IN3170/4170, Spring 2023

Philipp Häfliger
hafliger@ifi.uio.no

Excerpt of Sedra/Smith Chapter 7: Integrated CMOS Amplifier Basics

Content

Thevenin and Norton Equivalent (book 1.1)

Small Signal Analysis and Transistor Model (book 6.2)

CMOS Common Source (CS) Amplifiers and FET Intrinsic Gain (book 7.3)

Common Gate (CG) Configuration (book 7.4-7.5)

Cascode Configuration (CG and CS in series) (book 7.4-7.5)

Common Drain Stage/Source Follower (Book 6.3)

Improved Current Mirrors/Sources (book 7.6)

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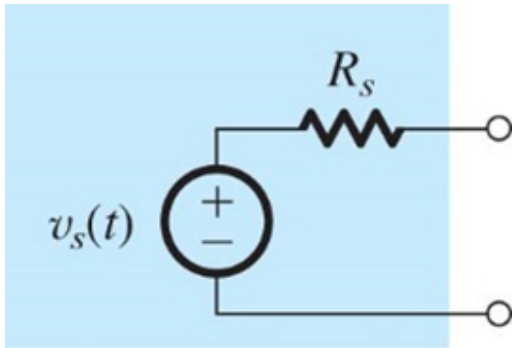
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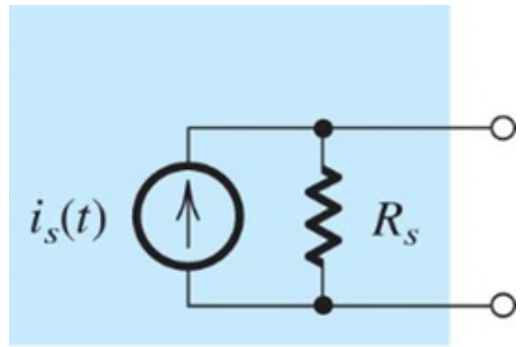
Common Drain Stage/Source Follower (Book 6.3)

Improved Current Mirrors/Sources (book 7.6)

Thévenin and Norton (1/3)



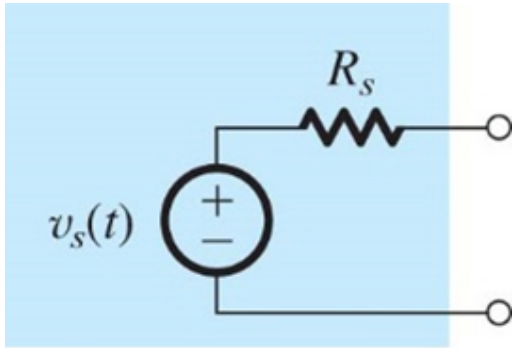
(a)



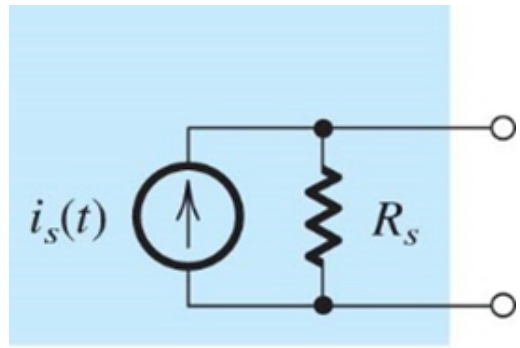
(b)

Two *equivalent* models of signal sources (e.g. a sensor or other *transducer*): a) is called the The Thévenin form and b) the Norton form. Note: Outputresistance R_S is the same in both models while $v_S(t) = i_S(t)R_S$. A more general model would consider an output *impedance*, i.e. including output capacitance/inductance rather than just a resistance.

Thévenin and Norton (2/3)



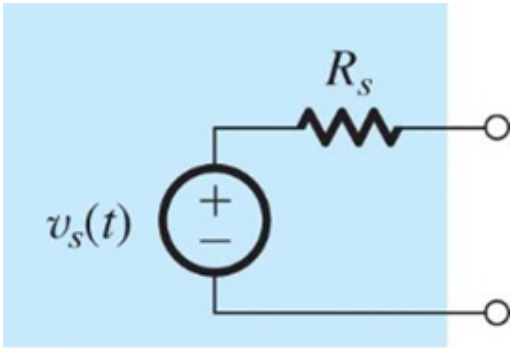
(a)



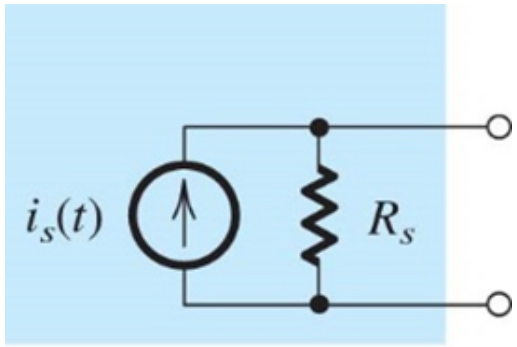
(b)

In general any 'black box' two terminal circuit of passive linear components can be expressed as Thévenin or Norton equivalent circuit, with an equivalent resitance R_S (or impedance Z_S) and (optionally) a signal or DC source. One often talks about the 'impedance seen in this node' where the second node implicitly is Gnd.

Thévenin and Norton (3/3)



(a)



(b)

To get the resistance in this node, stop the time varying signal (if any and if possible) and supply a test current i_X or test voltage v_X between the nodes and measure/compute the voltage or current respectively. R_S is then $\frac{\delta v_X}{\delta i_X}$.

To get the signal source, for the Thevenin equivalent, measure/compute the output voltage (voltage-signal) for open circuit between the two terminals and for the Norton form, measure/compute the current (or current-signal) for a short circuit between the terminals.

The Thevenin can be transformed into the Norton form or vice versa by using $v_S(t) = R_S i_S(t)$.

Example: output resistance of MOSFET

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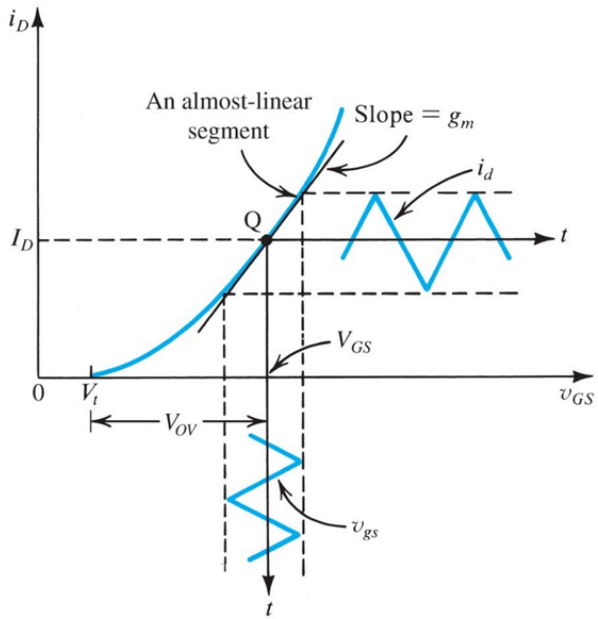
Improved Current Mirrors/Sources (book 7.6)

Intermezzo: Signal Naming Convention in the Book

The book uses upper case letters with upper case indices for large signal variables and DC point of operation for linearised signal approximations (that you find using large signal analysis). Lower case letters with upper case indices to denominate linearised signals around a point of operation, i.e. with both DC (point of operation) and inearized (!) AC (i.e. small signal) components. Lower case letters with lower case indices are small signal components that move around zero, i.e. with the DC point of operation removed. The latter is equivalent to the AC component for completely linear (!) systems. So for example:

$$i_D \approx I_D + i_d$$

Small Signal Analysis $i_D(v_{GS})$



Equivalent to a 1^{st} order Taylor expansion, the small signals only model a linear approximation around a point of operation.

$$\begin{aligned} I_D(V_{GS}) &\approx i_D(v_{GS}) \\ &= I_D(V_{GS}) + \underbrace{(v_{GS} - V_{GS})}_{v_{gs}} * \underbrace{\frac{dI_D}{dV_{GS}}}_{g_m} \end{aligned}$$

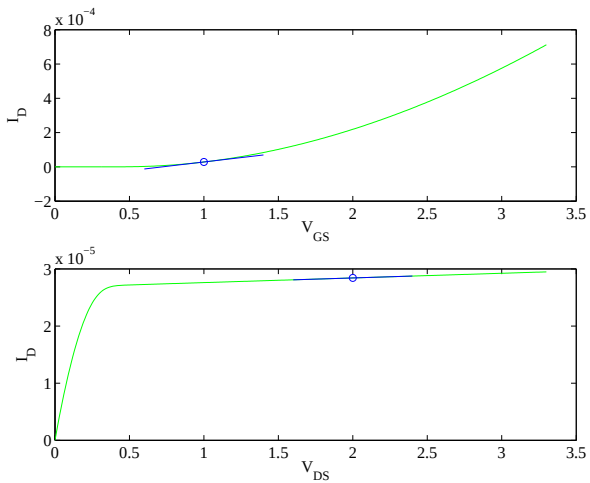
For strong inversion, active region:

$$g_m = \frac{dI_D}{dV_{GS}} = k_n V_{OV} = \sqrt{2 k_n I_D}$$

For weak inversion, active region:

$$g_m = \frac{dI_D}{dV_{GS}} = \frac{I_D}{V_T}$$

Small Signal Analysis $i_D(v_{DS})$



Again a 1^{st} order Taylor expansion, i.e. a linear approximation around a point of operation:

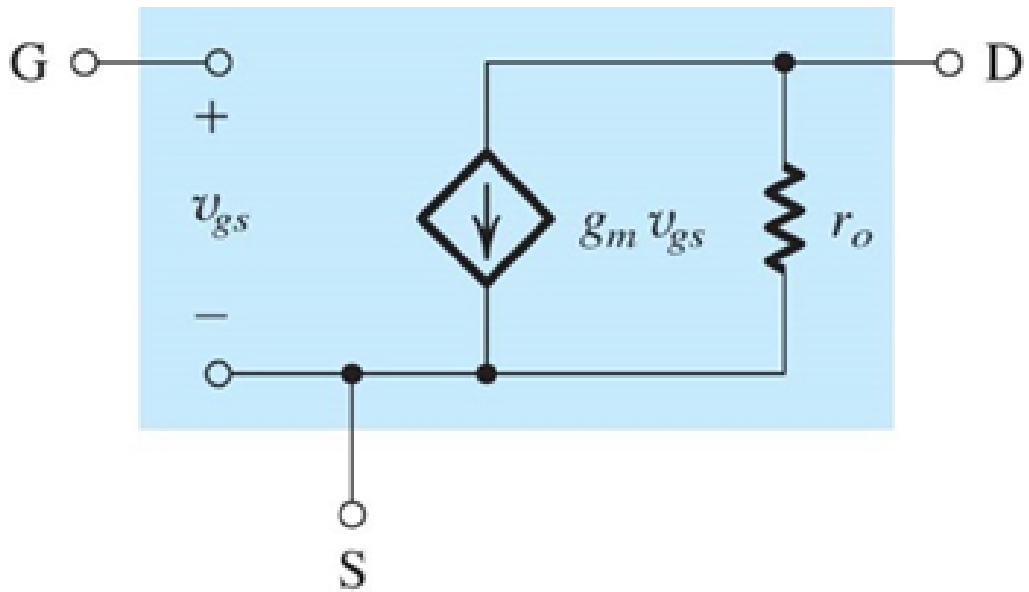
$$\begin{aligned} i_D(v_{DS}) &\approx i_D(v_{DS}) \\ &= I_D(V_{DS}) + \underbrace{(v_{DS} - V_{DS})}_{v_{ds}} * \underbrace{\frac{dI_D}{dV_{DS}}}_{\frac{1}{r_o}} \end{aligned}$$

For active region (both strong and weak inversion):

$$r_o = \frac{dv_{DS}}{di_D} = \frac{1}{\lambda I'_D} = \frac{V_A}{I'_D} = \frac{V'_A L}{I'_D} \quad (5.26/27)$$

MOSFET Small Signal Circuit Model

Replacing circuit components with *small signal circuit models* is a way of using circuit analysis instead of maths to analyse the linearised behaviour around a point of operation. Small signal circuit models model only the deviations from the DC point of operation, i.e. they assume all DC points of operation are zero. So they only model for example v_{gs} but NOT v_{GS}!

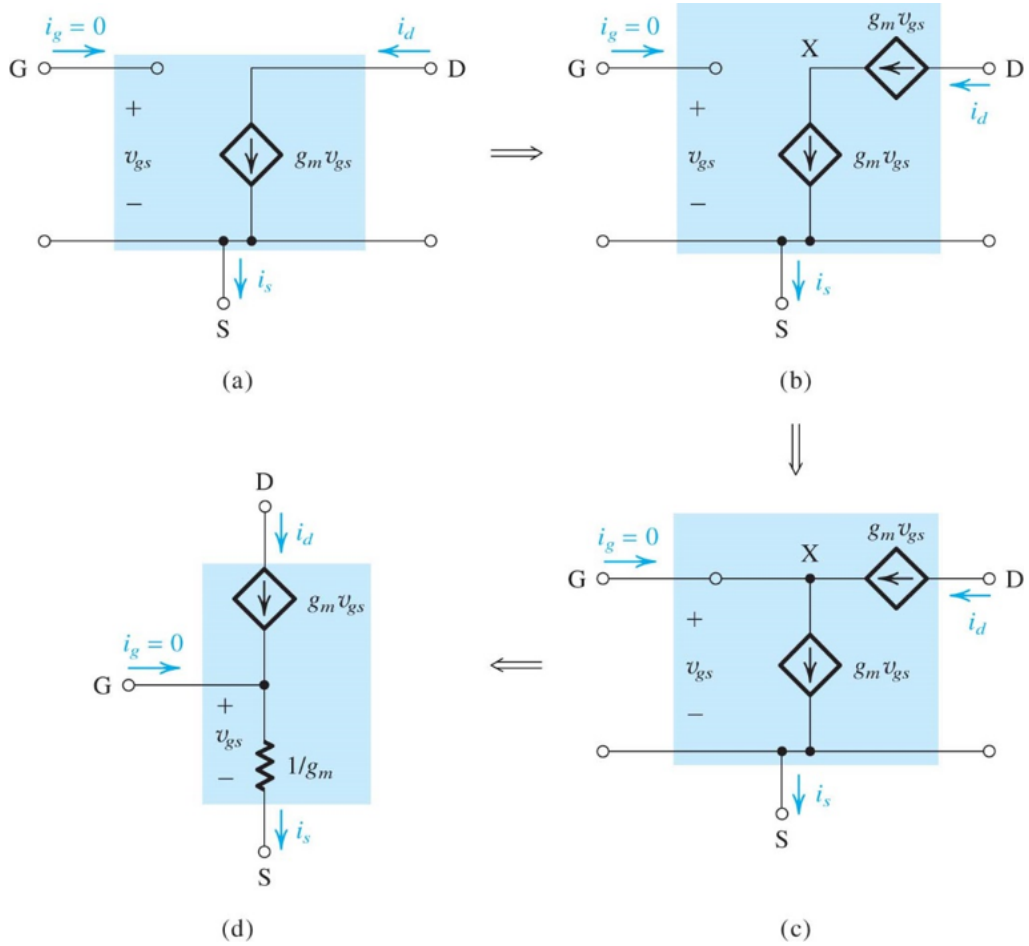


(b)

This is the equivalent small signal circuit of a MOSFET in the active region, low frequency variant, i.e. neglecting capacitors

MOSFET T-Model

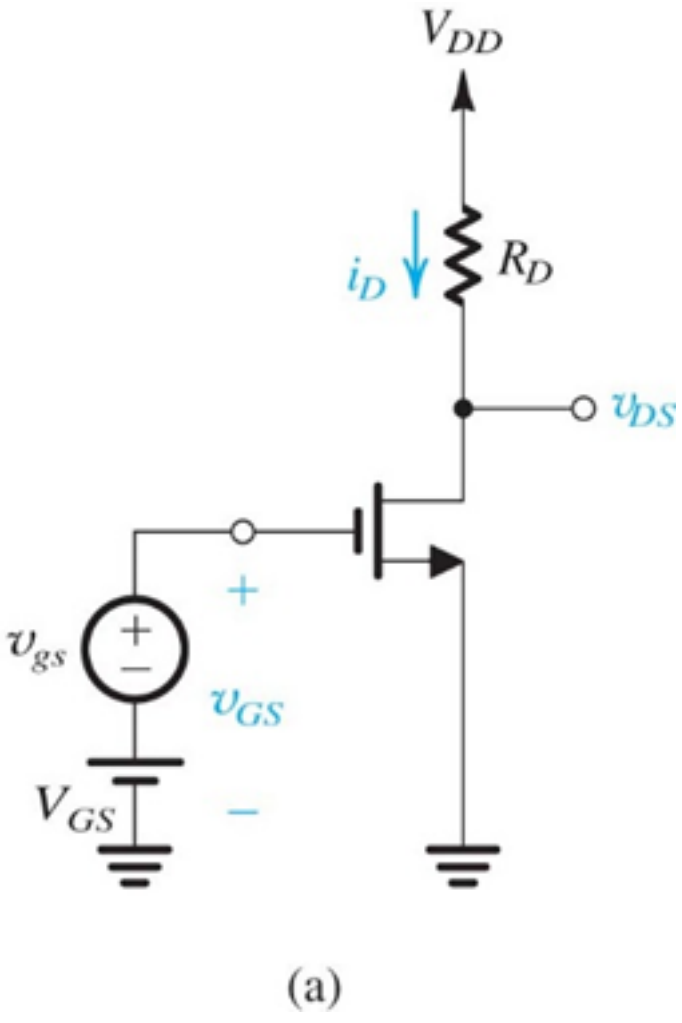
The previous model variant is called hybrid-π and is preferred by the lecturer. The book sometimes uses the equivalent T-model, which might simplify analysis in some cases.



Applying the Small Signal Equivalent Circuits

1. Set all constant voltage sources (including DC biases) to Gnd
2. Set all constant current sources (including DC biases) to zero/open circuit
3. Substitute transistors with small signal equivalent circuit

Example: Common Source Amplifier



(a)

$$A = -g_m(R_D||r_o)$$

$$R_O = R_D||r_o$$

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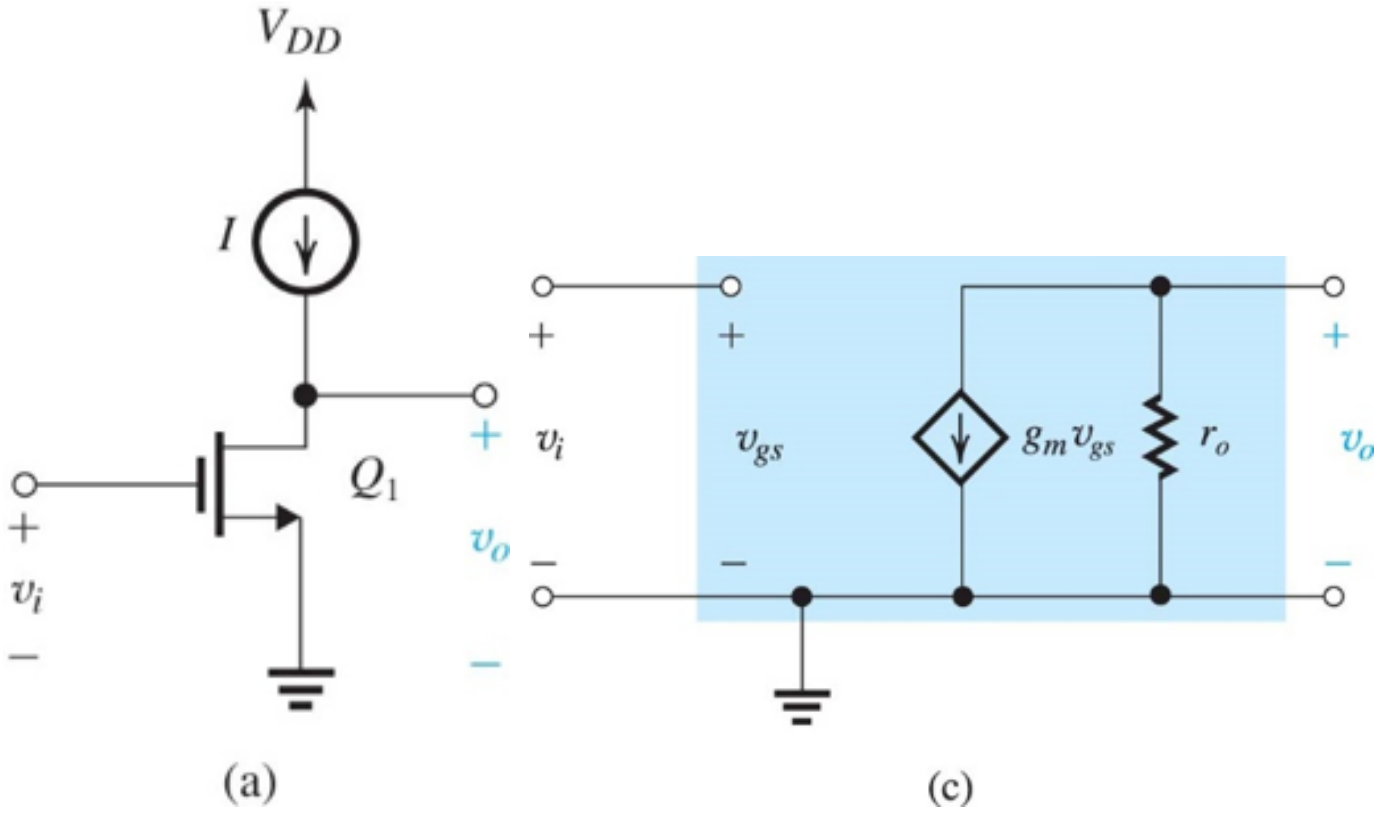
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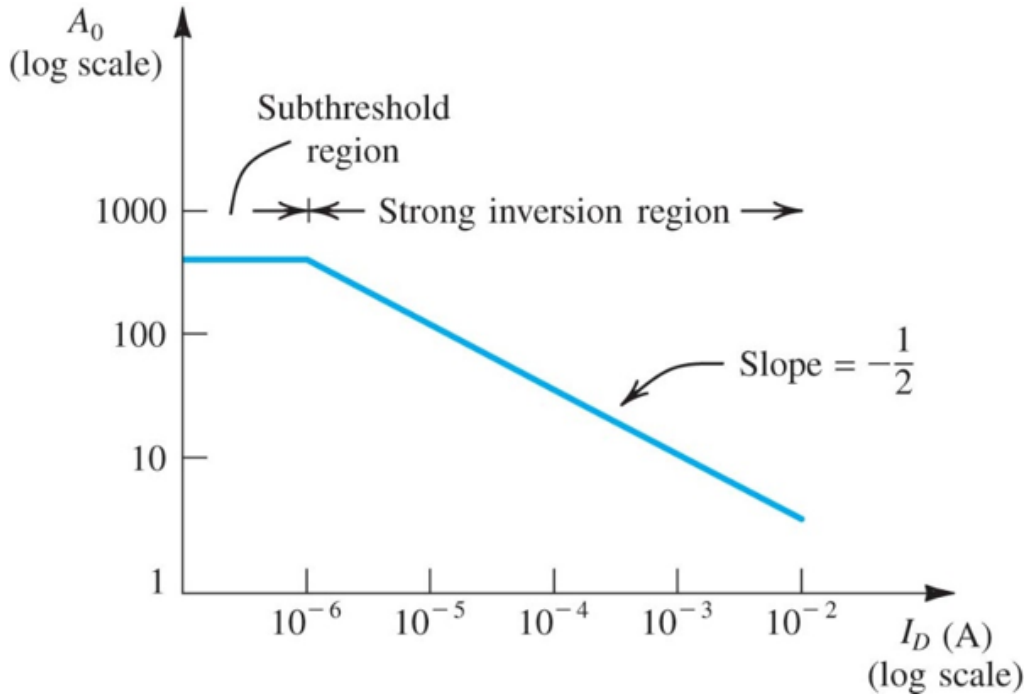
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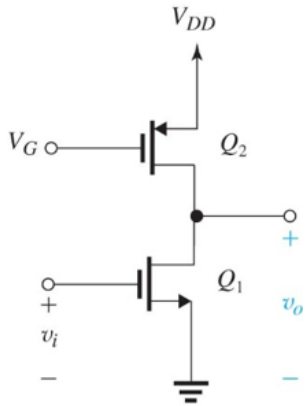
Intrinsic Gain from Small Signal



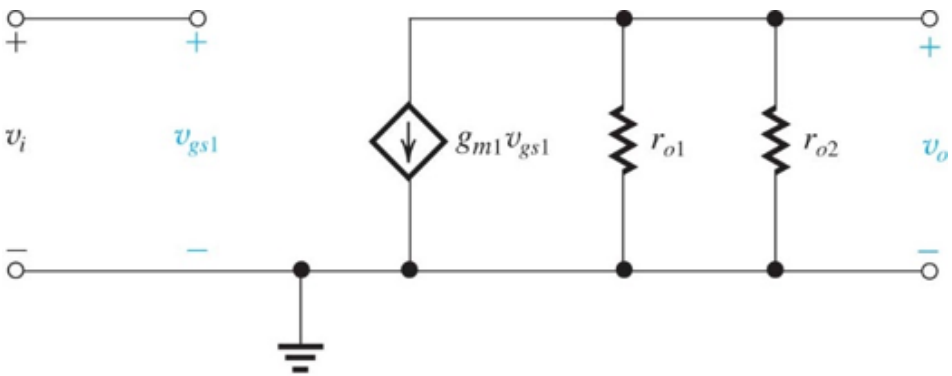
Intrinsic Gain vs Bias Current



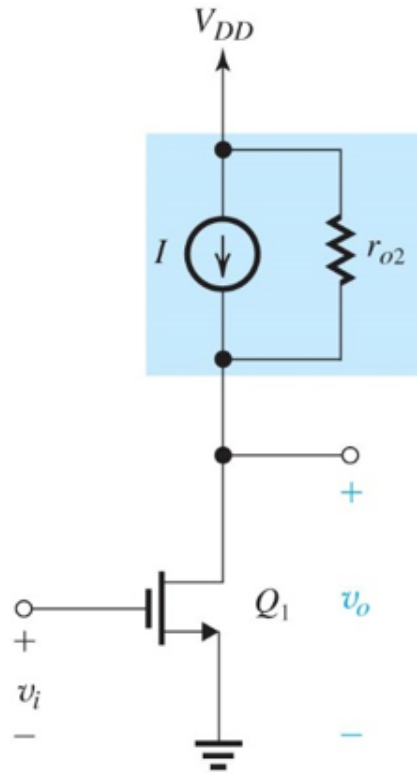
CS Amplifier with Current-Source Load



(a)

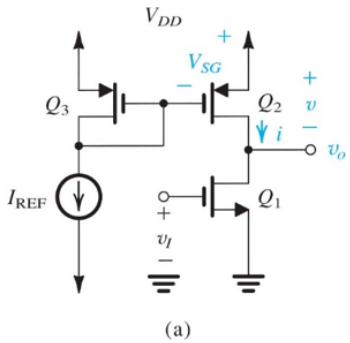


(c)

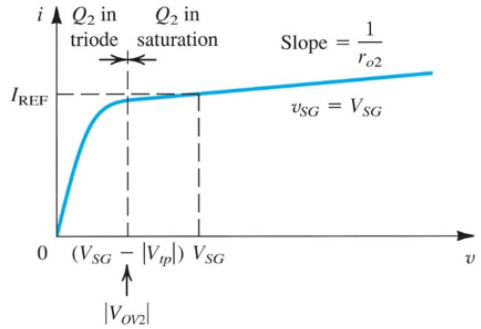


(b)

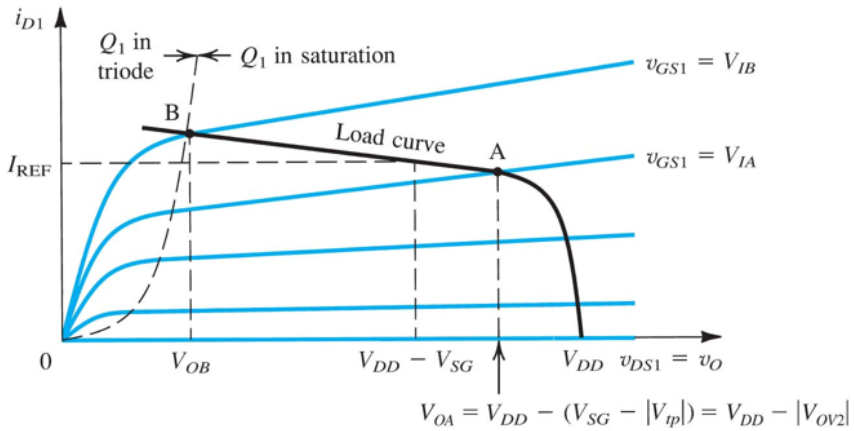
CS Amplifier Analysis



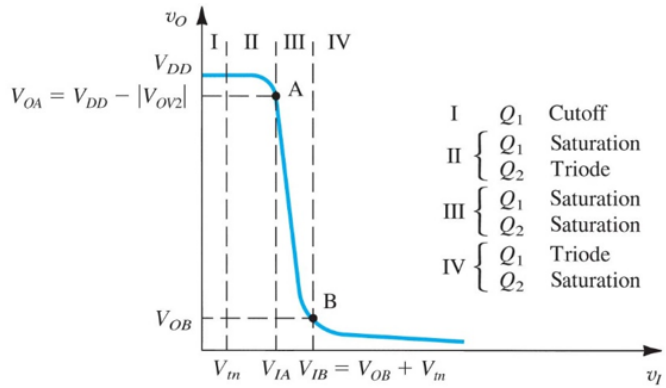
(a)



(b)



(c)

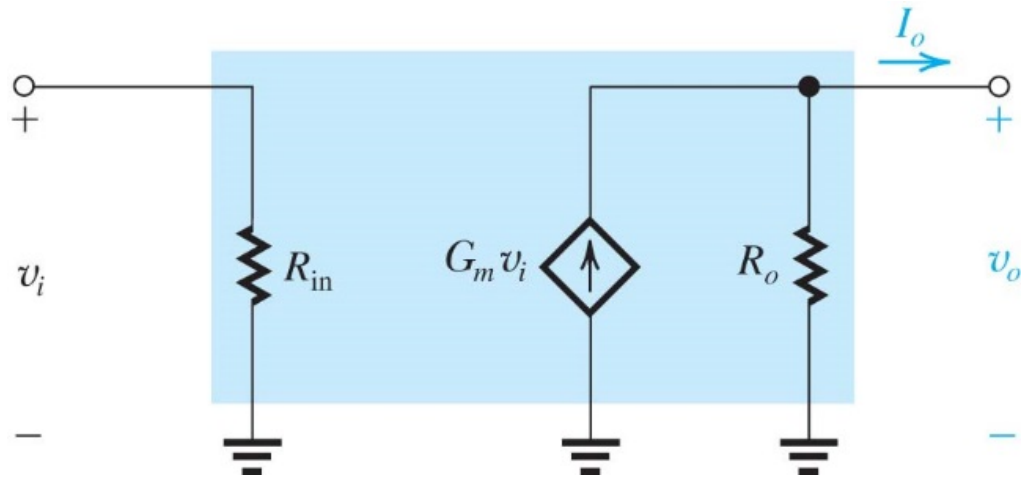


(d)

CS summary

R_{in}	∞
R_o	r_o
G_m	g_m
A	$-g_m(r_o \| \| R_L)$

In a general amplifier model:

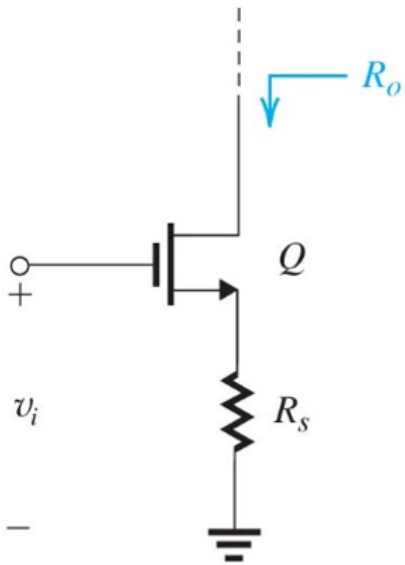


In general we want to find:

$\frac{v_i}{i_i} = R_{in}$ while v_o connected via R_L to Gnd
$\frac{v_o}{i_o} = R_o$ while v_i connected via R_S to Gnd
$\frac{i_o}{v_i} = G_m$ while v_o connected to Gnd
$\frac{v_o}{v_i} = A$ while v_o connected via R_L to Gnd

CS with source degeneration no R_L

Good as current source, e.g in current mirror, but not so good as amplifier.



$$R_o = R_s + r_o + g_m r_o R_s$$
$$R_o \simeq (1 + g_m R_s)\, r_o$$

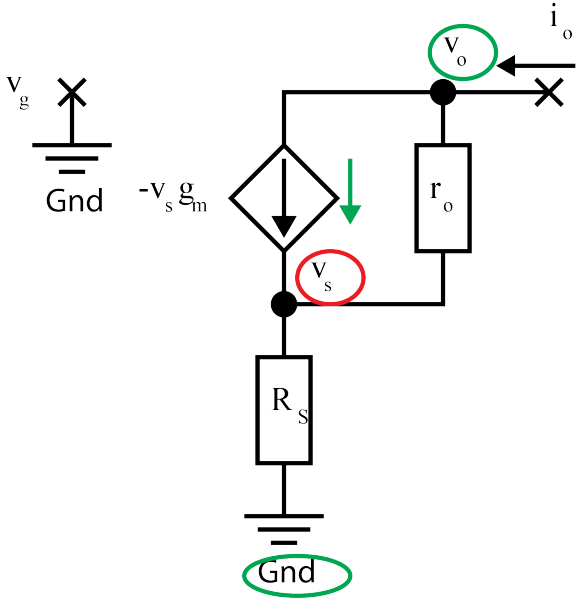
$$R_O \approx (1 + g_m R_S) r_o$$

$$g_m \rightarrow G_m = \frac{g_m}{1 + g_m R_s}$$

Much higher output resistance. Still no net-increase in gain!

Derivation of R_O (1/3)

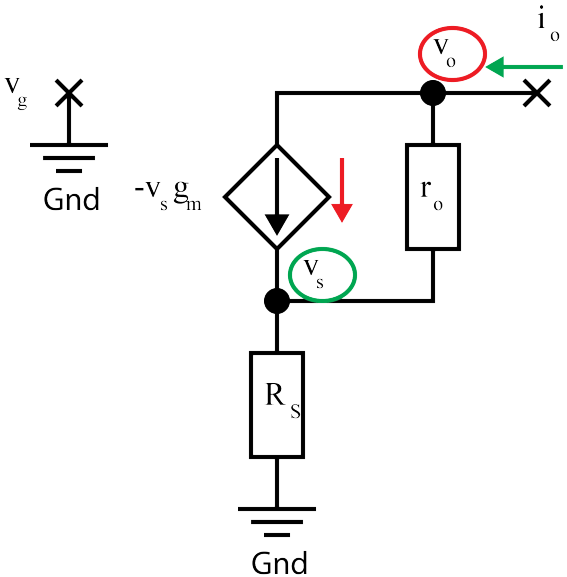
Kirchoff for node v_s:



$$v_s\left(\frac{1}{R_S}+\frac{1}{r_o}\right)=(0-v_s)g_m+0\frac{1}{R_S}+v_o\frac{1}{r_o}$$

Derivation of R_O (2/3)

Kirchoff for node v_o:



$$v_o \frac{1}{r_o} + (0 - v_s) g_m = v_s \frac{1}{r_o} + i_o$$

Derivation of R_O (3/3)

From node v_s:

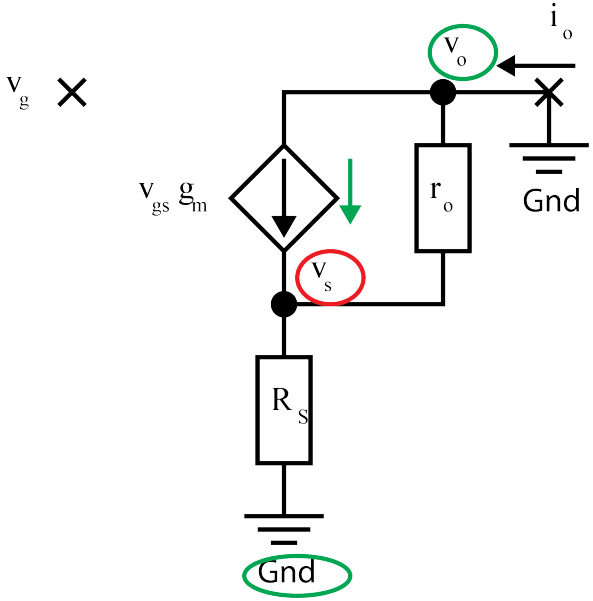
$$v_s = v_o \frac{\frac{1}{r_o}}{\frac{1}{R_S} + \frac{1}{r_o} + g_m}$$

Substitute into equation for node v_o:

$$\begin{aligned}
v_o \frac{1}{r_o} - v_s g_m &= v_s \frac{1}{r_o} + i_o \\
i_o &= v_o \frac{1}{r_o} - v_s (g_m + \frac{1}{r_o}) \\
\frac{i_o}{v_o} = \frac{1}{R_o} &= \frac{1}{r_o} - \left(\frac{\frac{1}{r_o}}{\frac{1}{R_S} + \frac{1}{r_o} + g_m} \right) (g_m + \frac{1}{r_o}) \\
&= \frac{\frac{1}{r_o}\left(\frac{1}{R_S} + \frac{1}{r_o} + g_m\right) - \frac{1}{r_o}(g_m + \frac{1}{r_o})}{\left(\frac{1}{R_S} + \frac{1}{r_o} + g_m\right)} \\
&= \frac{1}{(r_o + R_S + r_o R_S g_m)} \overset{R_S << r_o}{\approx} \frac{1}{r_o(1 + R_S g_m)}
\end{aligned}$$

Derivation of G_m (1/3)

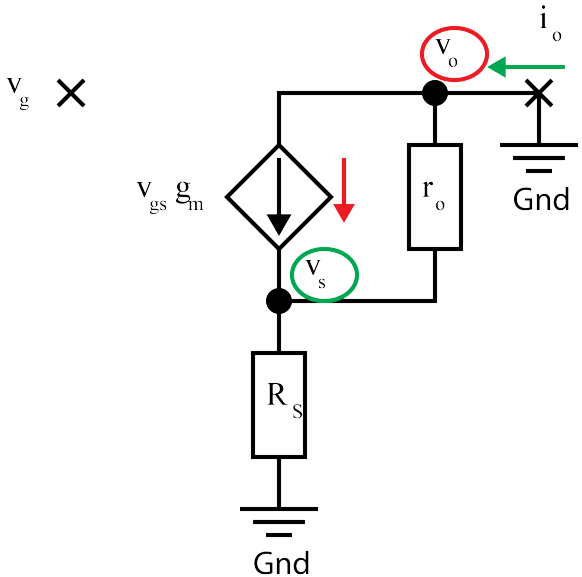
Kirchoff for node v_s:



$$v_s\left(\frac{1}{R_S}+\frac{1}{r_o}\right)=(v_g-v_s)g_m+0\frac{1}{R_S}+0\frac{1}{r_o}$$

Derivation of G_m (2/3)

Kirchoff for node v_o:



$$0\frac{1}{r_o} + (v_g - v_s)g_m = v_s\frac{1}{r_o} + i_o$$

Derivation of G_m (3/3)

From node v_s:

$$v_s = v_g \frac{g_m}{\left(\frac{1}{R_S} + \frac{1}{r_o} + g_m\right)}$$

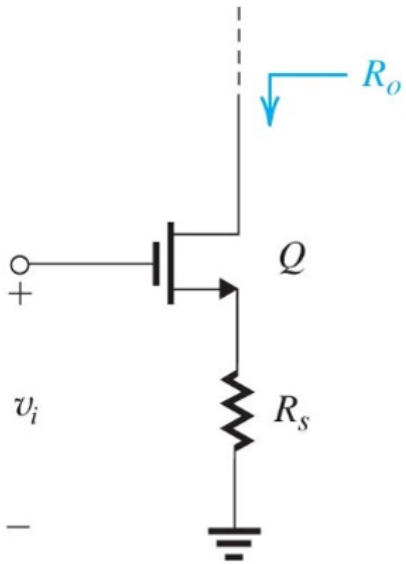
Substitute into equation for node v_o:

$$\begin{aligned}
i_o &= v_g g_m - v_s\left(g_m + \frac{1}{r_o}\right) \\
G_m &= \frac{i_o}{v_g} = g_m - \frac{g_m}{\left(\frac{1}{R_S} + \frac{1}{r_o} + g_m\right)}\left(g_m + \frac{1}{r_o}\right) \\
&= g_m \frac{\left(\frac{1}{R_S} + \frac{1}{r_o} + g_m\right) - \left(g_m + \frac{1}{r_o}\right)}{\left(\frac{1}{R_S} + \frac{1}{r_o} + g_m\right)} \\
&= g_m \frac{1}{\left(1 + \frac{R_S}{r_o} + R_S g_m\right)}
\end{aligned}$$

CS with source degeneration summary

R_I	∞
R_o	$r_o + R_S(g_m r_o + 1) \approx r_o(1 + R_S g_m)$
G_M	$g_m \frac{1}{1+R_S\left(g_m+\frac{1}{r_o}\right)} \approx g_m \frac{1}{1+R_S g_m}$
A	$-G_M(R_o \| \| R_L) = g_m r_o \frac{R_L}{R_L+R_o}$

CS with source degeneration with R_L



$R_o = R_s + r_o + g_m r_o R_s$

$R_o \simeq (1 + g_m R_s)\, r_o$

$$A_v = -G_m R_O \frac{R_L}{R_L + R_O}$$

$$\approx -g_m r_o \frac{R_L}{R_L + (1 + g_m R_S) r_o}$$

... and more degradation to A_v when load $R_L < g_m R_S r_o$!

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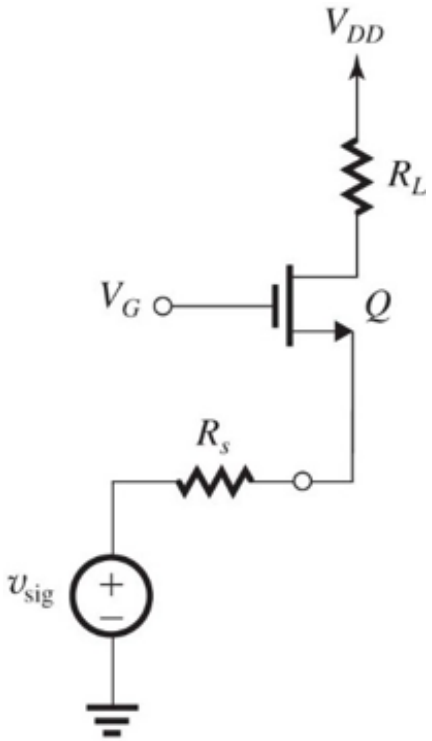
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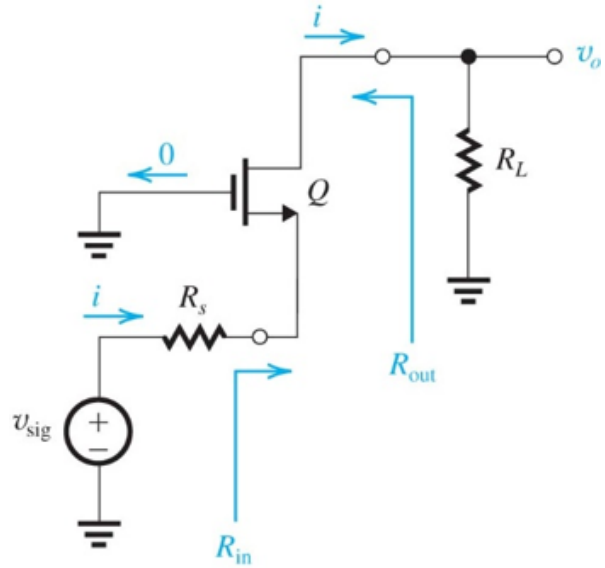
Common Drain Stage/Source Follower (Book 6.3)

Improved Current Mirrors/Sources (book 7.6)

CG amplifier

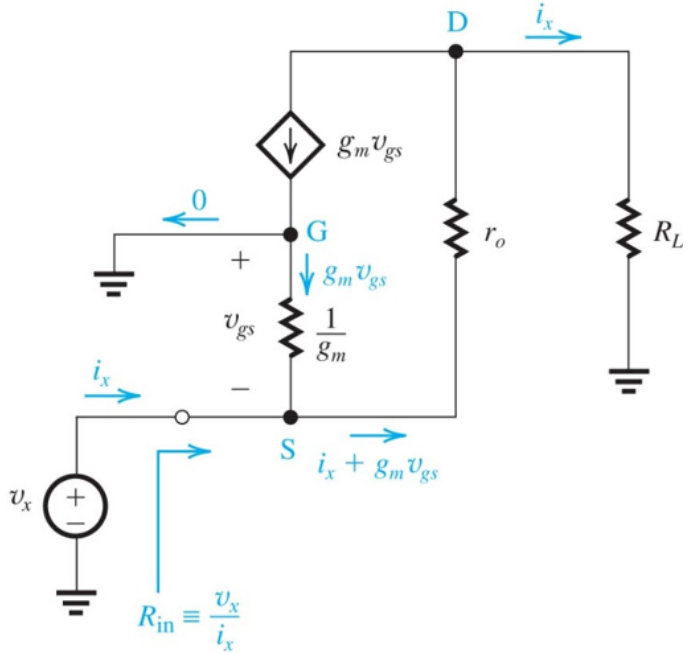


(a)



(b)

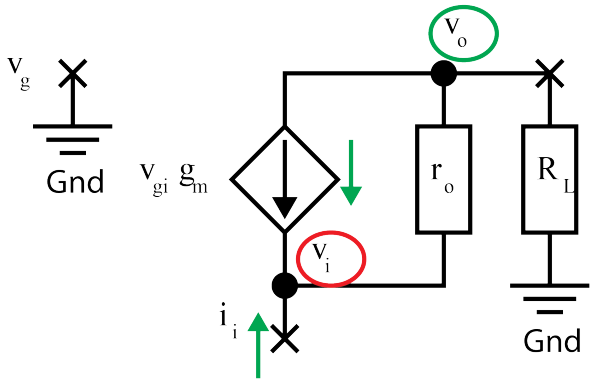
R_{in} of a CG amplifier (book)



$$v_x = (i_x + g_m v_{gs}) r_o + i_x R_L \Rightarrow R_{in} = \frac{r_o + R_L}{1 + g_m r_o} \approx \frac{1}{g_m} + \frac{R_L}{g_m r_o} \quad (7.53, 7.54)$$

Derivation of R_i (Philipp) (1/3)

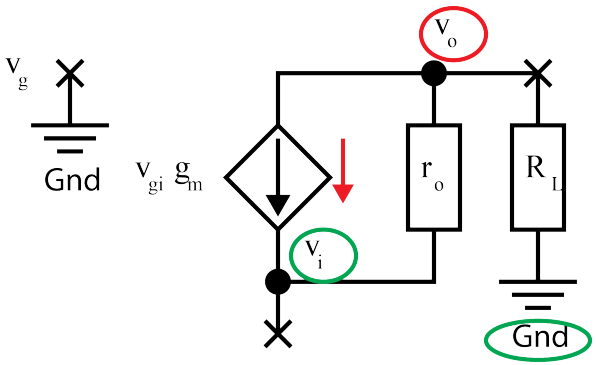
Kirchoff for node v_s(i.e. v_i):



$$v_i \frac{1}{r_o} = i_i - v_i g_m + v_o \frac{1}{r_o}$$

Derivation of R_i (Philipp) (2/3)

Kirchoff for node v_o:



$$v_o \left(\frac{1}{r_o} + \frac{1}{R_L} \right) - v_i g_m = v_i \frac{1}{r_o} + 0 \frac{1}{R_L}$$

Derivation of R_i (Philipp)(3/3)

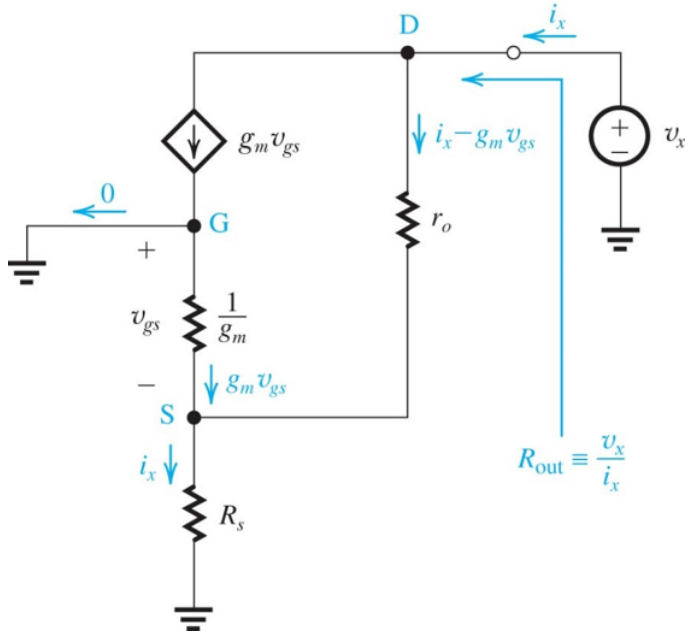
From node v_o:

$$v_o = v_i \left(\frac{\frac{1}{r_o} + g_m}{\frac{1}{r_o} + \frac{1}{R_L}} \right)$$

Substitute into equation for node v_i:

$$\begin{aligned} i_i &= v_i \left(\frac{1}{r_o} + g_m \right) - v_o \frac{1}{r_o} \\ i_i &= v_i \left[\left(\frac{1}{r_o} + g_m \right) - \left(\frac{\frac{1}{r_o} + g_m}{\frac{1}{r_o} + \frac{1}{R_L}} \right) \frac{1}{r_o} \right] \\ \frac{i_i}{v_i} = \frac{1}{R_i} &= \left(\frac{g_m(\frac{1}{r_o} + \frac{1}{R_L})}{\frac{1}{r_o} + \frac{1}{R_L}} \right) + \left(\frac{\frac{1}{r_o}(\frac{1}{R_L} - g_m)}{\frac{1}{r_o} + \frac{1}{R_L}} \right) \\ &= \frac{\frac{g_m}{R_L} + \frac{1}{R_L r_o}}{\frac{1}{r_o} + \frac{1}{R_L}} = \frac{g_m r_o + 1}{R_L + r_o} \end{aligned}$$

R_{out} of a CG amplifier

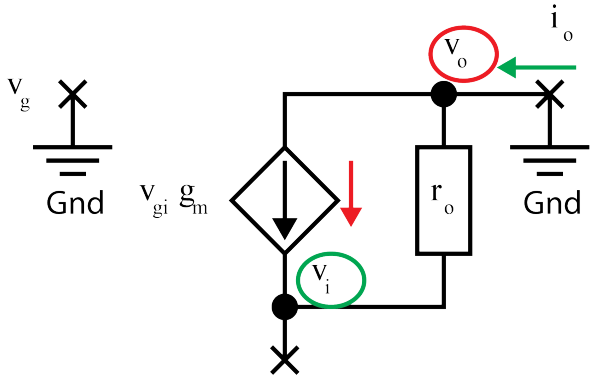


$$v_x = (i_x - g_m v_{gs}) r_o + i_x R_S \;\Rightarrow\; R_{out} \approx r_o + (g_m r_o) R_s \quad (7.58)$$

(Philipp: see derivation of CS with source degeneration. It's exactly the same!)

Derivation of G_m (Philipp) (1/2)

Kirchoff for node v_o:



$$0\frac{1}{r_o} - v_i g_m = i_o + v_i \frac{1}{r_o}$$

$$\frac{i_o}{v_i} = G_m = -(g_m + \frac{1}{r_o})$$

CG summary

R_I no R_L	∞
R_I	$\frac{r_o+R_L}{1+g_m r_o} \approx \frac{1}{g_m} + \frac{R_L}{g_m r_o}$
R_O no R_S	r_o
R_O	$r_o + R_S(g_m r_o + 1) \approx r_o(1 + R_S g_m)$
$G_M = \frac{i_o}{v_i}$	$-(\frac{1}{r_o} + g_m)$
$G_M = \frac{i_o}{v_{sig}}$	$-\frac{g_m+\frac{1}{r_o}}{1+R_S\left(g_m+\frac{1}{r_o}\right)}$
$A_0 = \frac{v_o}{v_i} = \frac{v_o}{v_{sig}} = -G_M R_O$	$(1 + g_m r_o) \approx g_m r_o$
$A = -G_M(R_O\|\|R_L)$	$(1 + g_m r_o)\frac{R_L}{R_L+R_O}$

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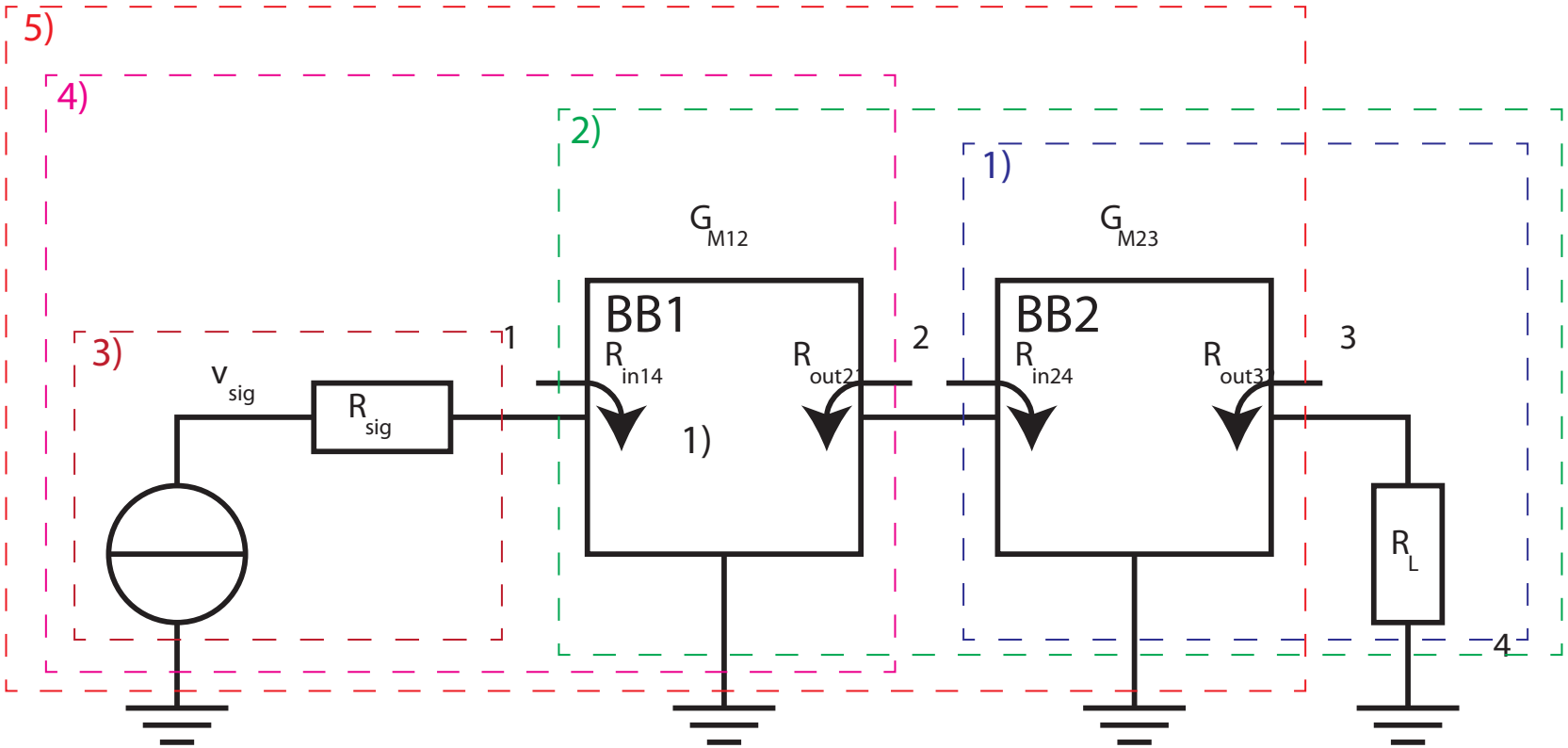
Common Gate (CG) Configuration (book 7.4-7.5)

Cascode Configuration (CG and CS in series) (book 7.4-7.5)

Common Drain Stage/Source Follower (Book 6.3)

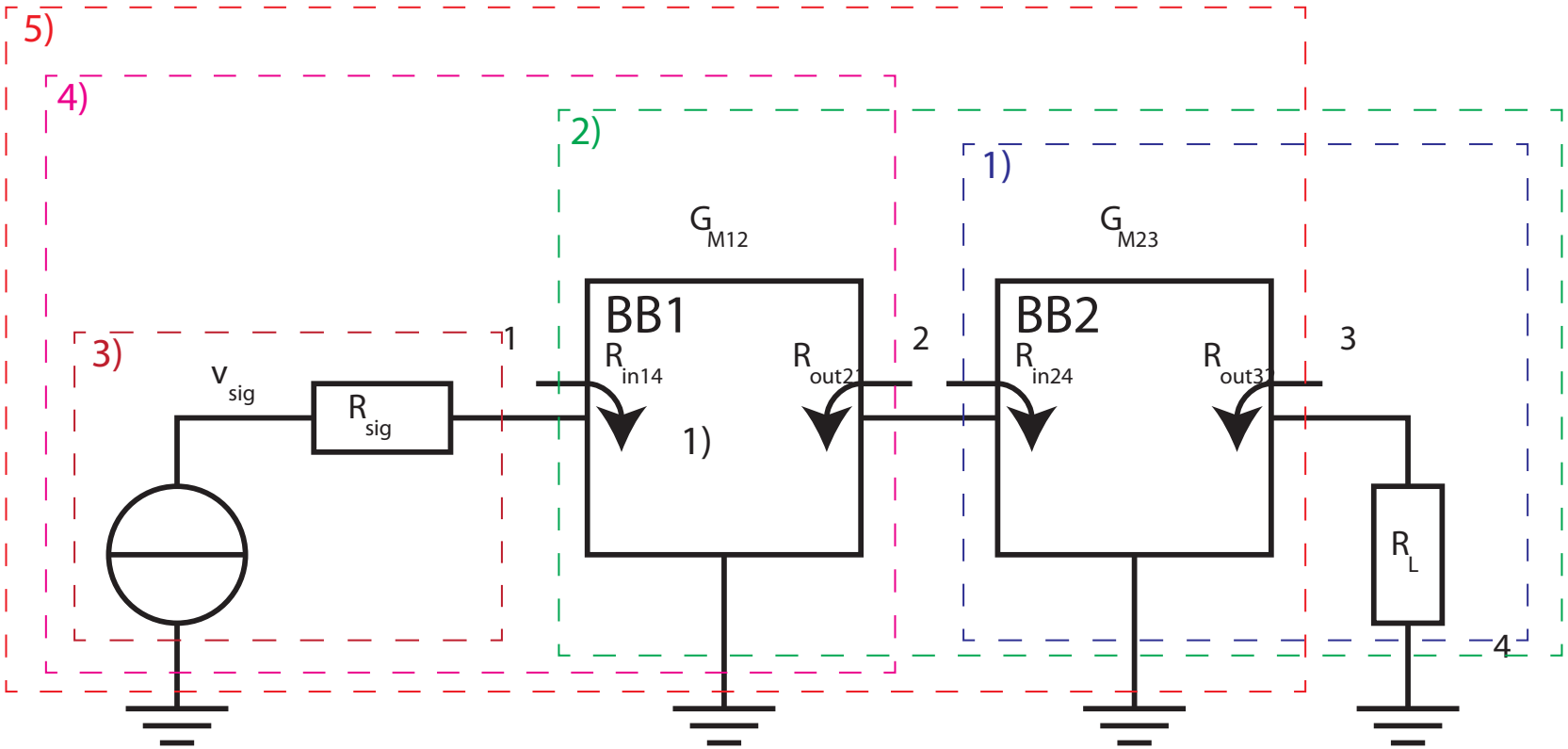
Improved Current Mirrors/Sources (book 7.6)

Cascading Filters in General, total A (1/3)



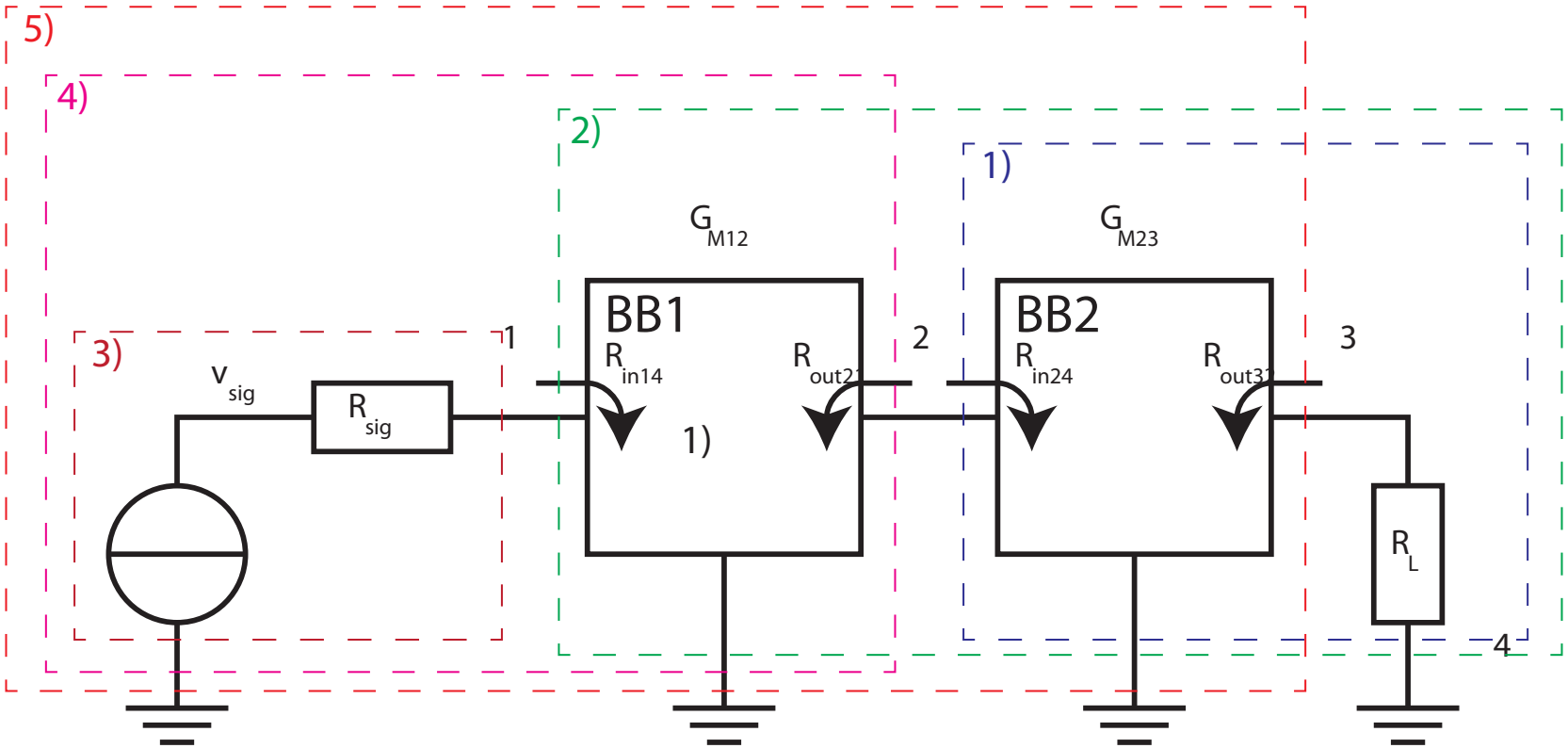
1. Compute Norton/Thévenin equivalent small signal resistance R_{in24} between nodes 2 and 4
2. Compute small signal R_{in14} between nodes 1 and 4 replacing 1) with R_{in24}

Cascading Filters in General, total A (2/3)



3. Compute small signal gain $A_1 = \frac{v_1}{v_{sig}} = v_{sig} \frac{R_{in14}}{R_{in14}+R_{sig}}$
4. Compute small signal intrinsic (!) transconductance G_{M12} and output resistance R_{out21} with ideal voltage source input $v_1 = A_1 v_{sig}$ and $A_2 = \frac{v_2}{v_1} = G_{M12} \frac{R_{in24}}{R_{in24}+R_{out21}}$

Cascading Filters in General, total A (3/3)



5. Compute small signal transconductance G_{M23} and output resistance R_{out32} with ideal voltage source input $v_2 = A_1 A_2 v_{sig}$ and $A_3 = \frac{v_3}{v_2} = G_{M23} \frac{R_L}{R_L + R_{out32}}$

$$A = \frac{v_3}{v_{sig}} = \frac{R_{in14}}{R_{in14} + R_{sig}} A_1 A_2 A_3 \qquad (1)$$

Cascode gain stage as CS and CG in series (1/2)

$$A_{cas} = A_{CS}A_{CG} = A_1 A_2 \tag{2}$$

$$A_{CS} = A_1 = g_{m1} r_{o1} \frac{R_{I2}}{R_{I2} + r_{o1}} \tag{3}$$

$$A_{CG} = A_2 = -(1 + g_{m2} r_{o2}) \frac{R_L}{R_L + r_{o2}} \tag{4}$$

For A_{CG} we have set $R_S = 0$ as we set $v_1 = v_i A_{CS}$, i.e. an ideal voltage source with $R_S = 0$ ...

$$A_{cas} = \underbrace{-g_{m1} r_{o1}(1 + g_{m2} r_{o2})}_{A_{cas0}} \frac{R_{I2}}{R_{I2} + r_{o1}} \frac{R_L}{R_L + r_{o2}} \tag{5}$$

$$= A_{cas0} \frac{\frac{r_{o2}+R_L}{1+g_{m2} r_{o2}} R_L}{\left(\frac{r_{o2}+R_L}{1+g_{m2} r_{o2}} + r_{o1}\right)(R_L + r_{o2})} \tag{6}$$

$$= A_{cas0} \frac{R_L}{R_L + r_{o1} + r_{o2} + g_{m2} r_{o1} r_{o2}} \tag{7}$$

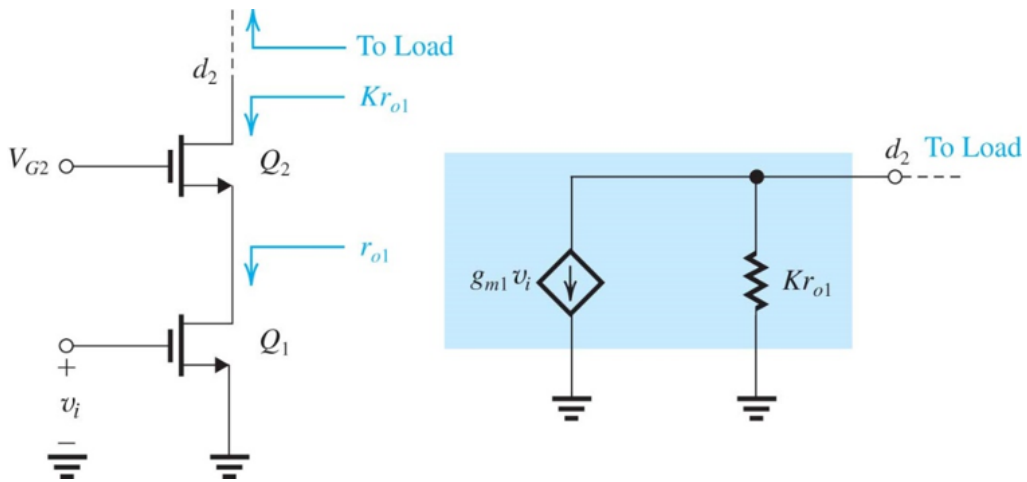
Cascode gain stage as CS and CG in series (2/2)

According to (7)

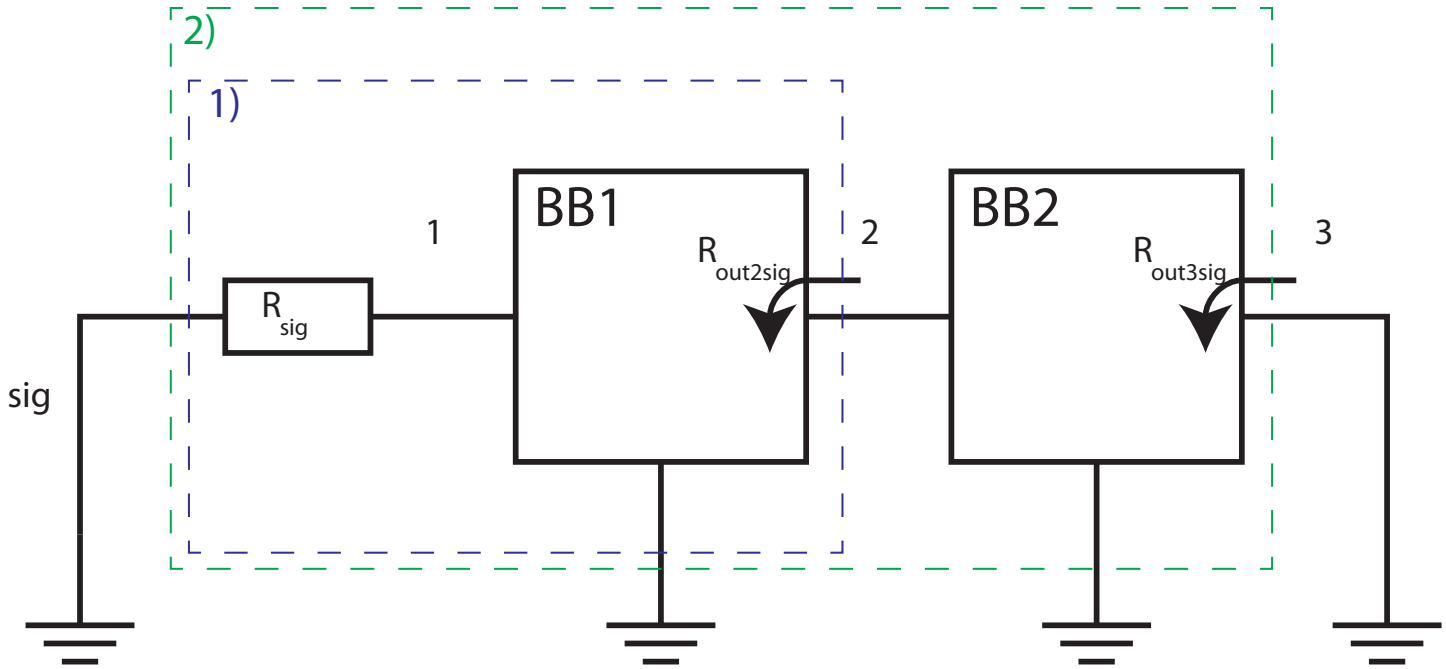
R_L	$A_{cas} = A_{cas0} * \ldots$
∞	1
$g_{m2} r_{o2}^2$	$\approx \frac{r_{o2}}{r_{o1}+r_{o2}} \approx \frac{1}{2}$
r_{o2}	$\approx \frac{1}{g_{m2} r_{o2}}$

Cascode Amplifier

The book does a slightly different approach looking at the undivided small signal analysis of both CS and GC stages together, arguing that the total transconductance $G_M \approx g_{m1}$ And that the total output resistance is much increased to $R_O \approx g_{m2} r_{o1} r_{o2}$ giving the same net result.

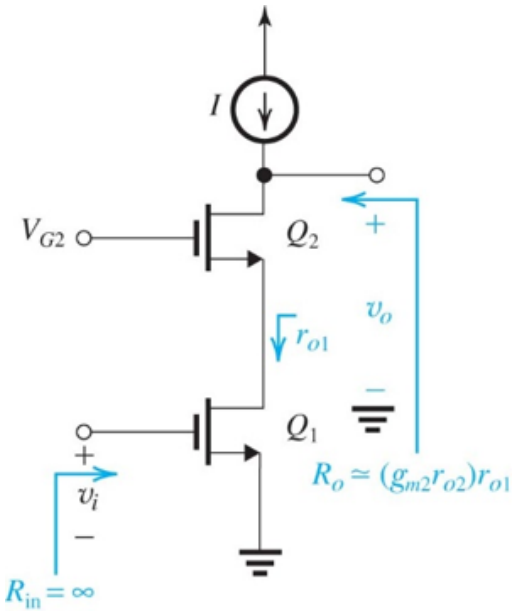


Cascading Filters in General, total R_O

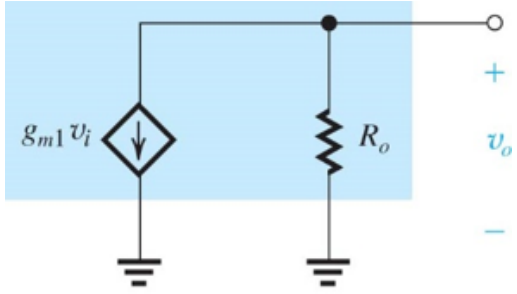


1. Compute Norton/Thévenin equivalent small signal resistance $R_{out2sig}$ between nodes 2 and v_{sig} (for small signal purposes $v_{sig} = 0$)
2. Compute small signal $R_{out3sig}$ between nodes 4 and v_{sig} using $R_{out2sig}$

Cascode Amplifier with Infinite Load Resistance



(a)



(b)

Alternative: Do small signal analysis without separating into stages

confirmimng with global (!) G_M and R_O

$$G_M = \frac{g_{m1}(g_{m2} + \frac{1}{r_{o2}})}{g_{m2} + \frac{1}{r_{o2}} + \frac{1}{r_{o1}}} \quad (8)$$

$$R_O = r_{o2} + r_{o1}(g_{m2} r_{o2} + 1) \quad (9)$$

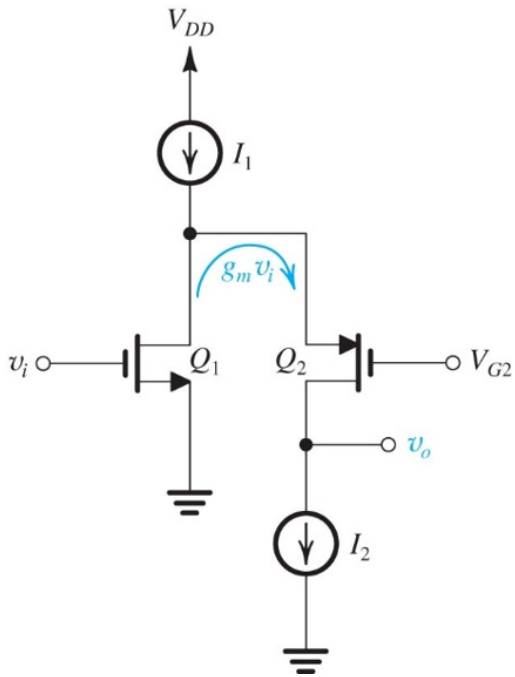
$$G_M R_O = \frac{g_{m1}(g_{m2} + \frac{1}{r_{o2}})}{g_{m2} + \frac{1}{r_{o2}} + \frac{1}{r_{o1}}}(r_{o2} + r_{o1}(g_{m2} r_{o2} + 1)) \quad (10)$$

$$= g_{m1} g_{m2} \frac{(1 + \frac{1}{r_{o2} g_{m2}})}{g_{m2} + \frac{1}{r_{o2}} + \frac{1}{r_{o1}}} r_{o1} r_{o2}(\frac{1}{r_{o2}} + \frac{1}{r_{o1}} + g_{m2}) \quad (11)$$

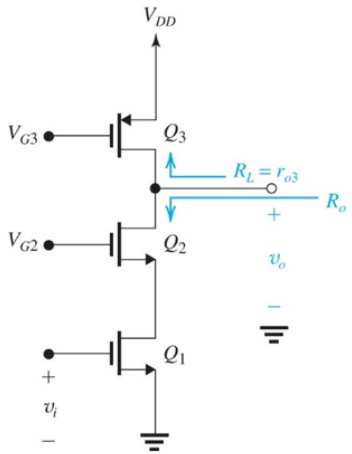
$$= g_{m1} g_{m2}(1 + \frac{1}{r_{o2} g_{m2}}) r_{o1} r_{o2} \quad (12)$$

$$= g_{m1} r_{o1}(1 + g_{m2} r_{o2}) \quad (13)$$

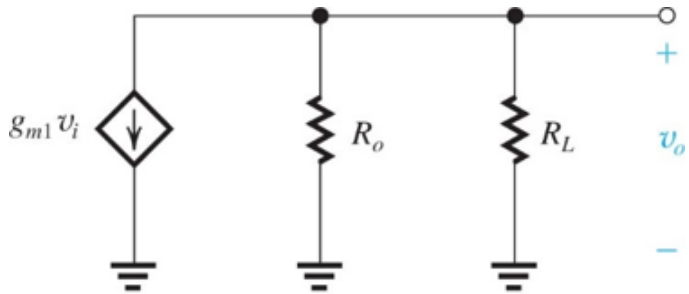
Folded Cascode with Infinite Load Resistance



Cascode Amplifier with Finite Load Resistance (1/2)



(a)

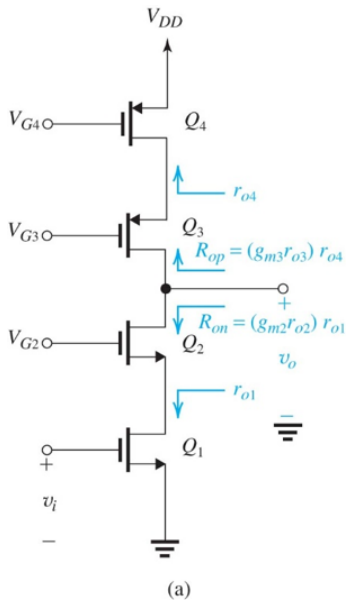


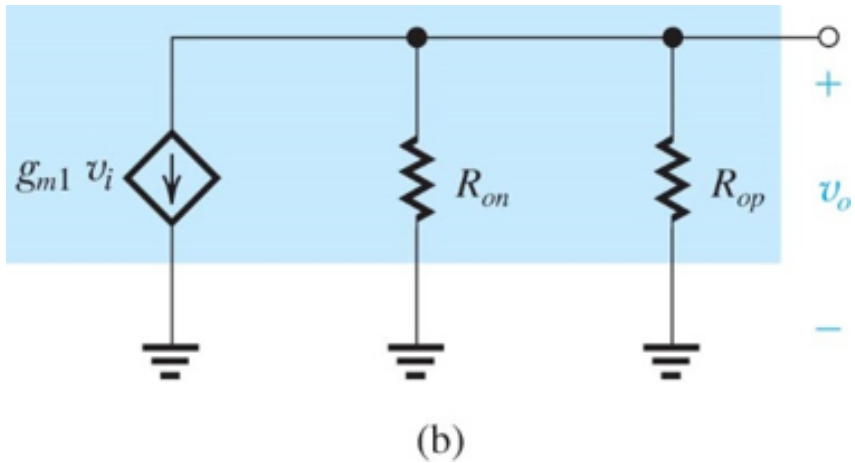
(b)

The load R_L must be of equal magnitude as R_O to get the benefit of the increased gain A_V! So here with a simple pFET we are back to square one.

Cascode Amplifier with Finite Load Resistance (2/2)

Better: employ a cascoded current source.





Dependency of Gain on R_L

Table 7.1 Gain Distribution in the MOS Cascode Amplifier for Various Values of R_L

Case	R_L	R_{in2}	R_{d1}	A_{v1}	A_{v2}	A_v
1	∞	∞	r_o	$-g_m r_o$	$g_m r_o$	$-(g_m r_o)^2$
2	$(g_m r_o) r_o$	r_o	$r_o/2$	$-\frac{1}{2}(g_m r_o)$	$g_m r_o$	$-\frac{1}{2}(g_m r_o)^2$
3	r_o	$\frac{2}{g_m}$	$\frac{2}{g_m}$	-2	$\frac{1}{2}(g_m r_o)$	$-(g_m r_o)$
4	0	$\frac{1}{g_m}$	$\frac{1}{g_m}$	-1	0	0

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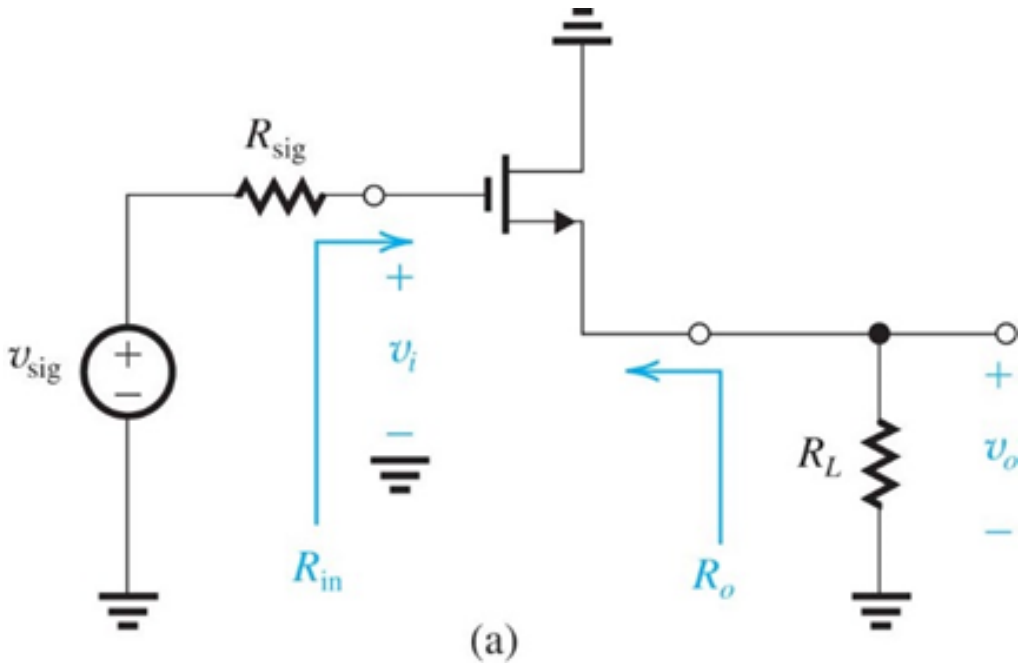
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Cascode Configuration (CG and CS in series) (book 7.4-7.5)

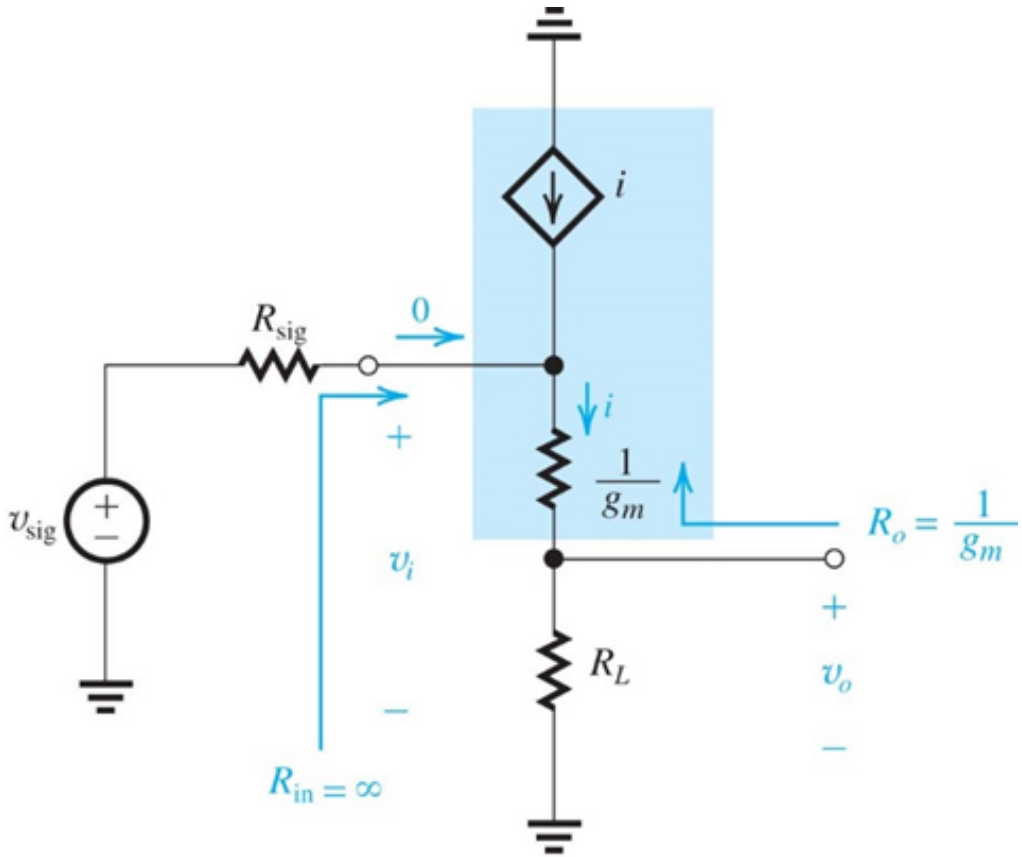
Common Drain Stage/Source Follower (Book 6.3)

Improved Current Mirrors/Sources (book 7.6)

Common Drain Amplifier/Source Follower



Common Drain Amplifier/Source Follower



(b)

$$A = \frac{R_L}{R_L + \frac{1}{g_m}}$$

$$R_O = \frac{1}{g_m}$$

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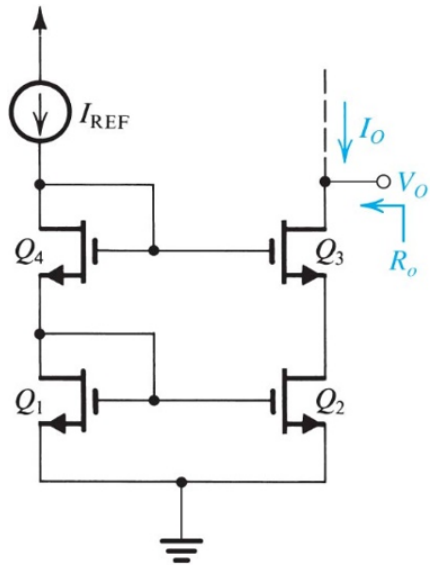
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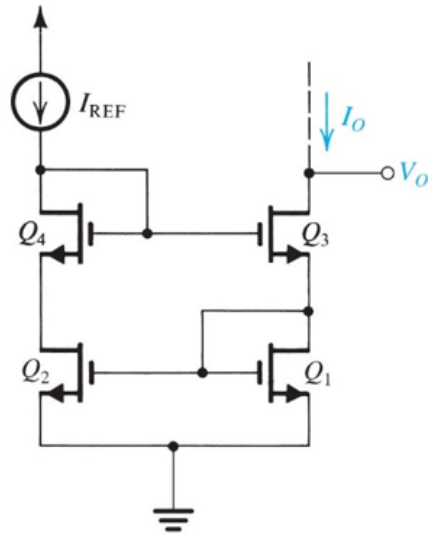
Improved Current Mirrors/Sources (book 7.6)

Cascode Current Mirror



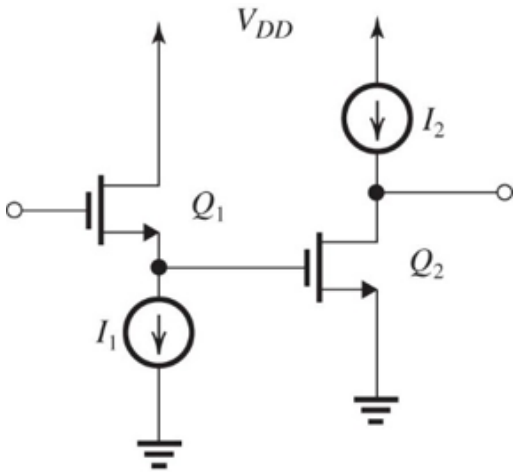
Increased output impedance, but quite a bit of output voltage headroom necessary ...

Modified Wilson MOS Current Mirror



(c)

A CD-CS Amplifier



(b)

Larger bandwidth than simple CS amplifier (explained later in chapter 9).