IN3170/4170, Spring 2023

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Excerpt of Sedra/Smith Chapter 8: Differential and Multistage CMOS Amplifier Basics

Content

The MOS Differential Pair (book 8.1)

Common Mode Rejection and Random DC offset (book 8.3-8.4)

Current Mirror Load (book 8.5)

Multi Stage Amplifiers (book 8.6)

Content

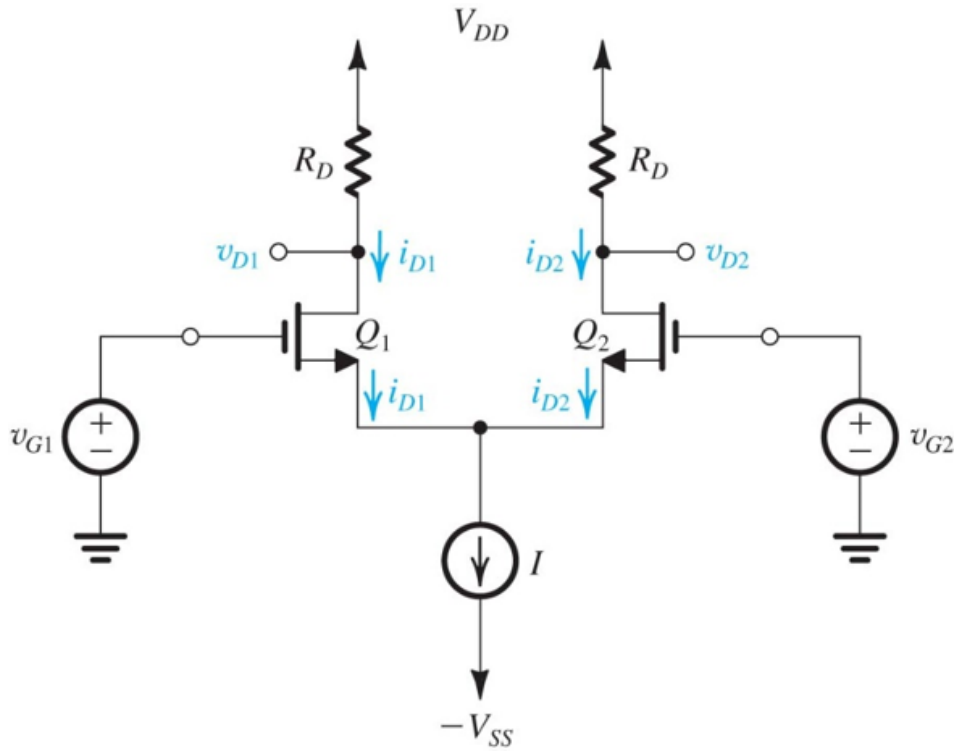
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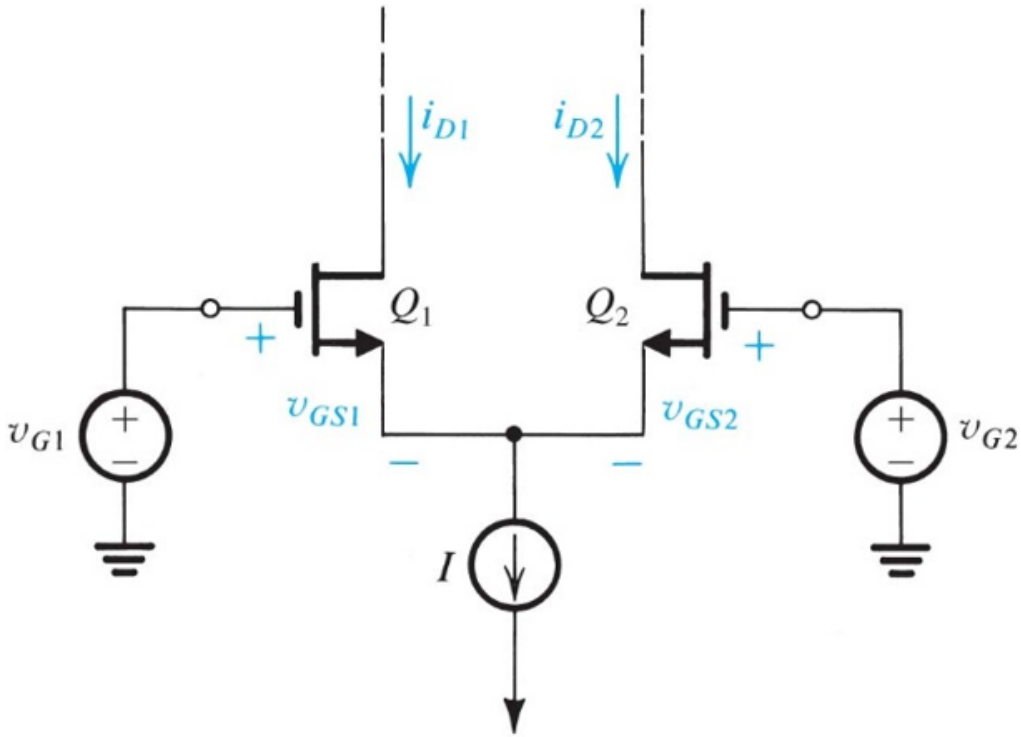
Multi Stage Amplifiers (book 8.6)

The differential pair with resistive loads



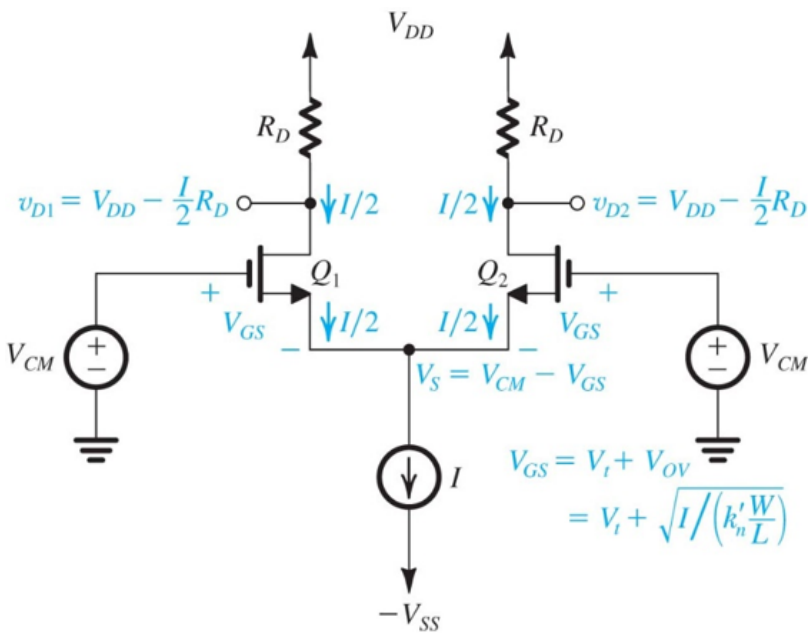
The resistors turn I_D/i_d linearly into voltage

Easier (in my opinion): look simply at i_d/I_D



Analysis for Common Mode Input

The common mode input V_{CM} is the large signal average of the two inputs. If too small or too big, the diff-pair does not work as intended. In-between the limits, it might also have a small influence on the behaviour (see later 'common mode rejection ratio' (CMRR)), leading to deviation from an ideal differential behaviour.



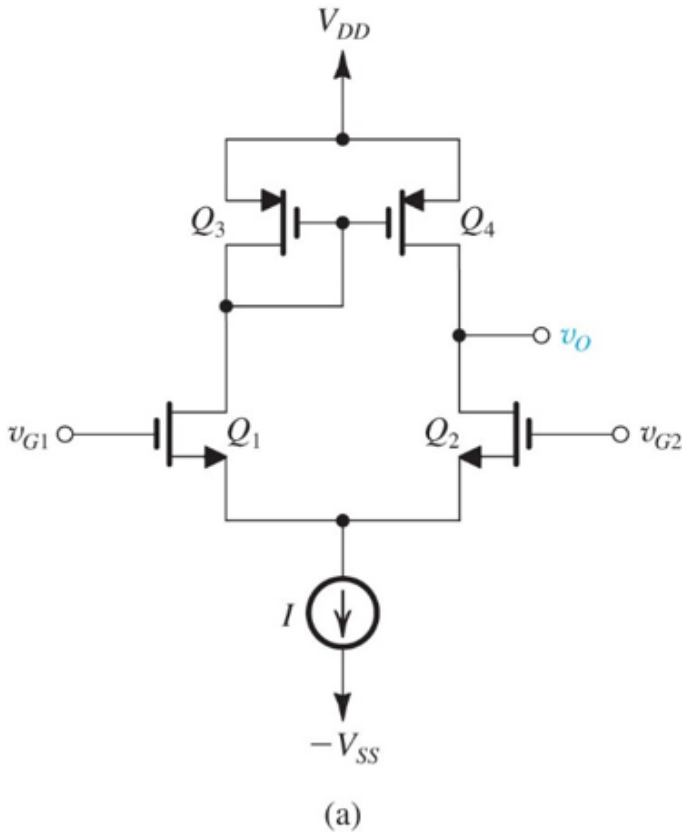
With ideal current source: common mode voltage V_{CM} has no effect in the nominal range of operation. Range limits:

$$V_{CMmax} = V_t + V_{DD} - \frac{I}{2}R_D \quad (8.7)$$

$$V_{CMmin} = -V_{SS} + V_{CS} + V_t + V_{OV} \quad (8.8$$

(Note: the book always writes $-V_{SS}$ for lower supply, i.e. always expresses V_{SS} as a positive number...)

With current mirror



(a)

When the output is connected to a voltage source, the output current becomes the difference of the two i_d

Large Signal, Weak Inversion

Simpler analysis in weak inversion:

$$I_b = I_1 + I_2 = I_S e^{\frac{-V_t - nV_S}{nV_T}} \left(e^{\frac{V_1}{nV_T}} + e^{\frac{V_2}{nV_T}} \right)$$

$$I_{out} = I_1 - I_2 = I_S e^{\frac{-V_t - nV_S}{nV_T}} \left(e^{\frac{V_1}{nV_T}} - e^{\frac{V_2}{nV_T}} \right)$$

$$\frac{I_{out}}{I_b} = \frac{e^{\frac{V_1}{nV_T}} - e^{\frac{V_2}{nV_T}}}{e^{\frac{V_1}{nV_T}} + e^{\frac{V_2}{nV_T}}}$$

$$I_{out} = I_b \frac{e^{\frac{V_1}{nV_T}} - e^{\frac{V_2}{nV_T}}}{e^{\frac{V_1}{nV_T}} + e^{\frac{V_2}{nV_T}}} = I_b \tanh \frac{V_1 - V_2}{2nV_T}$$

Small Signal, Weak Inversion

Since the slope of $\tanh x$ for $x = 0$ is 1, the slope of $I_b \tanh \frac{\Delta V}{2nV_T}$ with respect to ΔV (the transconductance g of this transamp) is:

$$g = \frac{I_b}{2nV_T}$$

The point where the linear approximation reaches $I_{out} = I_b$, i.e. an approximation of the limit of the linear region is thus at:

$$\Delta V_{in} = \pm 2nV_T \tag{1}$$

Note that his limit is independent of I_b, which is different for strong inversion!

Large Signal, Strong Inversion (1/2)

$$I_b = I_1 + I_2 = \frac{k_n}{2}(V_{OV1}^2 + V_{OV2}^2)$$

$$I_{out} = I_1 - I_2 = \frac{k_n}{2}(V_{OV1}^2 - V_{OV2}^2)$$

$$\frac{I_{out}}{I_b} = \frac{V_{OV1}^2 - V_{OV2}^2}{V_{OV1}^2 + V_{OV2}^2}$$

$$I_{out} = I_b \frac{V_{OV1}^2 - V_{OV2}^2}{V_{OV1}^2 + V_{OV2}^2}$$

Large Signal, Strong Inversion (2/2)

Rewrite with $\hat{V}_{OV} = \frac{V_{OV1}+V_{OV2}}{2}$ and $\Delta V_{OV} = V_{OV1} - V_{OV2}$

$$I_{out} = I_b \frac{2\Delta V_{OV}\hat{V}_{OV}}{\frac{1}{2}\left(\Delta V_{OV}^2 + (2\hat{V}_{OV})^2\right)}$$

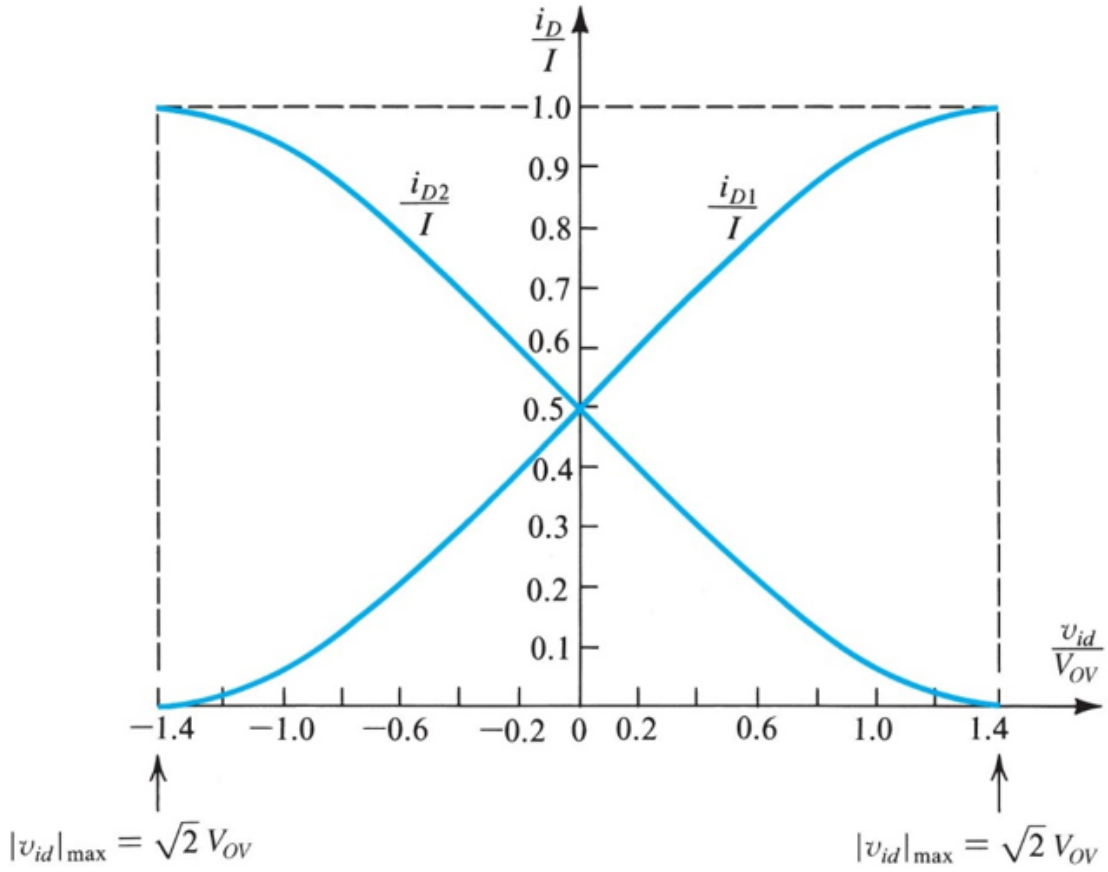
Note that this is not yet a closed solution as in the large signal world V_S, and thus $\hat{V}_{OV}$ depends on ΔV_{OV}. Still it can reveal a few properties:

Extrema where one transistor conducts the entire I_B is where $\Delta V_{OV} = 2\hat{V}_{OV}$ (since then one branch has $V_{OV2} = 0$) and $I_B = k_n \Delta V_{OV}^2$. It follows that at that point (i.e. the extrema of an estimate for the linear region):

$$\Delta V_{OV} = \pm\sqrt{\frac{I_B}{k_n}} = \pm\sqrt{2}V_{OV0} \quad (8.10)$$

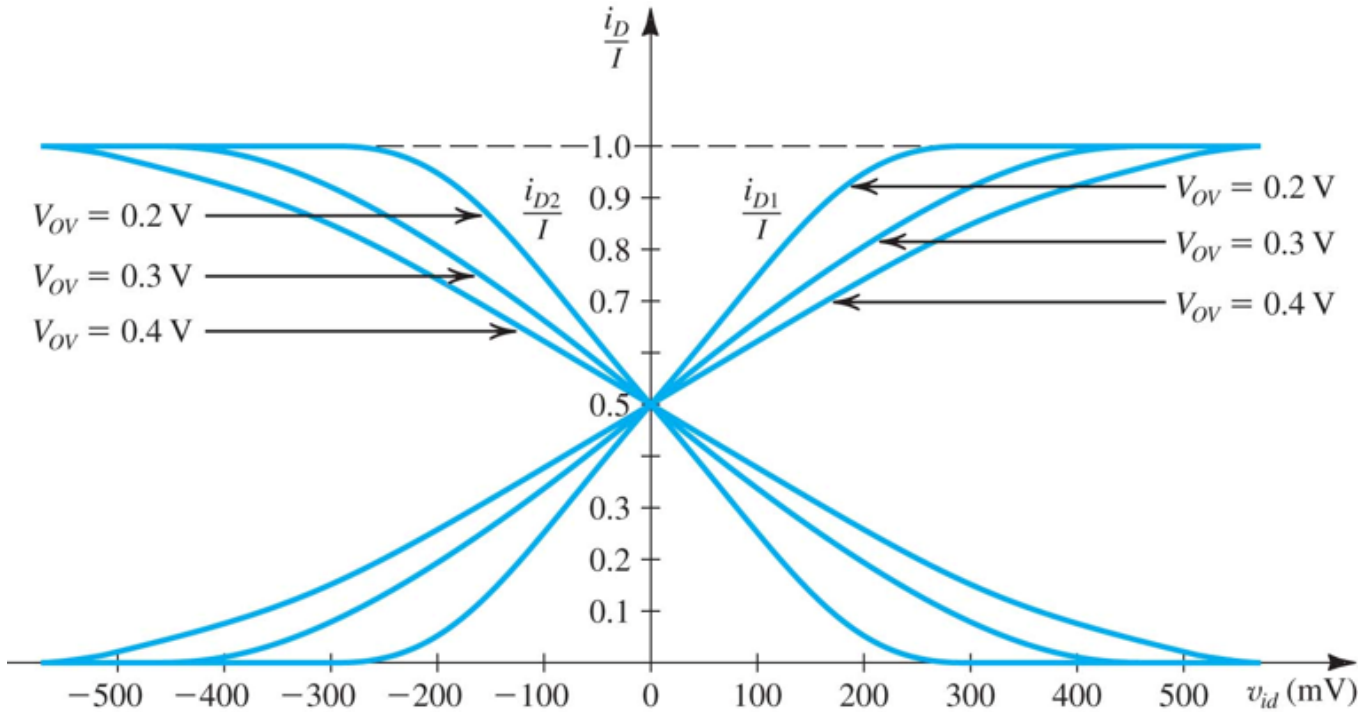
Where V_{OV0} is the overdrive voltage for $\Delta V_{OV} = 0$, i.e. when both inputs are equal. (Note: The book writes only V_{OV} for that!)

Normalized I/V Curves and Ranges



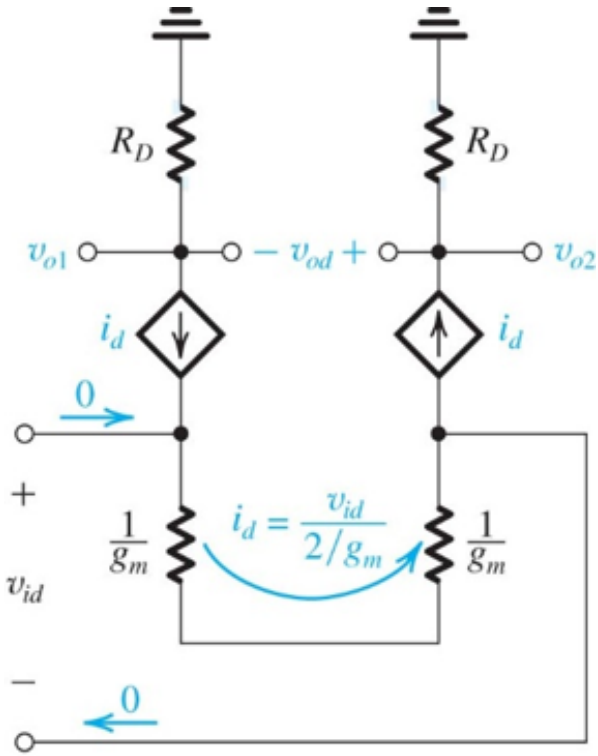
I/V Curves for different V_{OV} respectively $\frac{W}{L}$

(This is only valid for strong inversion)



Note that here in strong inversion (!) the linear range depends on I_B, i.e. on V_{OV}

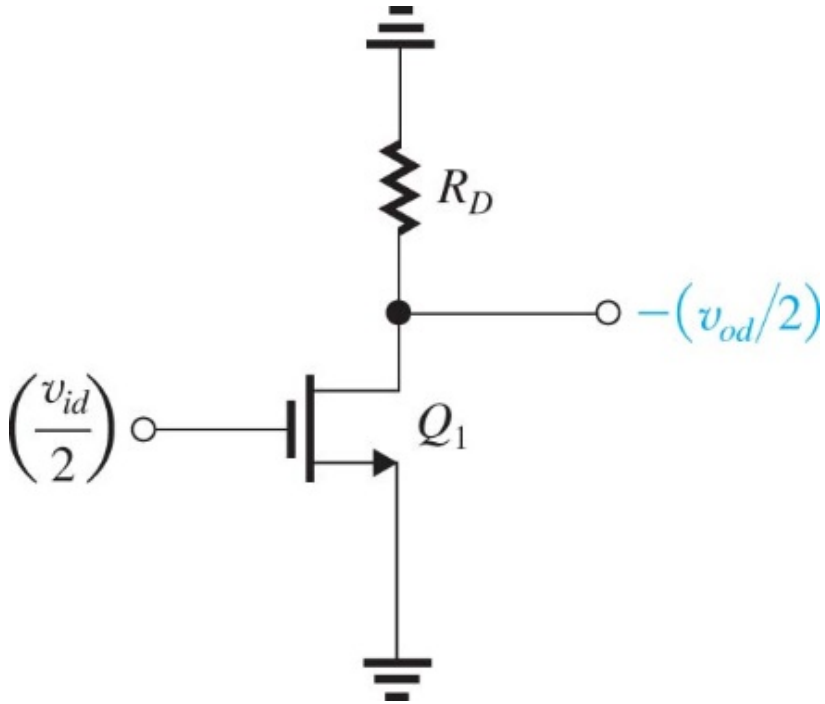
Small Signal Analysis on the Half Circuit (1/2)



(b)

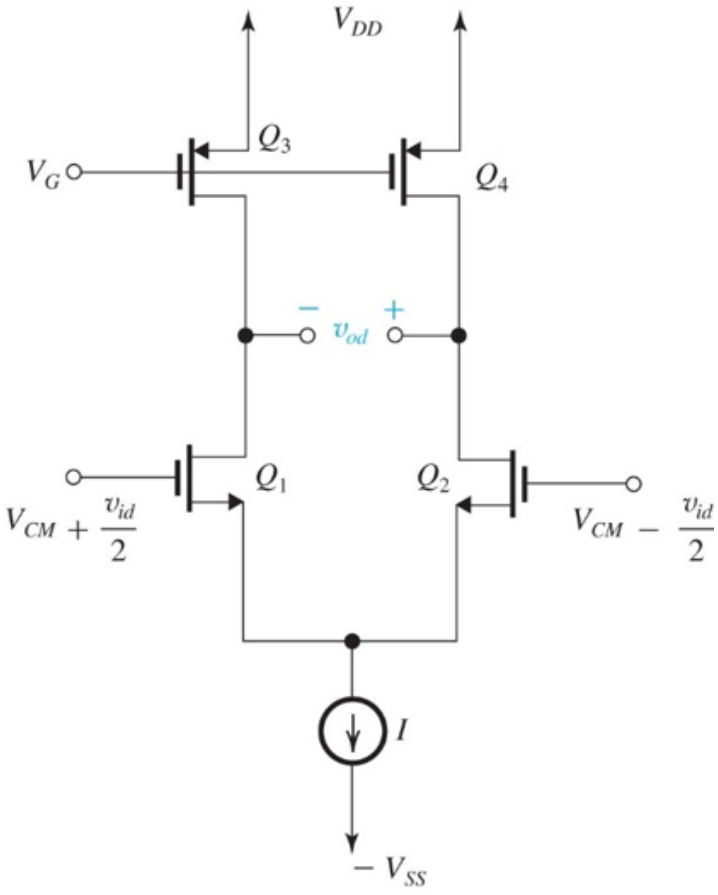
Assuming a 'balanced' input, i.e. $v_{g1} = -v_{g2} = \frac{v_{id}}{2}$.
This results in a virtual small signal Gnd at the source of the transistors.

Small Signal Analysis on the Half Circuit (2/2)

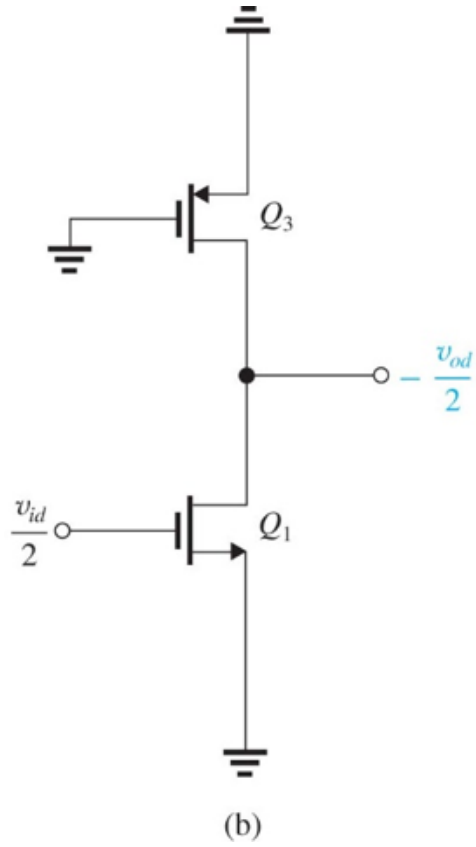


Thus one can look at the branches individually: It's the good old common source amp.

Current Source Load Differential Amplifier

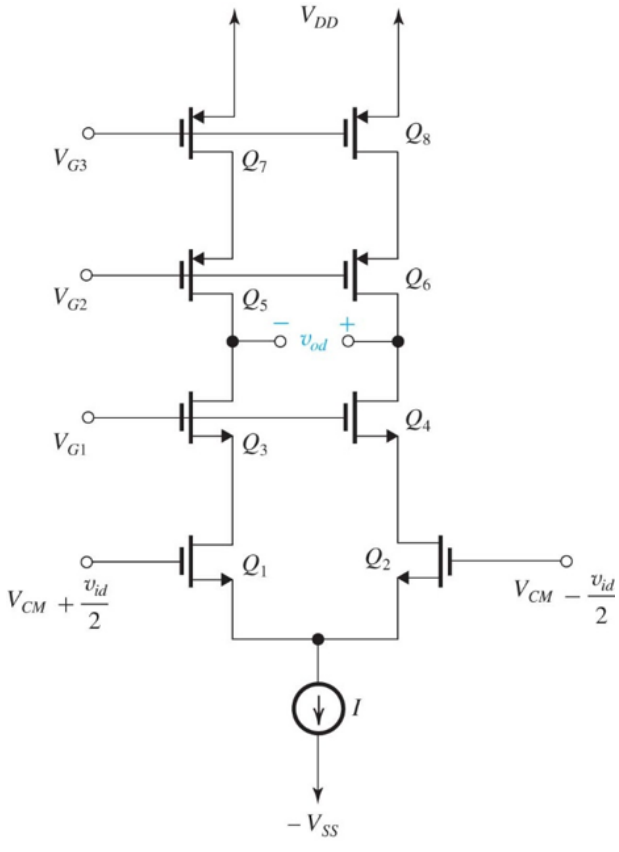


(a)

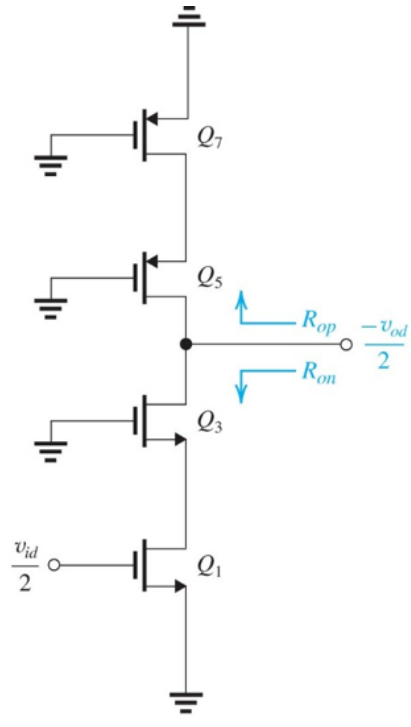


(b)

Cascode Differential Amplifier



(a)



(b)

Content

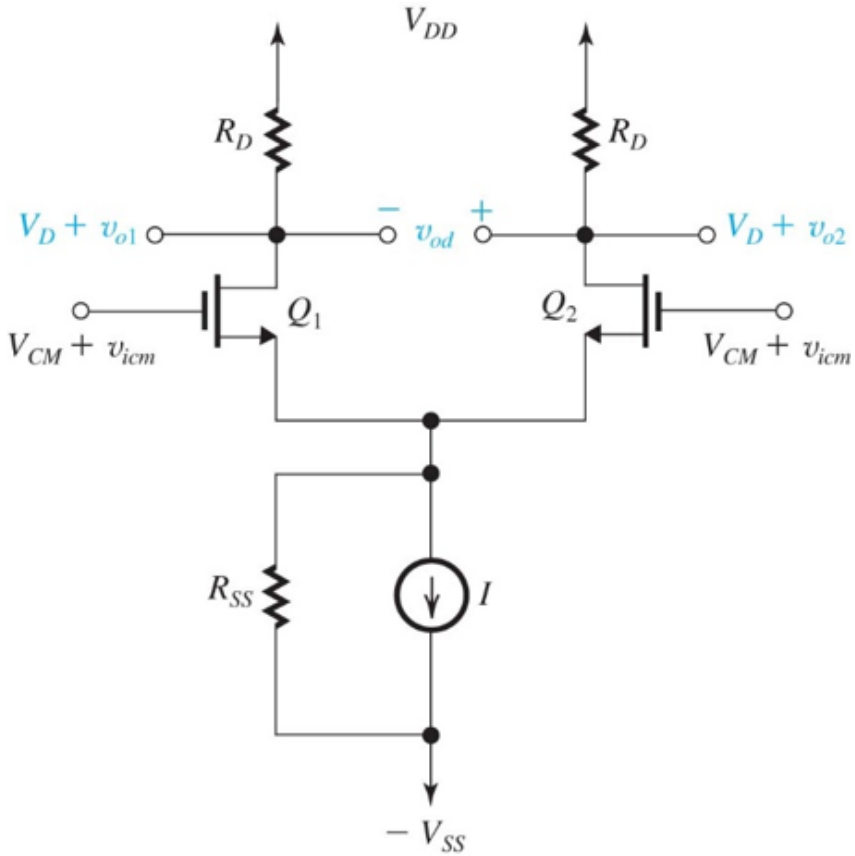
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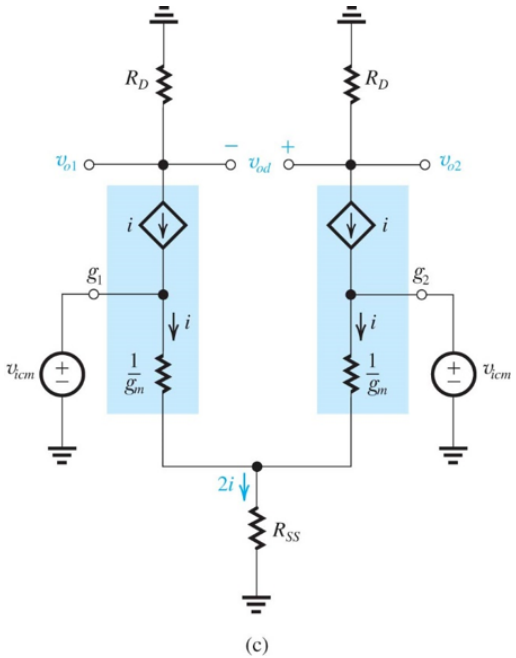
Multi Stage Amplifiers (book 8.6)

Common Mode Rejection



(a)

Common Mode Rejection



$$v_{icm} = \frac{i}{g_m} + 2iR_{SS}$$

$$i = \frac{v_{icm}}{\frac{1}{g_{m0}} + 2R_{SS}} \approx \frac{v_{icm}}{2R_{SS}}$$

And R_D converts i into the two outout voltages v_{o1} and v_{o2}. Note that the *difference* of currents is still 0, i.e. not affected by the common mode input. However, since a change in i, respectively a change in I_b, affects the transconductance, v_{icm} will influence the output difference if the differential input is not zero, and mismatch will lead to common mode gain, i.e. a DC offset with zero input difference that varies with v_{icm}.

Content

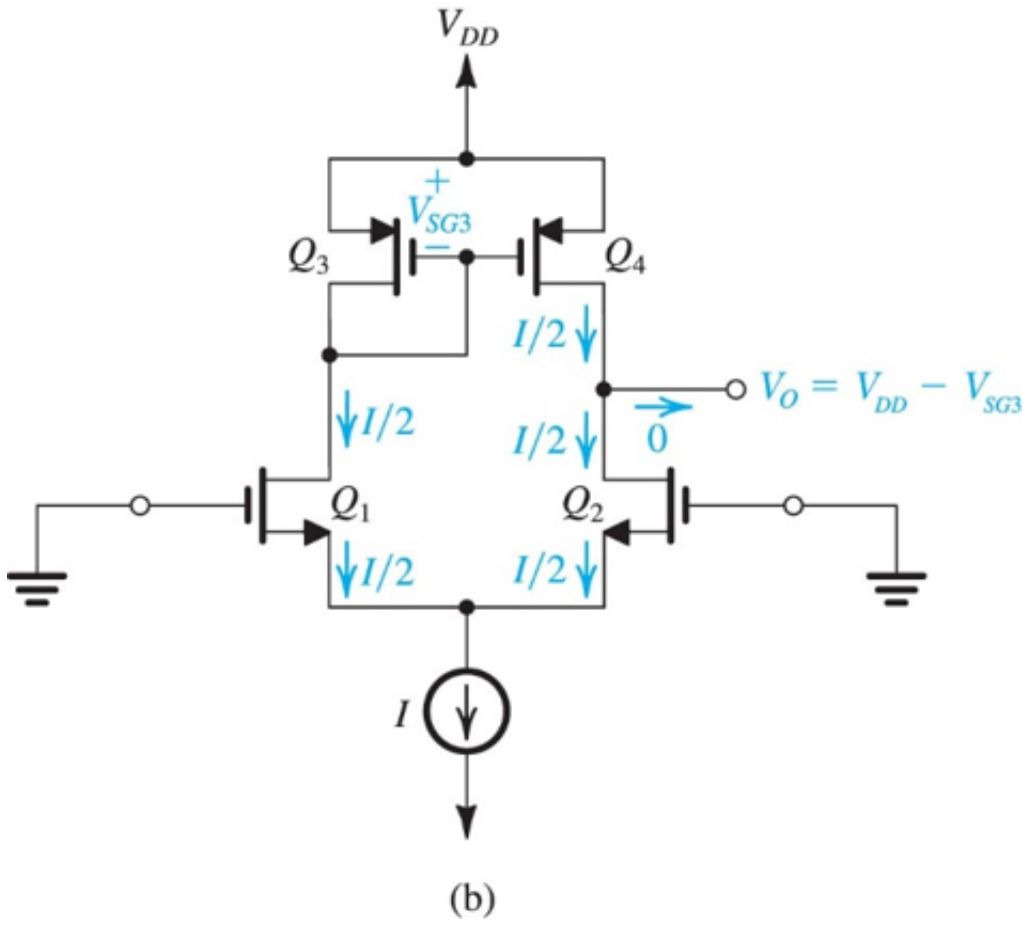
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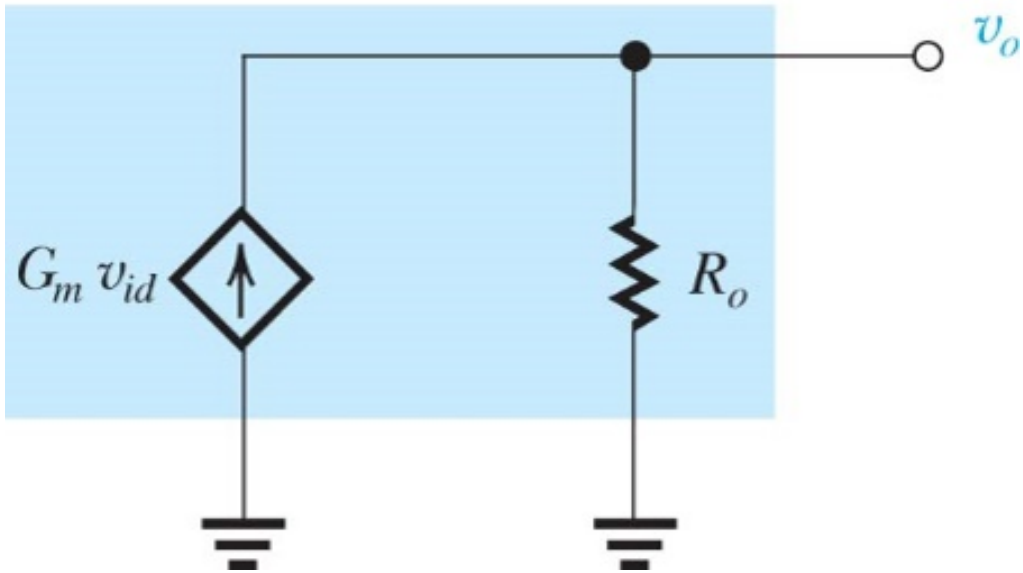
Multi Stage Amplifiers (book 8.6)

Systematic DC offset with current mirror load



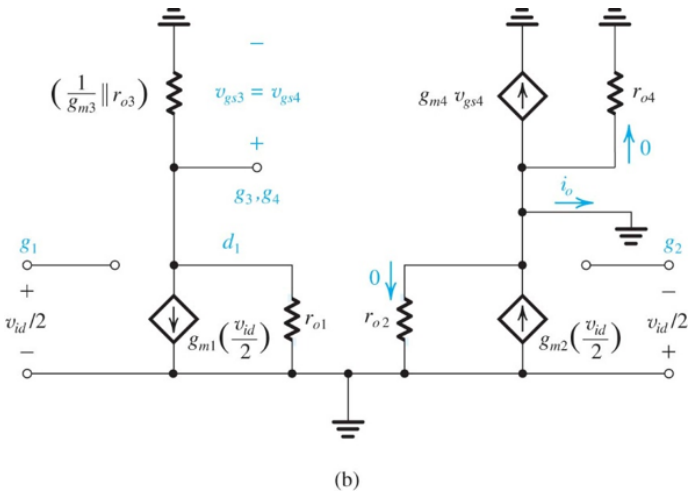
(b)

Output equivalent circuit



$$A_d = \frac{v_o}{v_{id}} = G_m R_o = g_{m1,2}(r_{o2}||r_{o4})$$

A more careful deduction of G_m



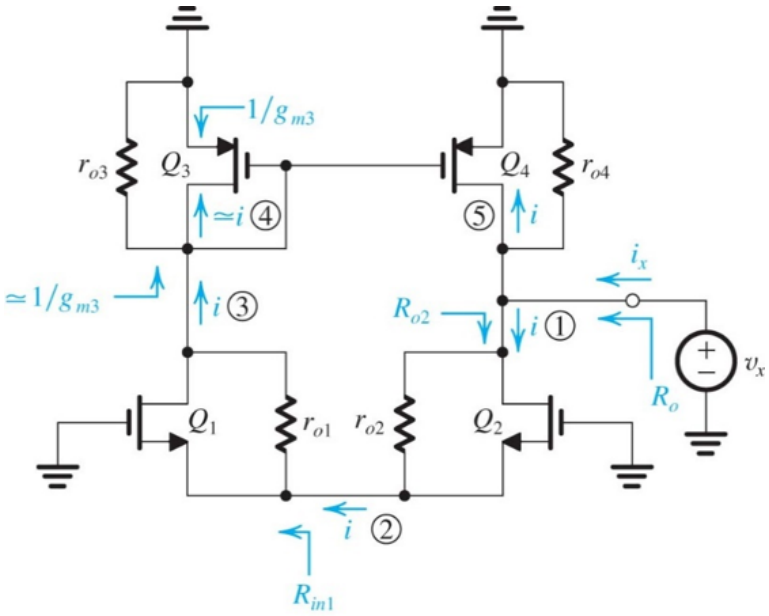
(b)

$$i_o = g_{m2}\frac{v_{id}}{2} - g_{m4}v_{gs4} \quad (8.132)$$

$$\begin{aligned} v_{gs3} &= -g_{m1}\frac{v_{id}}{2}\left(\frac{1}{g_{m3}} || r_{o3} || r_{o1}\right) \\ &\approx -\frac{g_{m1}}{g_{m3}}\frac{v_{id}}{2} \quad (8.134) \end{aligned}$$

$$i_o \approx g_m v_{id} \Rightarrow G_m = g_m$$

A more careful deduction of R_o



$$i = \frac{v_x}{R_{o2}}$$

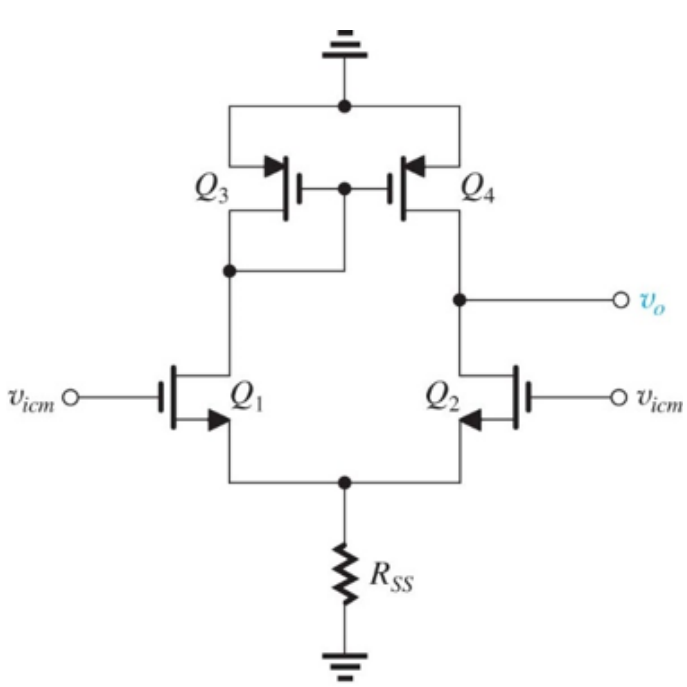
$$R_{in1} = \frac{r_{o1} + R_L}{g_{m1} r_{o1}} \approx \frac{1}{g_{m1}}$$

$$\begin{aligned} R_{o2} &= R_{in1} + r_{o2} + g_{m2} r_{o2} R_{in1} \\ &\approx 2r_{o2} \quad (8.135) \end{aligned}$$

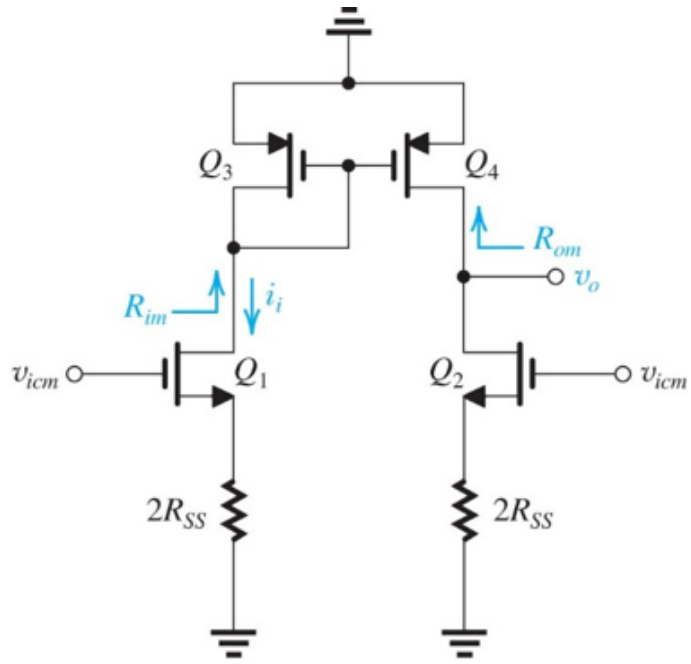
$$i_x = i + i + \frac{v_x}{r_{o4}} = 2\frac{v_x}{2r_{o2}} + \frac{v_x}{r_{o4}}$$

$$R_O = \frac{v_x}{i_x} = r_{o2} || r_{o4} \quad (8.136)$$

Common Mode Gain (1/2)

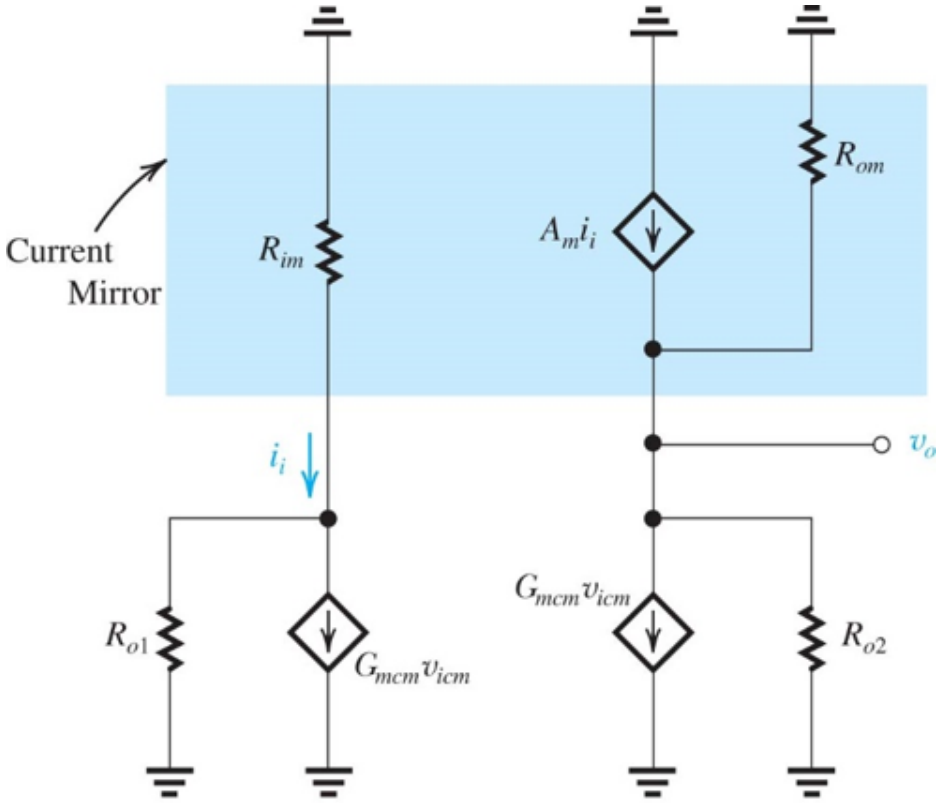


(a)

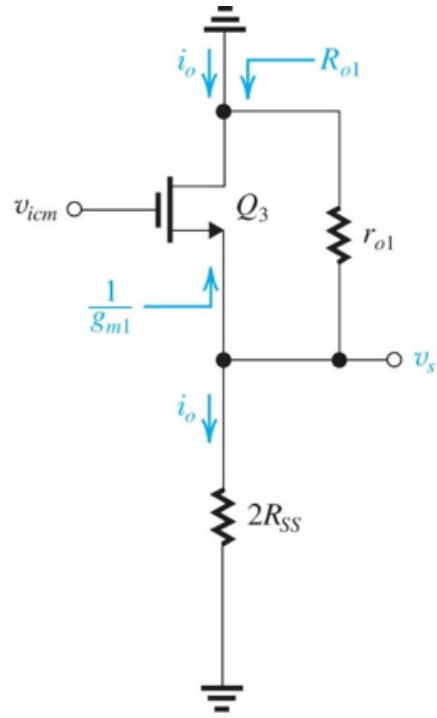


(b)

Common Mode Gain (2/2)

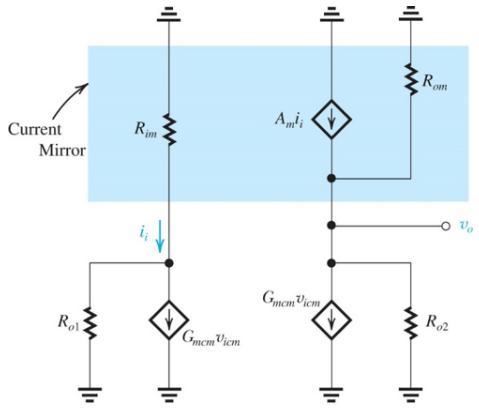


(c)

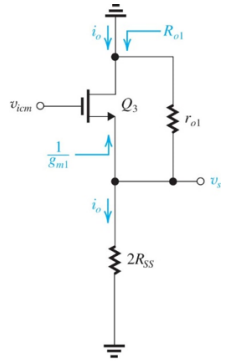


(d)

For General Current Gain



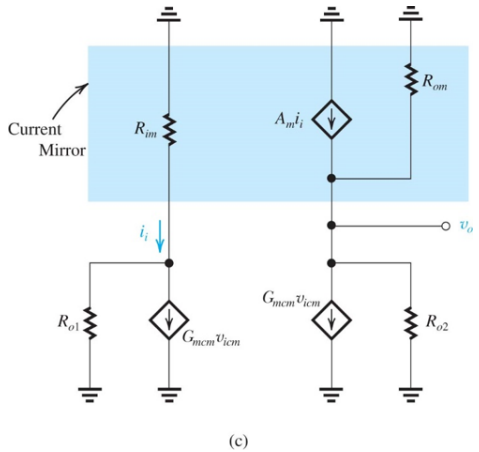
(c)



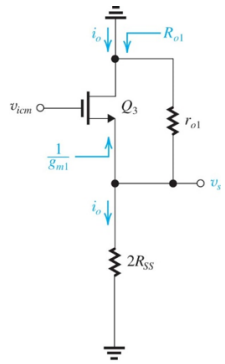
(d)

$$A_{cm} = \frac{v_o}{v_{icm}} = -(1 - A_m)G_{mcm}(R_{om}||R_{o2})$$

For Simple Current Mirror



(c)



(d)

$$A_m i_i = v_{gs3} g_{m4} \quad v_{gs3} = i_i R_{im} \quad R_{im} = \frac{1}{g_{m3}} || r_{o3}$$

$$A_m = \frac{1}{1 + \frac{1}{g_{m3} r_{o3}}}$$

Content

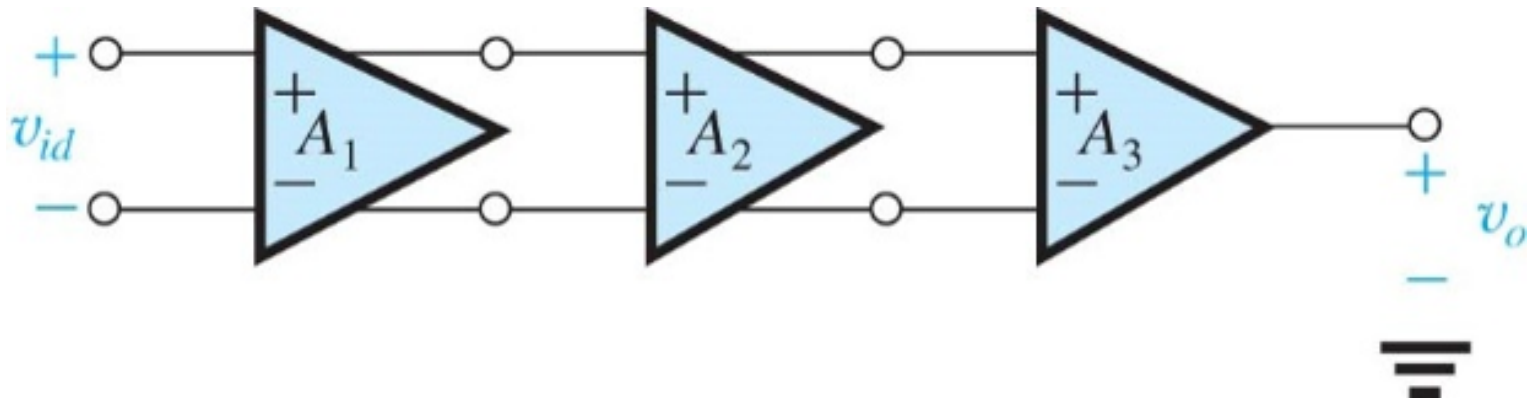
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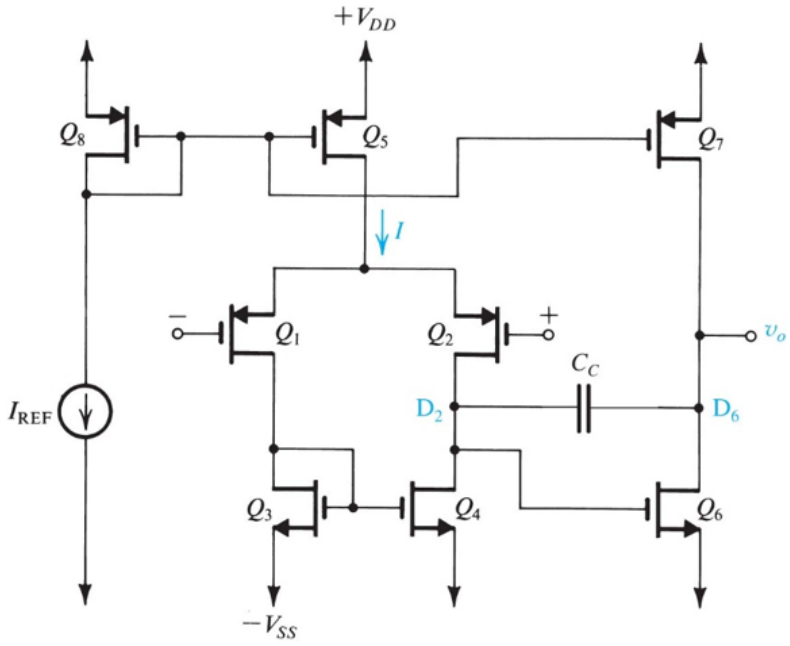
Multi Stage Amplifiers (book 8.6)

Internally All Differential Example



Has some advantages, foremost a better CMRR. And with single ended stages you have to care about 'hitting' the right input DC level of the next stage.

Two Stage CMOS op-amp Example



$$A = A_1 A_2 = g_{m1}(r_{o2}||r_{o4})g_{m6}(r_{o6}||r_{o7})$$

$$\frac{(W/L)_6}{(W/L)_4} = 2\frac{(W/L)_7}{(W/L)_5}$$