15/31.24) \\
&= 4.02\text{GHz} \\
&\approx \frac{1}{R_{sig} C_{gd}} \\
&= 1/822e3/(0.2e-15) \\
&= 6.08\text{GHz}
\end{aligned}$$

Rather wrong:

$$\begin{aligned}
\omega_t &= \omega_H * A_0 \\
&\approx \frac{1}{R_{sig}(C_{gd} + C_{gs})} \\
&= 1/822e3/(0.2e-15 + 9.65e-15) \\
&= 123.5\text{MHz}
\end{aligned}$$

Points:

1 for R_sig/Ao

1 for gm/Ao

1 for C_gd/C_eq

1 for C_gs

1 for w_p1 and w_t

Chapter 15

Task 10 (3p): Progressing from 0.18μm technology to 45nm technology, using the parameters of table 1, give a well founded and explained estimate on how the propagation delay of an inverter changes. Use the simple model from the book equations (15.54) to (15.57) for your argument:

$$t_{PHL/PLH} = 0.69 R_{N/P} C \quad (15.54, 15.55)$$

$$R_N \approx \frac{12.5}{(W/L)_n} \mathrm{k}\Omega \quad (15.56)$$

$$R_P \approx \frac{30}{(W/L)_n} \mathrm{k}\Omega \quad (15.57)$$

Task 10 Solution: Using that simple model the equivalent resistances do not change for a scaled inverter layout, i.e. W/L remain the same.

Surface is scaled by a factor $45^2/180^2$=0.0625. C_{ox} is increased from 8.5μF/μm^2 to 25μF/μm^2 i.e. by a factor 2.94. So a rough estimateif C is dominated by $C_{ox} * W * L$ would be that C and thus the gate delay is scaled by a factor 2.94*0.0625=0.184.

If you'd want to be picky one could add minor contributions as well. C_{OV} for a linearly scaled inverter is scaled with 45/180*0.5/0.35=0.3571 so less consequently C_{gd} and a small part of C_{gs} scales by this factor only. Another influence would be the wire capacitance, which behaves similarly to $C_{ox} * W * L$ where the area also scales with 0.0625 and there is a wire capacitance per area that is also increased, probably similarly as C_{ox}. Consequently the effect of C_{OV} is dampended some more by the contribution of C_W. Thus the factor 0.184 will likely overestimate the delay reduction slightly. For a more accurate estimate by how much one would need to consider a particular layout of an inverter and would need info on C_W.

points:

1 for reduction in area in numbers (0.5 for just the principle)
1 for increase in C_ox in numbers (0.5 for just the principle)
1 for reduction in C_ov or good explanation what is more important

Also 0.5 if understanding just that C is the decisive factor here

Chapter 10/Transfer Functions and Bode Plots

Task 11 (1p): Consider a amplifier with a transfer function $A(s)$ that has two poles at the same frequency ω_{p1}:

$$A(s) = A_0 \frac{1}{(1 + \frac{s}{\omega_{p1}})^2}$$

Where is its unity gain frequency ω_t in terms of A_o and ω_{p1}?

Task 12 (3p): For the transfer function in the previous task let us assume that A_0 =80dB and the two poles ω_{p1} =1Mrad. Now let's introduce another single pole ω_{p2} with the aim to reduce the previous unity gain frequency ω_t by a

factor 2 to $\omega_t' = \frac{\omega_t}{2}$, at what frequency is that pole? Hint: it might help to draw the Bode magnitude plot and think of the problem graphically and solve it in the log-domain.

Solution:

$$\log(A_0) = 2(\log(\omega_t) - \log(\omega_{p1}))$$
$$A_0 = \frac{\omega_t^2}{\omega_{p1}^2}$$
$$\omega_t = \sqrt{A_0}\omega_{p1}$$

For ω_{p1} =1Mrad and A_0 =80dB=10000 ω_t = 100∗1Mrad=100Mrad.

To reduce ω_t by a factor 2 to $\omega_t' = \omega_t/2$ with another pole ω_{p2} one can draw a line from the 0dB point at ω_t' with slope -60dB/dec and see where it intersects the -40dB/dec line decending from (ω_{p1}, A_0). That's where the new pole needs to be. (If it would not intersect one could also look for the intersection point with the vertical line $x = \omega_{p1}$ and draw a -20dB/dec line from that point and see where it intersects with $y = A0$ and place the new pole there.) Numerically one has to solve:

$$\log_{10}(A_0) = 2(\log_{10}(\omega_{p2}) - \log_{10}(\omega_{p1})) + 3(\log_{10}(\omega_t/2) - \log_{10}(\omega_{p2}))$$
$$4 = 2 * (x - 6) + 3 * (7.70 - x)$$
$$x = -12 + 23.3 - 4 = 7.3$$
$$10^{7.3} = 20\text{Mrad/s}$$

points:
2 for correct drawing
1 for correct numbers

Grading

$$0.2 * \text{<lab2 score>} + 0.2 * \text{<lab3 score>} + 0.6 * \text{<written exam score>} \quad (1)$$

0-35%	F
35-45%	E
45-55%	D
55-65%	C
65-75%	B
75-100%	A

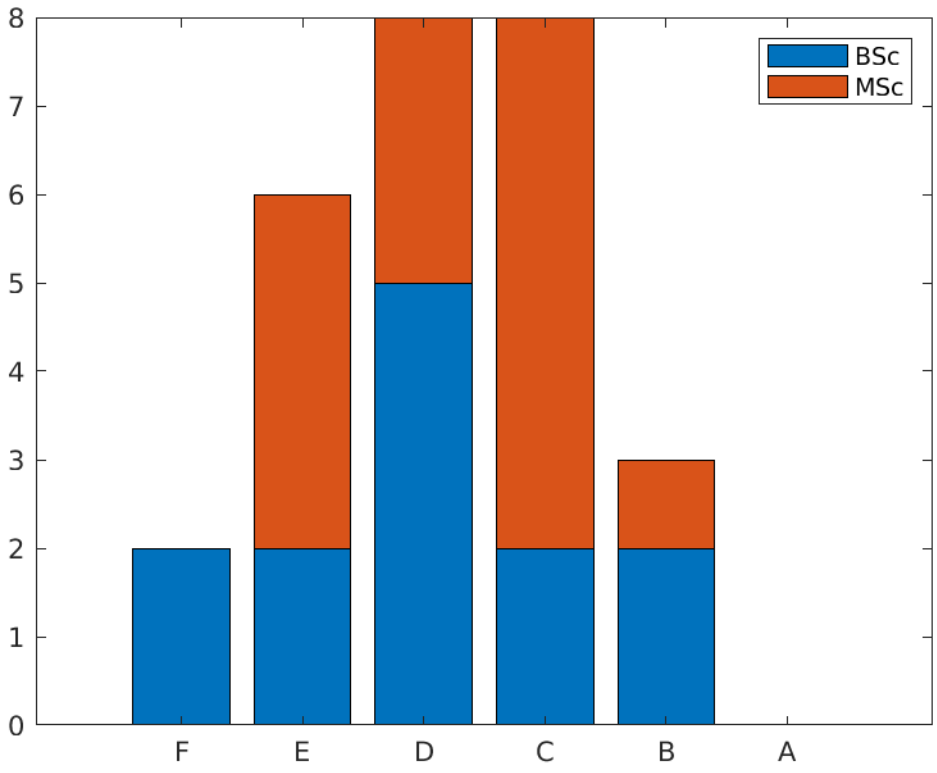


Figure 7: Grade distribution INF3410/4411 Fall 2018