IN3170/4170, Spring 2023

Philipp Häfliger
hafliger@ifi.uio.no

Excerpt of Sedra/Smith Chapter 9 & 10: Frequency Response of Basic CMOS Amplifiers and Feedback

Content

Intermezzo: Transfer Function and Bode Plot

High Frequency Small Signal Model of MOSFETs (book 9.2)

High Frequency Response of CS Amplifiers (book 9.3)

Toolset for Frequency Analysis and Complete CS Analysis (9.4)

CG and Cascode HF response (Book 9.5)

General Feedback and Stability Structure (book 10.1, 10.8, 10.9)

Differential Amp HF Analysis (book 9.7)

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Interrupt: Transfer Function and Bode Plot

'Interrupt' means that a topic is not explicitly discussed in the book.

Transfer Function

A transfer function describes the input output relationship of a linear signal filter by describing what happens to pure tone input signals (i.e. sine waves). For each pure tone input frequency the output is a pure tone of the same frequency with the amplitude multiplied with a gain and the phase shifted by a certain angle. Conveniently, complex number arithmetic can be used where to define a transfer function $\mathrm{H}(\mathrm{j}\omega)$ for a pure tone $\sin(\omega t)$ where the magnitude of the transfer function $|\mathrm{H}(\mathrm{j}\omega)|$ expresses the gain and the angle of the resulting complex number $\angle \mathrm{H}(\mathrm{j}\omega)$ is the phase shift.
And then conveniently, if $X(j\omega)$ is the spectrum of a signal $x(t)$ and $Y(j\omega)$ is the spectrum of the resulting output signal $y(t)$ from the filter, then can be found very simply from $H(j\omega)$ and $X(j\omega)$ by simple multiplication in each frequency:

$$Y(j\omega) = X(j\omega) * H(j\omega)$$

Transfer Function, H(jω)

The transfer function $H(s)$ or $H(j\omega)$of a linear filter is

- the Laplace transform of its impulse reponse $h(t)$.
- the Laplace transform of the differential equation describing the I/O realtionship that is then solved for $\frac{V_{out}(s)}{V_{in}(s)}$
- (this lecture!!!) the circuit diagram solved quite normally for $\frac{V_{out}(s)}{V_{in}(s)}$ by putting in impedances $Z(s)$ for all linear elements according to some simple rules.

Impedances of Linear Circuit Elements

resistor:	R
capacitor:	$\frac{1}{sC}$
inductor:	sL

Ideal sources (e.g. the $i_d = g_m v_{gs}$ sources in small signal models of FETs) are left as they are.

Transfer Function in Root Form

Transfer functions $H(s)$ for linear electronic circuits can be written as dividing two polynomials of s.

$$H(s) = \frac{a_0 + a_1 s + ... + a_m s^m}{1 + b_1 s + ... + b_n s^n}$$

$H(s)$ is often written as products of first order terms in both nominator and denominator in the following *root form*, which is conveniently showing some properties of the Bode-plots. More of that later.

$$H(s) = a_0 \frac{(1 + \frac{s}{z_1})(1 + \frac{s}{z_2})...(1 + \frac{s}{z_m})}{(1 + \frac{s}{\omega_1})(1 + \frac{s}{\omega_2})...(1 + \frac{s}{\omega_n})}$$

Bode Plots

Plots of magnitude (e.g. $|H(s)|$) in dB and phase e.g. $\angle H(s)$ or ϕ) vs. $\log(\omega)$.

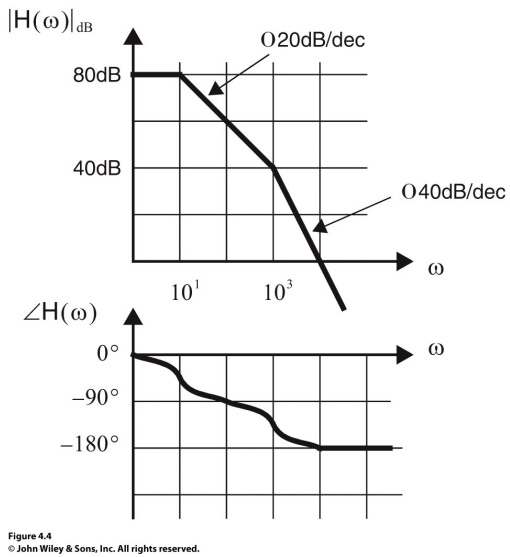


Figure 4.4
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In general for transfer functions with only real poles and no zeros (pure low-pass): a) $\omega \to 0_+$ $|H(s)|$ is constant at the low frequency gain and $\angle H(s) = 0^o$ b) for each pole as ω increases the slope of $|H(s)|$ increses by $-\frac{20dB}{\text{decade}}$ c) each pole contributes -90^o to the phase, but in a smooth transistion so that at a frequency exactly at the pole it is exactly -45^o

General rules of thumb to use real zeros and poles for Bode plots (1/3)

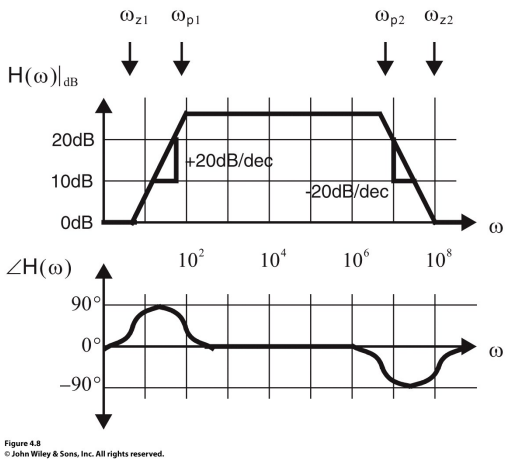


Figure 4.8
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a) find a frequency ω_{mid} with equal number k of zeros and poles where

$z_1, \ldots, z_k, \omega_1, \ldots, \omega_k < \omega_{mid} \Rightarrow$

$$|H(s_{mid})| \approx K \frac{|z_{(k+1)}| \ldots |z_m|}{|\omega_{(k+1)}| \ldots |\omega_n|}$$

where $K = \frac{a_m}{b_n}$

$$\angle H(s_{mid}) \approx 0^o$$

and the gradient of both $|H(s)|$ and $\angle H(s)$ is zero

General rules of thumb to use real zeros and poles for Bode plots (2/3)

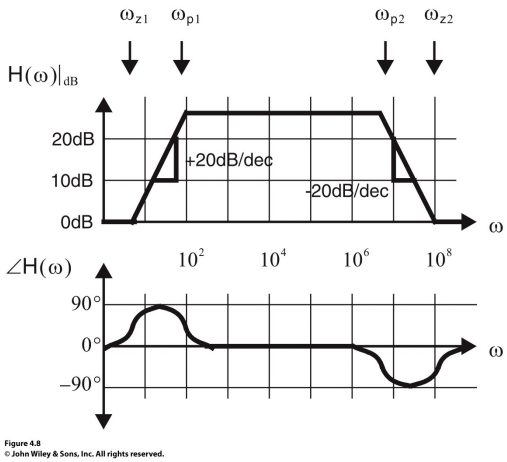


Figure 4.8
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b) moving from ω_{mid} in the magnitude plot at each $|\omega_i|$ add -20dB/decade to the magnitude gradient and for each $|z_i|$ add +20dB/decade

c) moving from ω_{mid} in the phase plot to higher frequencies at each ω_i add -90^o to the phase in a smooth transition (respectively -45^o right *at* the poles) and vice versa towards lower frequencies.

General rules of thumb to use real zeros and poles for Bode plots (3/3)

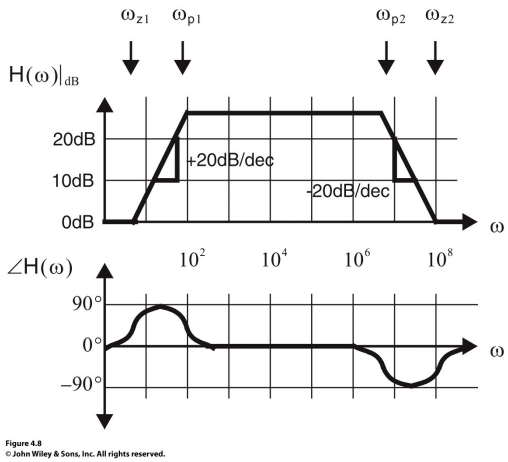


Figure 4.8
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d) For the zeros towards higher frequencies if the nominater is of the form $(1 + \frac{s}{z_i})$ add $+90^o$ and if its of the form $(1 - \frac{s}{z_i})$ (refered to as right half plain zero as the solution for s of $0 = (1 - \frac{s}{z_i})$ is positive) add -90^o to the phase in a smooth transition (i.e. respectively $\pm 45^o$ right at the zeros) and vice versa towards lower frequencies.

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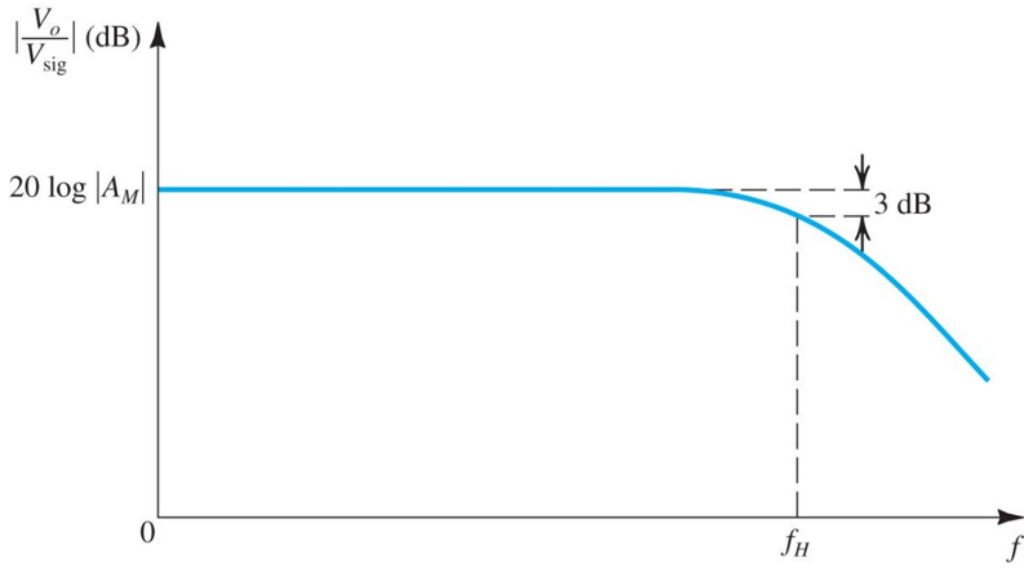
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MOSFET IC transfer function



BW and GB(P)

$$\text{GB}=\text{BW} * A_M$$

GB: gain bandwidth product, BW: bandwidth, A_M: mid-band gain. For low pass filters (and mostly for this course) $A_M = A_{DC}$. There is usually a trade-off between BW and A_M. If this trade off is inversely proportional, the GB is constant, e.g. in opamp feedback configurations.

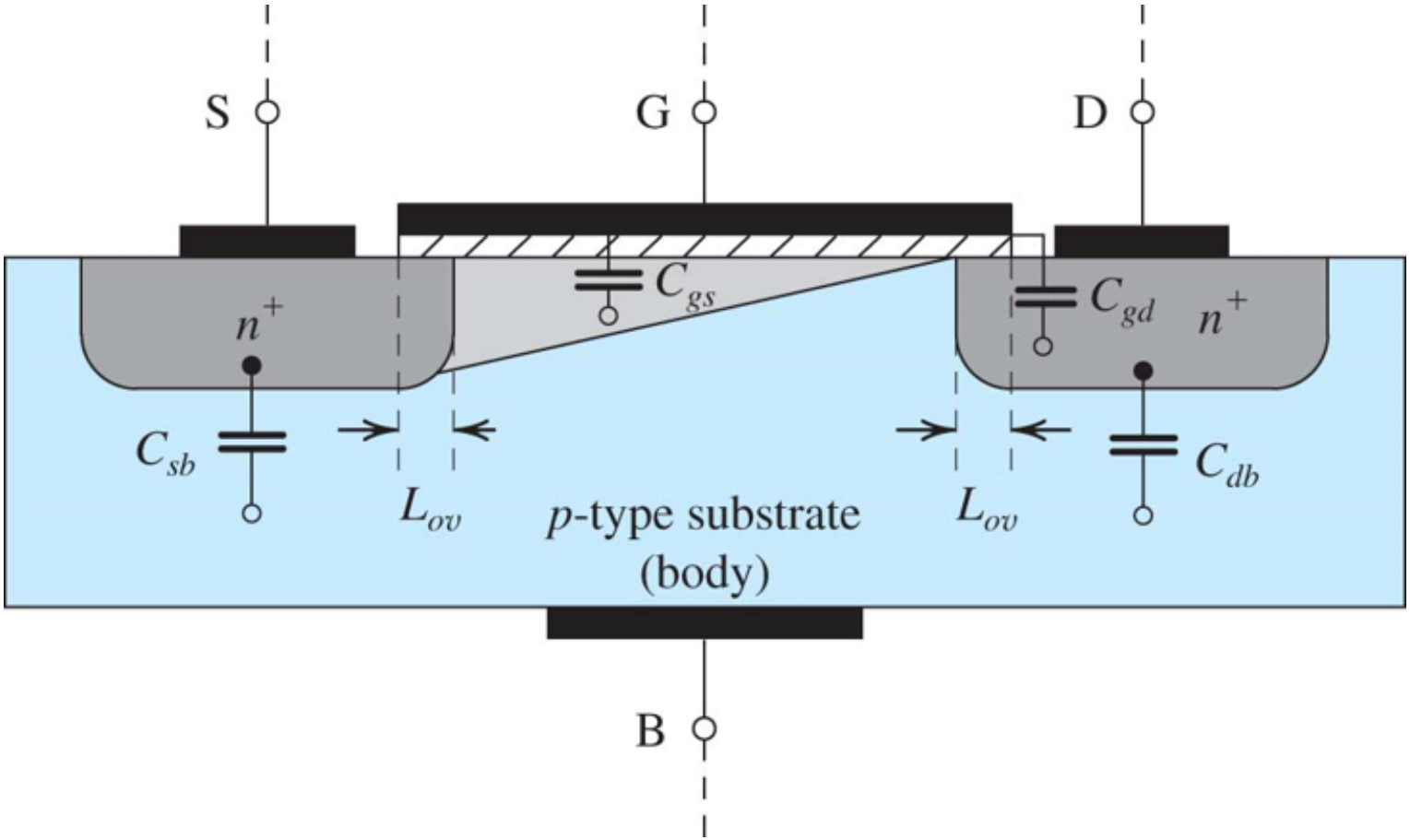
BW and GB(P) for CMOS integrated Circuits

For integrated circuits which normally have a pure low-pass characteristics (i.e. no explicit AC-coupling at the input) you can substitute A_M with A_{DC}, i.e. the gain at DC. And:

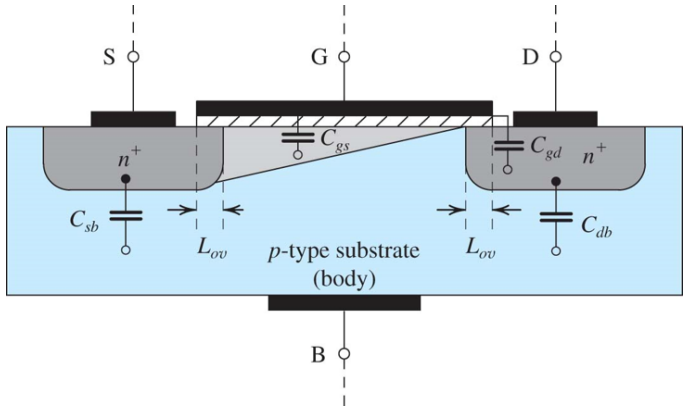
$$\text{BW} = f_H = f_{-3\text{dB}}$$

Where f_H is the high frequency cutoff and is the frequency at which point A_M i reduced by -3dB, i.e. the signal power is reduced by $\frac{1}{2}$, as $10 \log_{10} \frac{1}{2} = 3.0$

MOSFET 'Parasitic' Capacitances Illustration



MOSFET 'Parasitic' Capacitances Equations

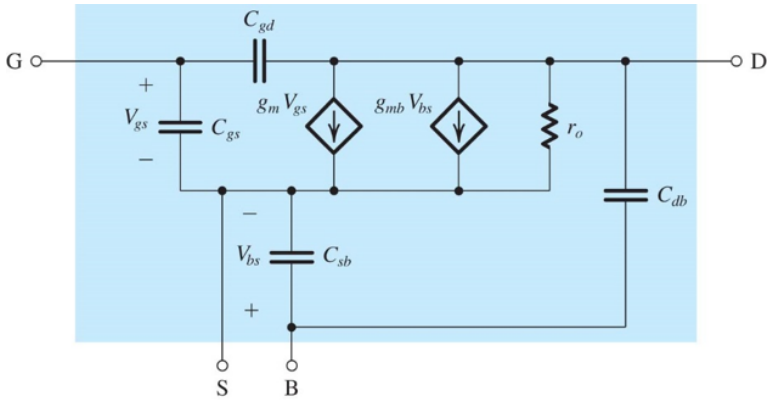


$$C_{gs} = C_{ox} W (\frac{2}{3} L + L_{ov}) \quad (9.22)$$

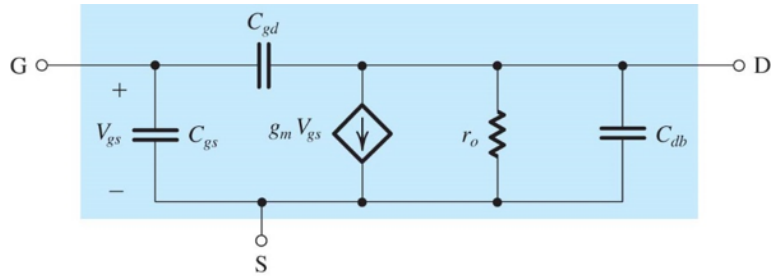
$$C_{gd} = C_{ox} W L_{ov} \quad (9.23)$$

$$C_{sb/db} = \frac{C_{sb0/db0}}{\sqrt{1 + \frac{V_{SB/DB}}{V_0}}} \quad (9.24/9.25)$$

High Frequency Small Signal Model (1/2)



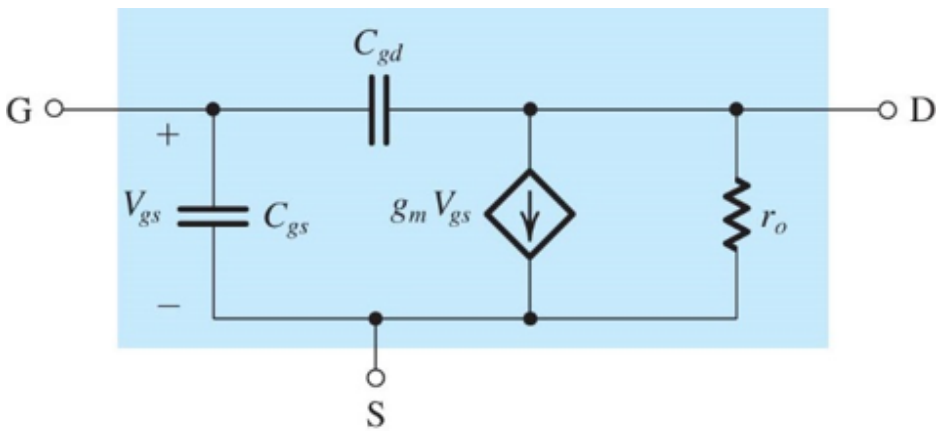
(a)



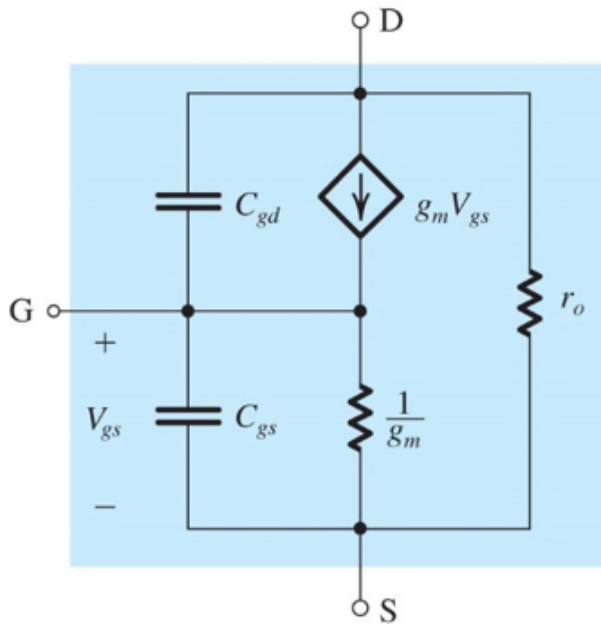
(b)

High Frequency Small Signal Model (2/2)

With only the two most relevant parasitic capacitors.



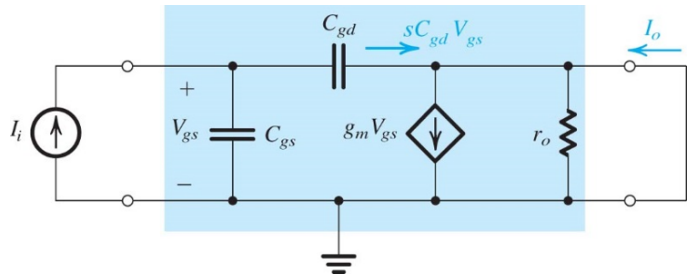
(c)



(d)

Unity Gain Frequency f_T

Short circuit current gain. A measure for the best case transistor speed.



(A short intro to a simple approach deriving Kirchhoff equations on white board)
Neglecting the current through C_{gd}:

$$\frac{i_o}{i_i} = \frac{g_m}{s(C_{gs} + C_{gd})} \quad (9.28)$$

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})} \quad (9.29)$$

Trade-off f_T vs A_o (i.e. GB)

$$f_T = \frac{g_m}{2\pi(C_{gs}+C_{gd})} \quad (9.29)$$

$$\underset{\text{strong inversion}}{\approx} \frac{3\mu_n V_{ov}}{4\pi L^2}$$

$$A_0 = g_m r_o \quad (7.40)$$

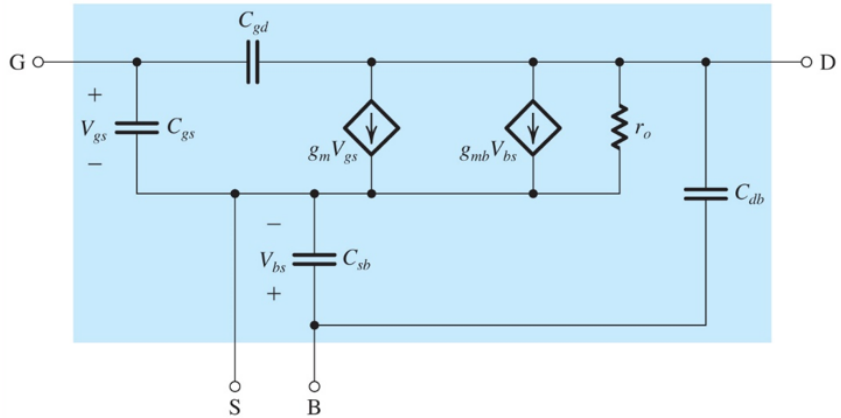
$$\underset{\text{strong inversion}}{\approx} \frac{2}{\lambda V_{ov}}$$

$$= \frac{2L}{\underbrace{\lambda L}_{\text{const}} V_{ov}}$$

Summary CMOS HF Small Signal Model

Table 9.1 The MOSFET High-Frequency Model

Model



Model Parameters

$$g_m = \mu_n C_{ox} \frac{W}{L} |V_{OV}| = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = \frac{2I_D}{|V_{OV}|}$$

$$g_{mb} = \chi g_m, \quad \chi = 0.1 \text{ to } 0.2$$

$$r_o = |V_A| / I_D$$

$$C_{gs} = \frac{2}{3} WLC_{ox} + WL_{ov} C_{ox}$$

$$C_{gd} = WL_{ov} C_{ox}$$

$$C_{sb} = \frac{C_{sb0}}{\sqrt{1 + \frac{|V_{SB}|}{V_0}}}$$

$$C_{db} = \frac{C_{db0}}{\sqrt{1 + \frac{|V_{DB}|}{V_0}}}$$

$$f_T = \frac{g_m}{2\pi (C_{gs} + C_{gd})}$$

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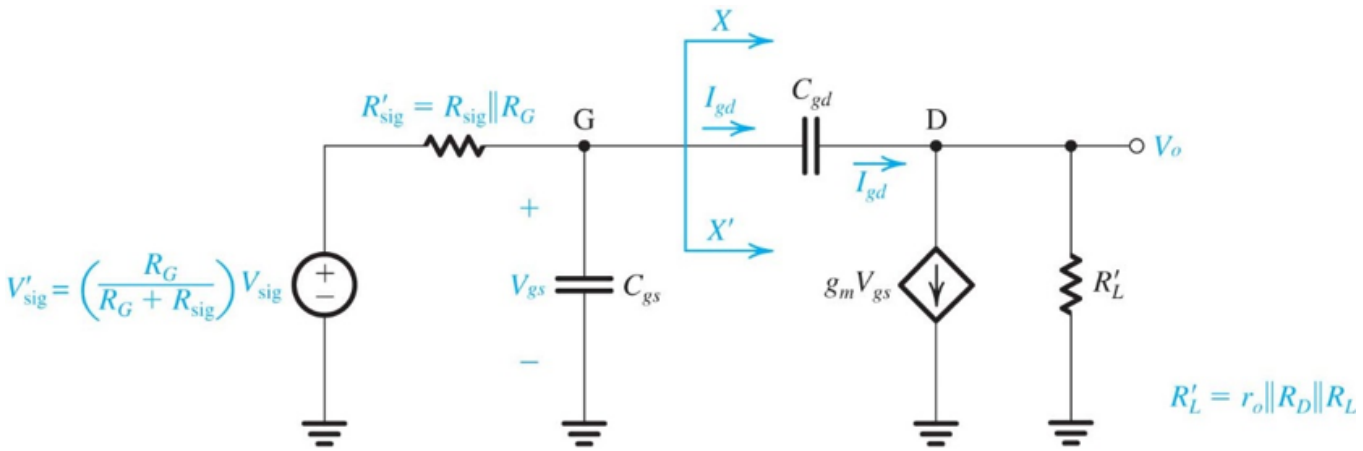
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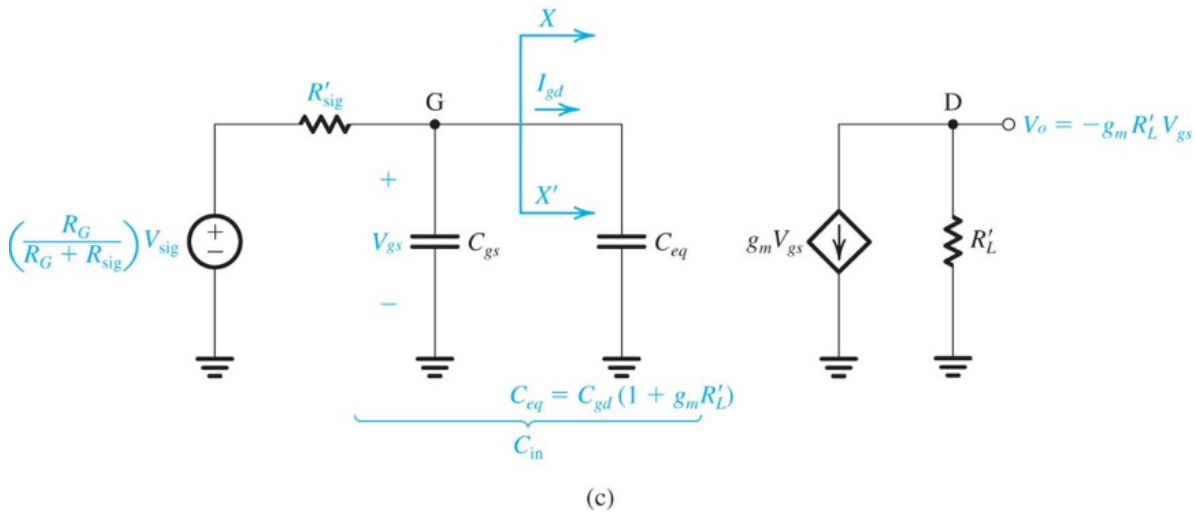
CS Amplifier HF small signal model



(b)

Using the Miller Effect

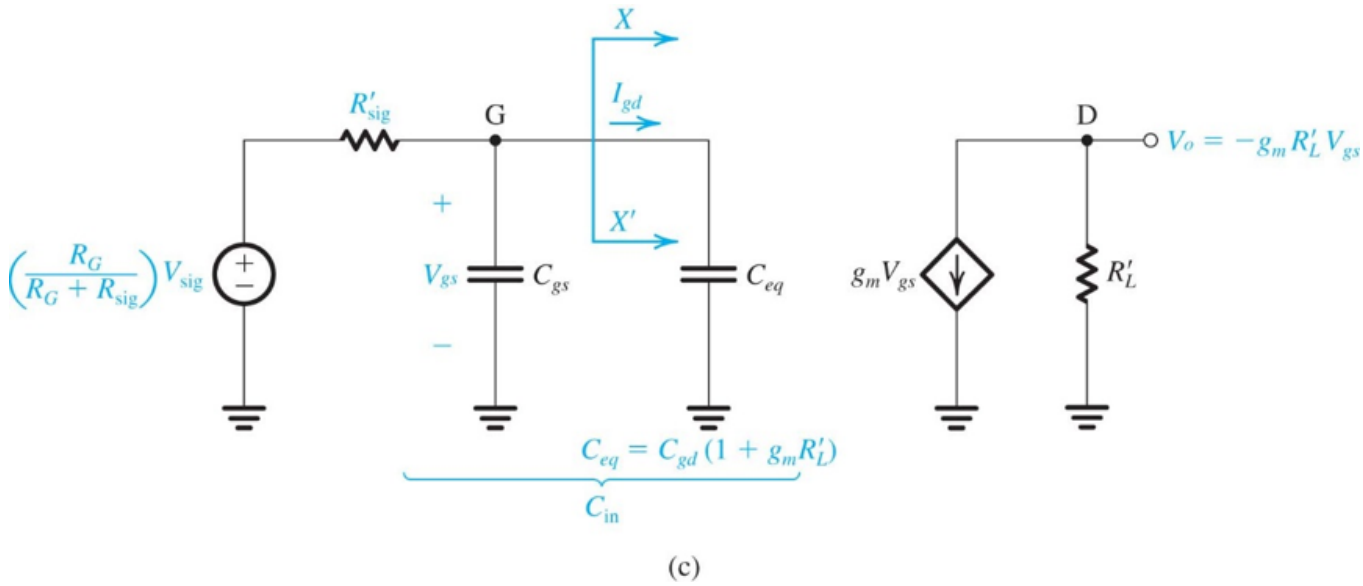
(some more intro to the Miller Effect on white board)



(c)

Note the simplyfying assumption that $v_o = g_m r_o v_{gs}$, i.e. neglecting feed forward contributions of i_{gd} which will still be very small around f_H and makes the following a quite exact approximation of the dominant pole's frequency $f_P \approx f_H$

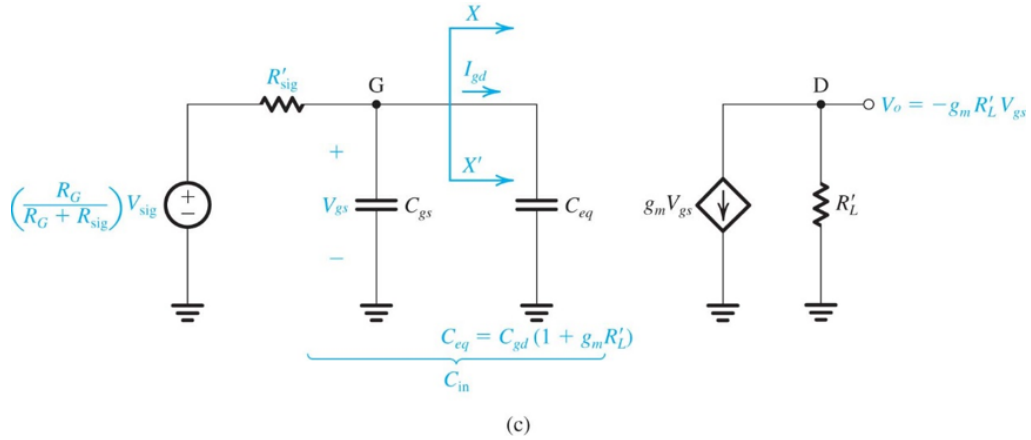
CS transfer function dominant R_{sig} (1/4)



(c)

$$v_{gs}(s(C_{gs} + C_{eq}) + \frac{1}{R'_{sig}}) = v_{sig}\frac{1}{R'_{sig}}$$

CS transfer function dominant R_{sig} (2/4)

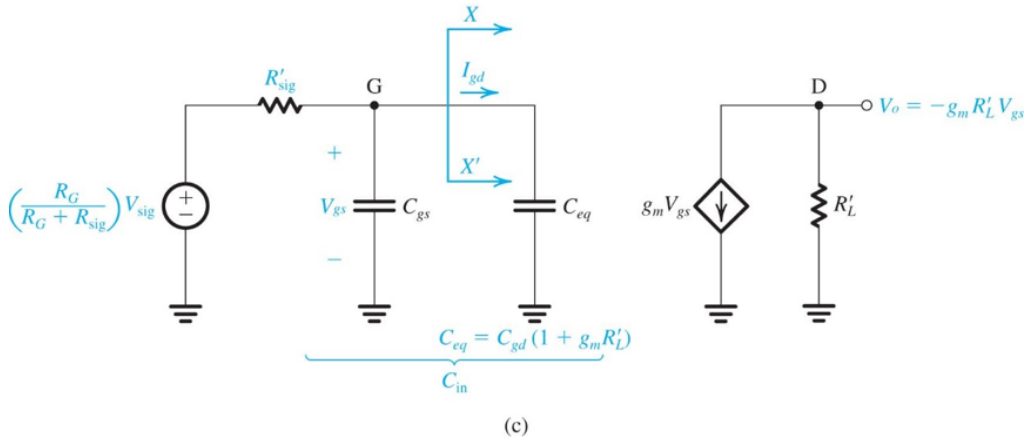


$$\frac{v_{gs}}{v_{sig}} = \frac{\frac{1}{R'_{sig}}}{s(C_{gs} + C_{eq}) + \frac{1}{R'_{sig}}}$$

$$= \frac{1}{1 + sR'_{sig}(C_{gs} + C_{eq})}$$

$$= \frac{1}{1 + \frac{s}{\omega_0}} \quad \text{where} \quad \omega_0 = \frac{1}{R'_{sig}(C_{gs} + C_{eq})}$$

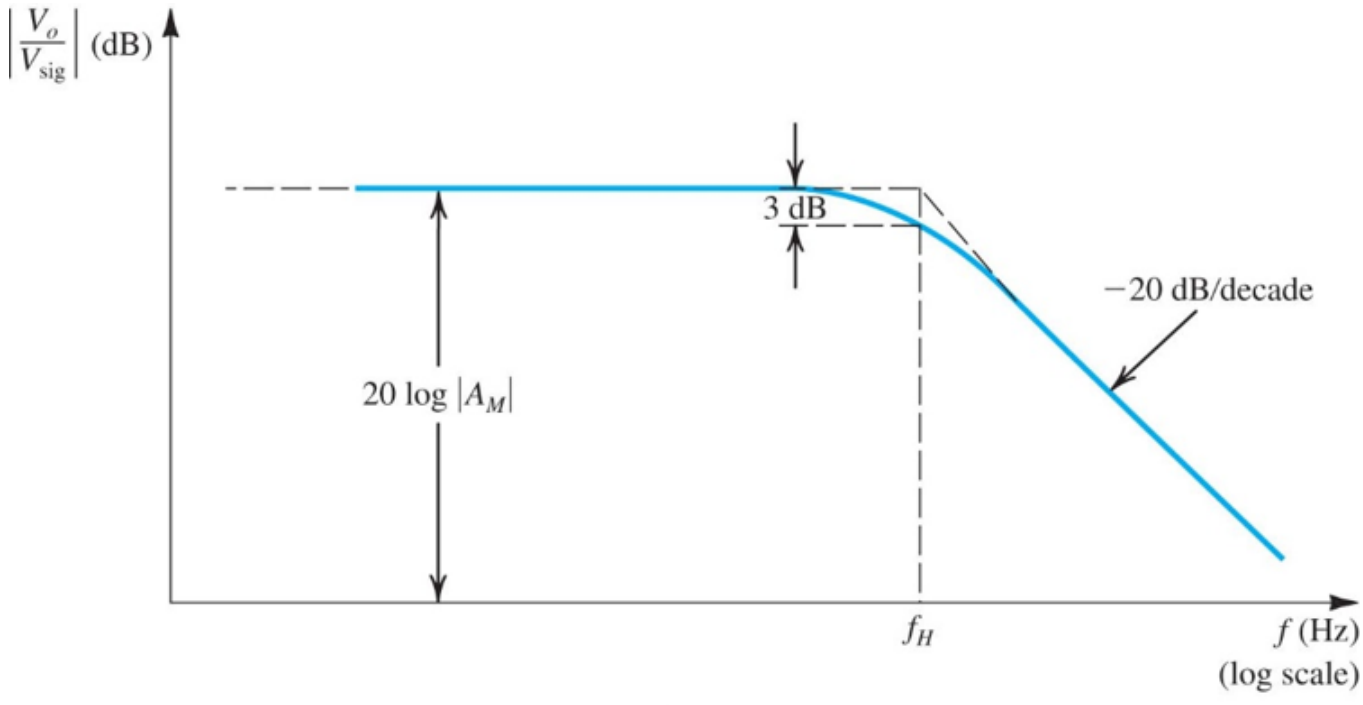
CS transfer function dominant R_{sig} (3/4)



$$\frac{v_o}{v_{sig}} = \frac{A_M}{1 + \frac{s}{\omega_0}} \quad (9.51)$$

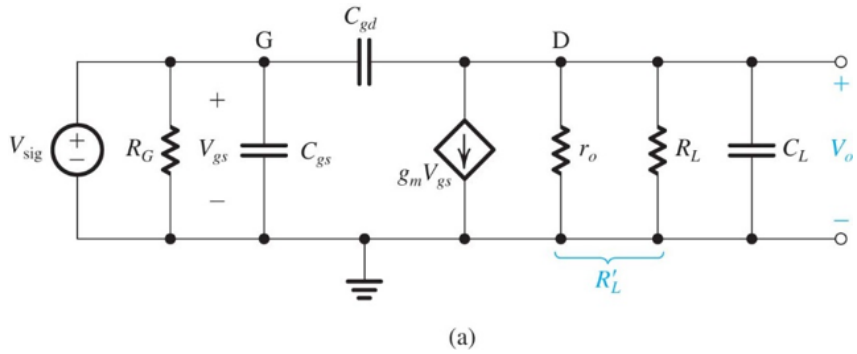
$$\omega_H = \omega_0 = \frac{1}{R'_{sig}(C_{gs} + C_{eq})} \quad (9.52)$$

CS transfer function dominant R_{sig}(4/4)



(d)

CS Frequency Response, Dominant C_L (1/2)

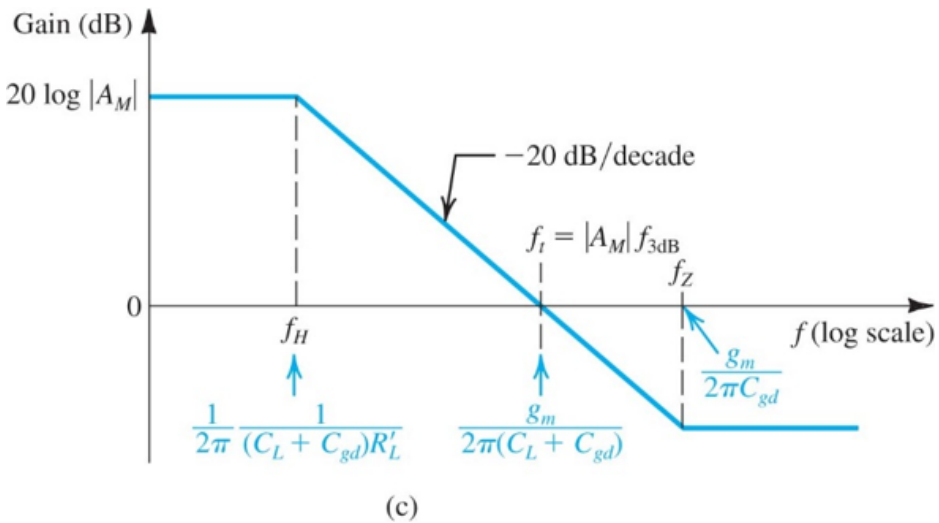


(a)

$$v_o(s(C_{gd}+C_L)+G_L)+g_m v_{gs}=v_{gs}sC_{gd}$$

$$\begin{aligned}\frac{v_o}{v_{gs}} &= \frac{sC_{gd}-g_m}{s(C_{gd}+C_L)+G_L}\\ &= -g_m R'_L \frac{1-s\frac{C_{gd}}{g_m}}{s(C_{gd}+C_L)R'_L+1} \quad (9.65)\end{aligned}$$

CS Frequency Response, Dominant C_L (2/2)



(c)

$$\omega_z = \frac{g_m}{C_{gd}} \quad (9.66)$$

$$\omega_p = \frac{1}{(C_{gd} + C_L)R'_L} \quad (9.67)$$

$$\frac{\omega_z}{\omega_p} = g_m R'_L \left(1 + \frac{C_L}{C_{gd}}\right) \quad (9.68)$$

$$\omega_t = \frac{g_m}{C_L + C_{gd}} \quad (9.69)$$

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Notation in this Book

$$A(s) = A_M F_H s$$

$$F_H(s) = \frac{(1 + \frac{s}{\omega_{z1}})...(1 + \frac{s}{\omega_{zn}})}{(1 + \frac{s}{\omega_{p1}})...(1 + \frac{s}{\omega_{pm}})}$$

Dominant Pole Approximation

If $\omega_{p1} < 4\omega_{p2}$ and $\omega_{p1} < 4\omega_{z1}$ then approximate with 1^{st} order low pass filter, i.e. single pole.

$$A(S) \approx \frac{1}{1 + \frac{s}{\omega_{p1}}}$$

$$\omega_H \approx \omega_{p1}$$

An Approximation Without a Dominant Pole

2^{nd} order example

$$|F_H(\omega_H)|^2 = \frac{1}{2} = \frac{(1+\frac{\omega_H^2}{\omega_{z1}^2})(1+\frac{\omega_H^2}{\omega_{z2}^2})}{(1+\frac{\omega_H^2}{\omega_{p1}^2})(1+\frac{\omega_H^2}{\omega_{p2}^2})}$$

$$= \frac{1+\omega_H^2\left(\frac{1}{\omega_{z1}^2}+\frac{1}{\omega_{z2}^2}\right)+\omega_H^4\left(\frac{1}{\omega_{z1}^2\omega_{z2}^2}\right)}{1+\omega_H^2\left(\frac{1}{\omega_{p1}^2}+\frac{1}{\omega_{p2}^2}\right)+\omega_H^4\left(\frac{1}{\omega_{p1}^2\omega_{p2}^2}\right)}$$

$$\Rightarrow \omega_H \approx \frac{1}{\sqrt{\frac{1}{\omega_{p1}^2}+\frac{1}{\omega_{p2}^2}-\frac{2}{\omega_{z1}^2}-\frac{2}{\omega_{z2}^2}}} \quad (9.76)$$

An Approximation Without a Dominant Pole

general:

$$\omega_H \approx \frac{1}{\sqrt{\frac{1}{\omega_{p1}^2} + \frac{1}{\omega_{p2}^2} \cdots + \frac{1}{\omega_{pm}^2} - \frac{2}{\omega_{z1}^2} - \frac{2}{\omega_{z2}^2} \cdots - \frac{2}{\omega_{zn}^2}}} \quad (9.77)$$

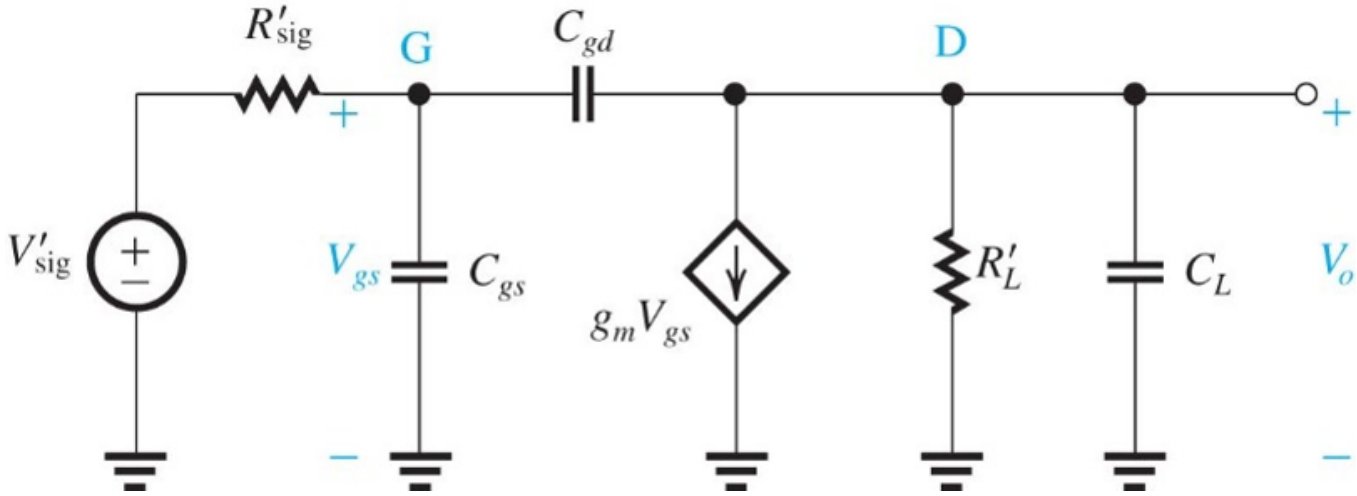
If ω_{p1} is much smaller than all other pole- and zero-frequencies this reduces to the dominant pole approximation.

Open-Circuit Time Constants Method

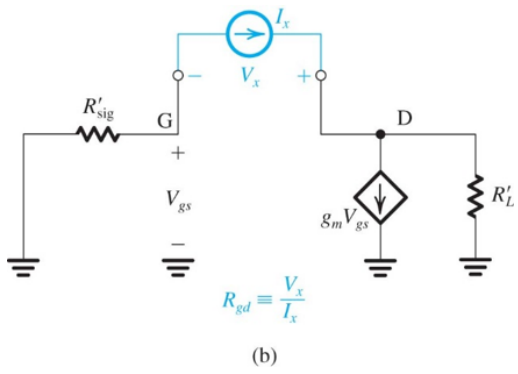
$$\omega_H \approx \frac{1}{\sum_i C_i R_i}$$

Where C_i are all capacitors in the circuit and R_i is the resistance seen by C_i when the input signal source is zeroed and all other capacitors are open circuited.

Open-Circuit Time Constants Method Example CS Amp



The Difficult One is R_{gd}



(b)

$$
\begin{aligned}
i_x &= -\frac{v_{gs}}{R_{sig}} \\
&= \frac{v_{gs} + v_x}{R_L} + v_{gs} g_m \\
&= \frac{v_x}{R_L} - i_x R_{sig} \left(\frac{1}{R_L} + g_m \right)
\end{aligned}
$$

$$
R_{gd} = \frac{v_x}{i_x} = [R_L + R_{sig} (1 + g_m R_L)]
$$

Open Circuit Time Constant

$$\begin{aligned}\tau_H &= R_{sig}C_{gs} + R_L C_L + [R_L + R_{sig}(1 + g_m R_L)]\, C_{gd} \\ &= R_{sig}[C_{gs} + (1 + g_m R_L)\, C_{gd}] + R_L[C_{gd} + C_L] \quad (9.88)\end{aligned}$$

Previously:

$$\omega_H = \frac{1}{R'_{sig}(C_{gs} + (1 + g_m R_L)C_{gd})} \quad (9.53)$$

$$\omega_H = \frac{1}{(C_{gd} + C_L)R_L} \quad (9.67)$$

Comparing Approximations

If you combine the prviously transfer functions for $\frac{v_{gs}}{v_{sig}}$ derived from (9.46) and $\frac{v_o}{v_{gs}}$ from (9.65) as $A(s) = \frac{v_{gs}}{v_{sig}} \frac{v_o}{v_{gs}}$ you get both of these previous ω_H as poles and can compute the combined ω_H according to (9.77):

$$\tau_H = \frac{1}{\omega_H} \approx \sqrt{[R'_{sig}(C_{gs} + C_{eq})]^2 + [(C_{gd} + C_L)R_L]^2} \quad (9.77)$$

So the geometric mean rather than the sum ...

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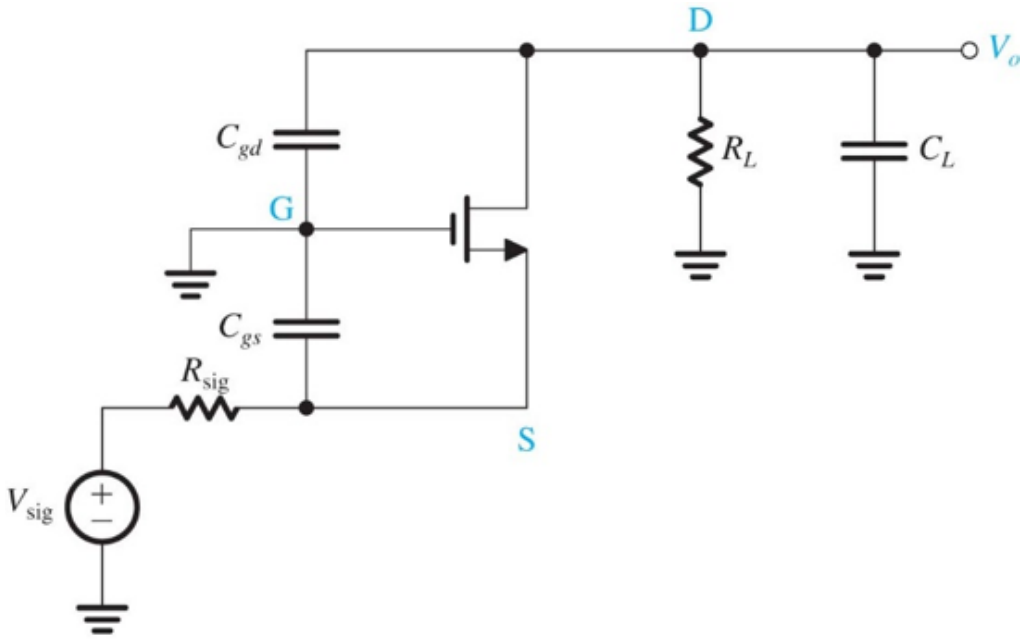
Toolset for Frequency Analysis and Complete CS Analysis (9.4)

CG and Cascode HF response (Book 9.5)

General Feedback and Stability Structure (book 10.1, 10.8, 10.9)

Differential Amp HF Analysis (book 9.7)

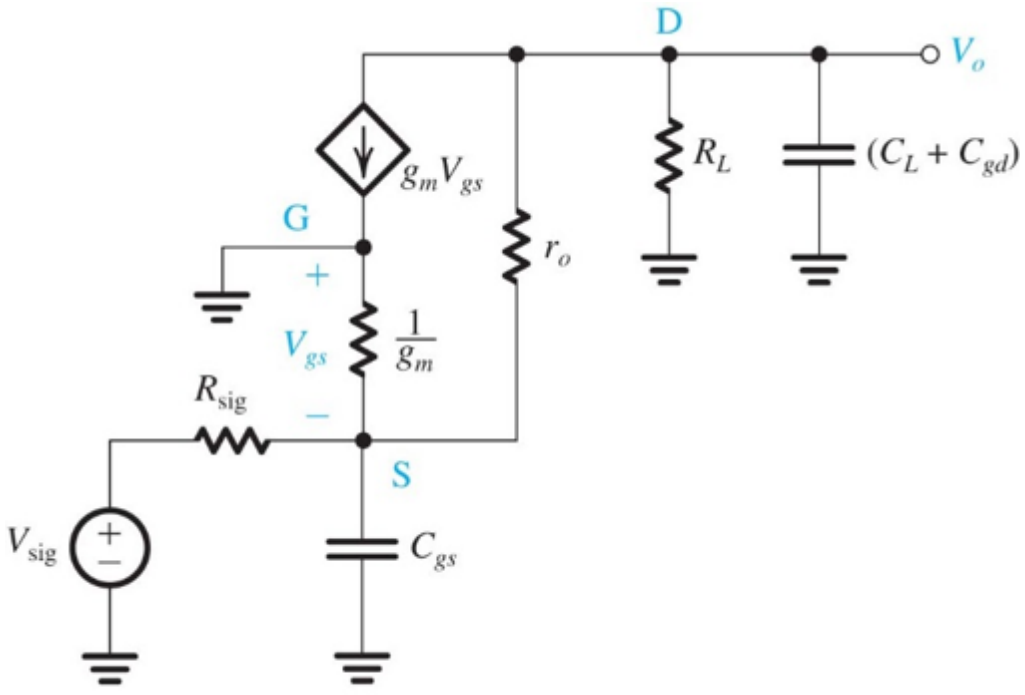
CG Amplifier HF Response



(a)

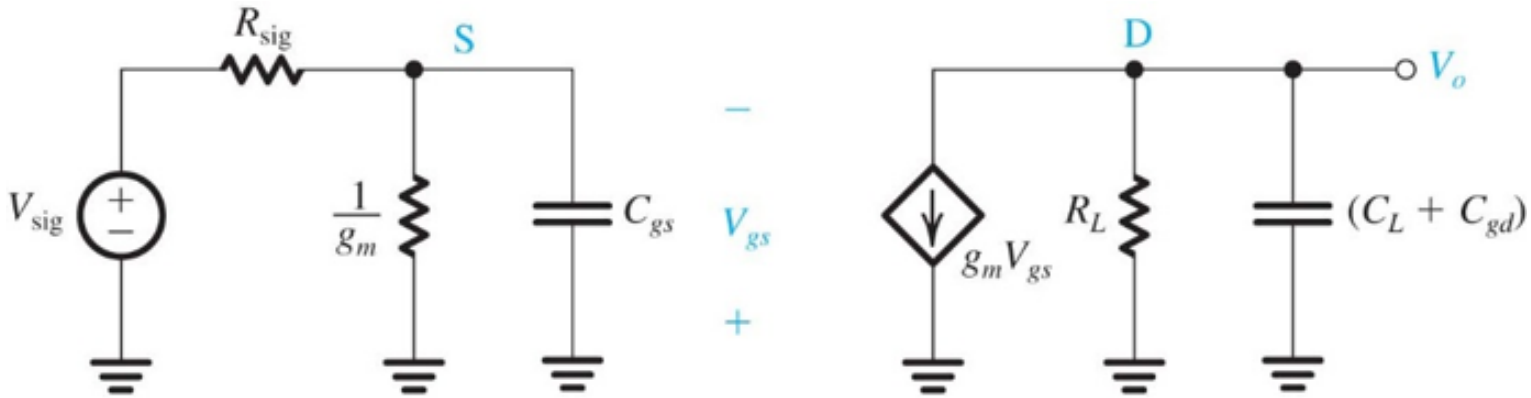
NOTE: no Miller effect!

CG Amplifier HF Response T-model



(b)

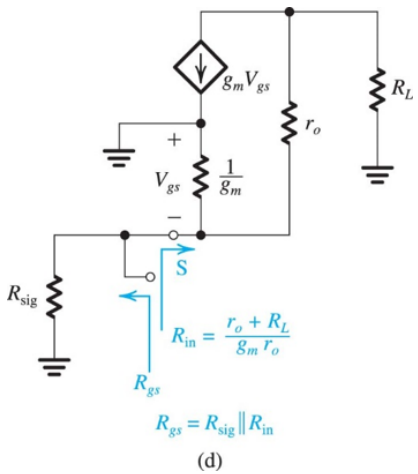
CG Amplifier HF Response without r_o



(c)

$$\tau_{p1} = C_{gs}\left(R_{sig}||\frac{1}{g_m}\right) \quad \tau_{p2} = (C_{gd} + C_L)R_L$$

CG Amplifier open circuit time-constant with r_o for C_{gs}



Kirchoff R_{in} for node v_s:

$$v_s(g_m + \frac{1}{r_o}) = v_o \frac{1}{r_o} + i_s$$

$$v_o(\frac{1}{r_o} + \frac{1}{R_L}) = v_s(g_m + \frac{1}{r_o})$$

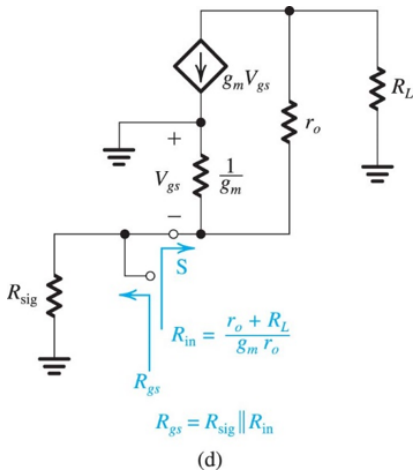
$$v_o = v_s \frac{g_m + \frac{1}{r_o}}{\frac{1}{r_o} + \frac{1}{R_L}}$$

combine: $v_s(g_m + \frac{1}{r_o}) = v_s \frac{g_m + \frac{1}{r_o}}{\frac{1}{r_o} + \frac{1}{R_L}} \frac{1}{r_o} + i_s$

$$v_s(g_m + \frac{1}{r_o} - \frac{g_m + \frac{1}{r_o}}{1 + \frac{r_o}{R_L}}) = i_s$$

$$v_s((g_m + \frac{1}{r_o})(1 - \frac{1}{1 + \frac{r_o}{R_L}})) = i_s$$

CG Amplifier open circuit time-constant with r_o for C_{gs}



Kirchoff R_{in} for node v_s (continued):

$$v_s((g_m + \frac{1}{r_o})(\frac{\frac{1}{R_L}}{\frac{1}{r_o} + \frac{1}{R_L}})) = i_s$$

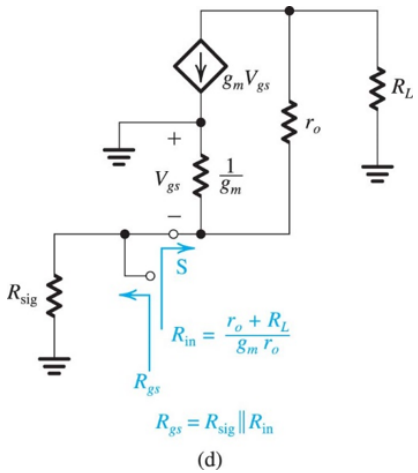
$$\frac{v_s}{i_s} = \frac{1}{g_m + \frac{1}{r_o}} \frac{\frac{1}{r_o} + \frac{1}{R_L}}{\frac{1}{R_L}}$$

$$R_{in} = \frac{R_L}{g_m + \frac{1}{r_o}}(\frac{1}{r_o} + \frac{1}{R_L})$$

$$R_{in} = \frac{R_L r_o}{g_m r_o + 1} \frac{R_L + r_o}{R_L r_o}$$

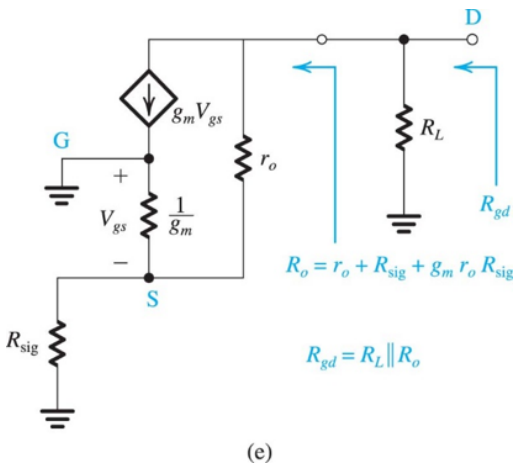
$$R_{in} = \frac{R_L + r_o}{g_m r_o + 1}$$

CG Amplifier open circuit time-constant with r_o for C_{gs}



$$\tau_{gs} = C_{gs}\left(R_{sig} || \frac{r_o + R_L}{g_m r_o}\right)$$

CG Amplifier open circuit time-constant with r_o for $C_{gd} + C_L$



(e)

Kirchoff for R_o and v_o:

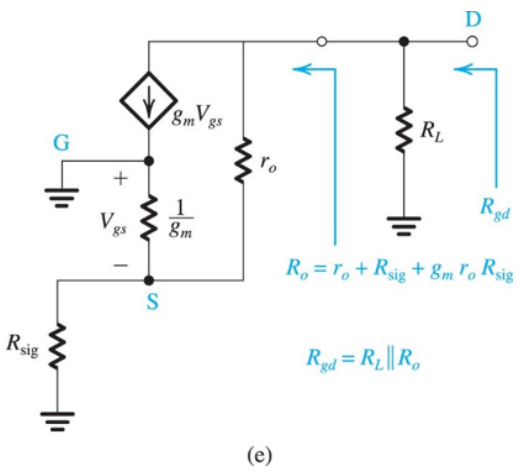
$$v_o \frac{1}{r_o} = i_o + v_s \frac{1}{r_o} + g_m v_s$$

$$v_s(g_m + \frac{1}{R_{sig}} + \frac{1}{r_o}) = v_o \frac{1}{r_o}$$

$$v_s = v_o \frac{\frac{1}{r_o}}{g_m + \frac{1}{R_{sig}} + \frac{1}{r_o}}$$

combine: $v_o \frac{1}{r_o} = i_o + v_o \frac{\frac{1}{r_o}}{g_m + \frac{1}{R_{sig}} + \frac{1}{r_o}} (\frac{1}{r_o} + g_m)$

CG Amplifier open circuit time-constant with r_o for $C_{gd} + C_L$



(e)

Kirchoff for R_o and v_o (continued):

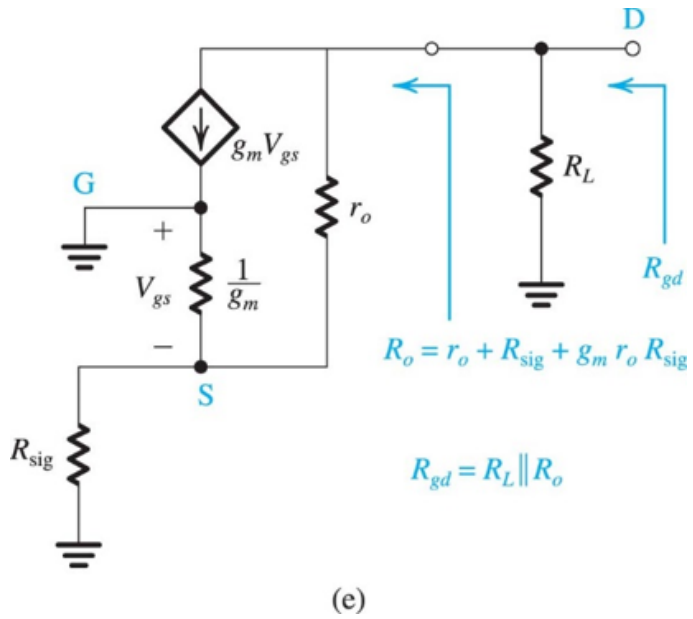
$$v_o \frac{1}{r_o}\left(1 - \frac{\frac{1}{r_o} + g_m}{g_m + \frac{1}{R_{sig}} + \frac{1}{r_o}}\right) = i_o$$

$$v_o \frac{1}{r_o}\left(\frac{\frac{1}{R_{sig}}}{g_m + \frac{1}{R_{sig}} + \frac{1}{r_o}}\right) = i_o$$

$$v_o\left(\frac{1}{g_m r_o R_{sig} + r_o + R_{sig}}\right) = i_o$$

$$R_o = g_m r_o R_{sig} + r_o + R_{sig}$$

CG Amplifier open circuit time-constant with r_o for $C_{gd} + C_L$



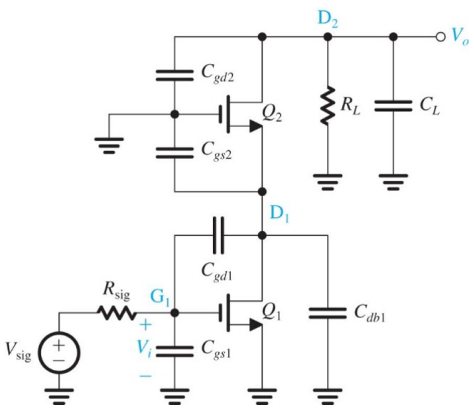
(e)

$$\tau_{gd} = (C_{gd} + C_L)\,(R_{sig}||(r_o + R_{sig} + g_m r_o R_{sig}))$$

CG Amplifier HF Response Conclusion

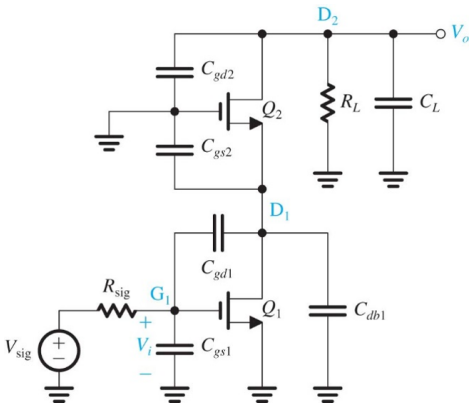
No Miller effect that would cause low impedance at high frequencies, but due to low input resistance the impedance is already low at DC $\Rightarrow$ low A_M for $R_{sig} > 0$

Cascode Amplifier HF Response



$$
\begin{aligned}
\tau_{gs1} &= C_{gs1} R_{sig} \\
\tau_{gd1} &= C_{gd1} \left[(1 + g_{m1} R_{d1}) R_{sig} + R_{d1} \right] \\
&\quad \text{where } R_{d1} = r_{o1} || \frac{r_{o2} + R_L}{g_{m2} r_{o2}} \\
\tau_{gs2} &= (C_{gs2} + C_{db1}) R_{d1} \\
\tau_{gd2} &= (C_L + C_{gd2})(R_L || (r_{o2} + r_{o1} + g_{m2} r_{o2} r_{o1})) \\
\tau_h &\approx \tau_{gs1} + \tau_{gd1} + \tau_{gs2} + \tau_{gd2}
\end{aligned}
$$

Cascode Amplifier HF Response

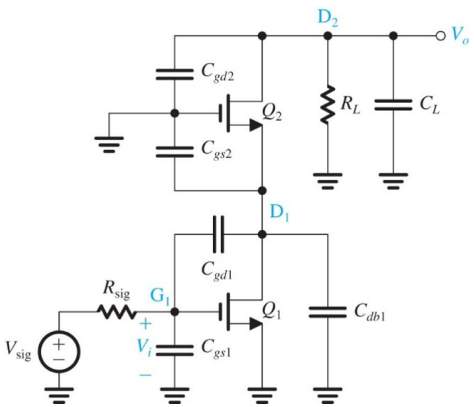


Rearranging τ_h grouping by the three nodes' resistors:

$$\begin{aligned}\tau_h \approx\ & R_{sig}\left[C_{gs1} + C_{gd1}(1 + g_{m1}R_{d1})\right] \\ & + R_{d1}(C_{gd1} + C_{gs2} + C_{db1}) \\ & + (R_L || R_o)(C_L + C_{gd2})\end{aligned}$$

Thus, if $R_{sig} > 0$ and terms with R_{sig} are dominant one can either get larger bandwidth at the same DC gain than a CS amplifier when $R_L \approx r_o$ or get more DC gain at the same bandwith than a CS amplifier when $R_L \approx g_m r_o^2$ or increase both bandwith and DC gain to less than their maximum by tuning R_L somewhere in-between.

Cascode Amplifier HF Response



Rearranging τ_h grouping by the three nodes' resistors:

$$\begin{aligned}\tau_h \approx\ & R_{sig}\left[C_{gs1} + C_{gd1}(1 + g_{m1}R_{d1})\right] \\ & + R_{d1}(C_{gd1} + C_{gs2} + C_{db1}) \\ & + (R_L||R_o)(C_L + C_{gd2})\end{aligned}$$

With $R_{sig} \approx 0$ one can trade higher BW for reduced A_{DC} or higher A_{DC} for reduced BW compared to a CS amp y tweaking R_L, keeping the unity gain frequency (i.e. the GB) constant.

Cascode vs CS

CS:

$$\begin{aligned} A_{DC} &= -g_m(r_o||R_L) \\ \tau_H &= R_{sig}\left[C_{gs} + (1 + g_m(r_o||R_L))\, C_{gd}\right] + (r_o||R_L)\left[C_{gd} + C_L\right] \end{aligned}$$

Cascode:

$$\begin{aligned} A_{DC} &= (-g_{m1}(R_O||R_L) \\ &\quad \text{where } R_O = g_{m2} r_{o2} r_{o1} \\ \tau_H &= R_{sig}\left[C_{gs1} + C_{gd1}(1 + g_{m1} R_{d1})\right] \\ &\quad + R_{d1}(C_{gd1} + C_{gs2} + C_{db1}) \\ &\quad + (R_L||R_o)(C_L + C_{gd2}) \\ &\quad \text{where } R_{d1} = r_{o1}||\frac{r_{o2} + R_L}{g_{m2} r_{o2}} \end{aligned}$$

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Intermezzo: Transfer Function and Bode Plot

High Frequency Small Signal Model of MOSFETs (book 9.2)

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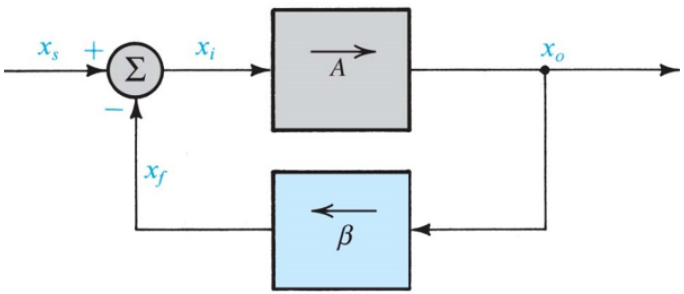
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- Phase margin
- Q-factor from 2nd order open loop gain
- Source follower HF response (Book 9.6), Q from closed loop gain

General Feedback Concept



$$A_f = \frac{A}{1 + A\beta}$$

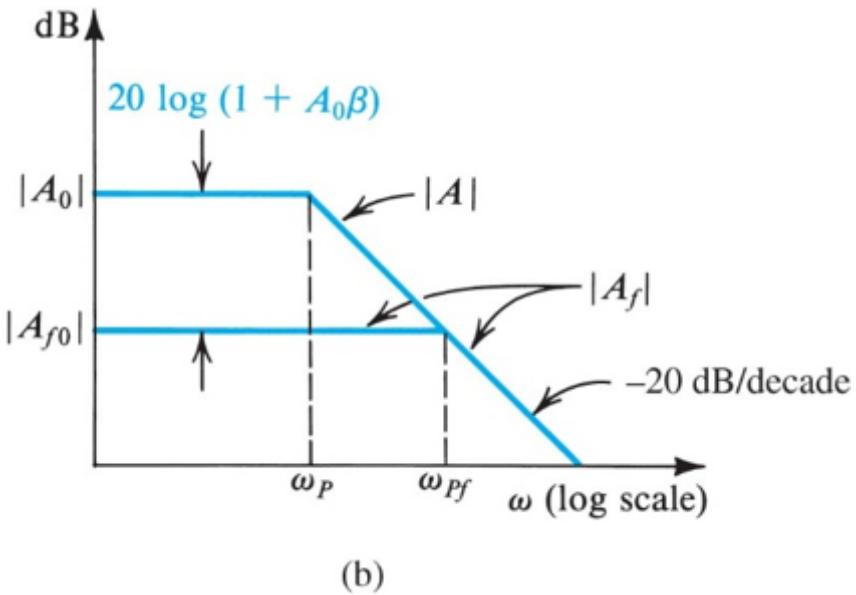
$$\approx \frac{1}{\beta} \text{ for } A\beta >> 1$$

A_f: closed loop gain

$A\beta$: loop gain

β normally considered to be free of any frequency dependency!)

A Has a Single Pole



(b)

$$A_f = \frac{\frac{A_0}{1+A_0\beta}}{1+\frac{s}{\omega_p(1+A_0\beta)}}$$

$$\omega_{pf} = \omega_p(1+A_0\beta)$$

Feedback can cause instability

Two methods to assess risk of instabilities discussed in the following:

- Phase margin derived from analysis or simulation of loop-gain
- Q-factor derived from transfer function

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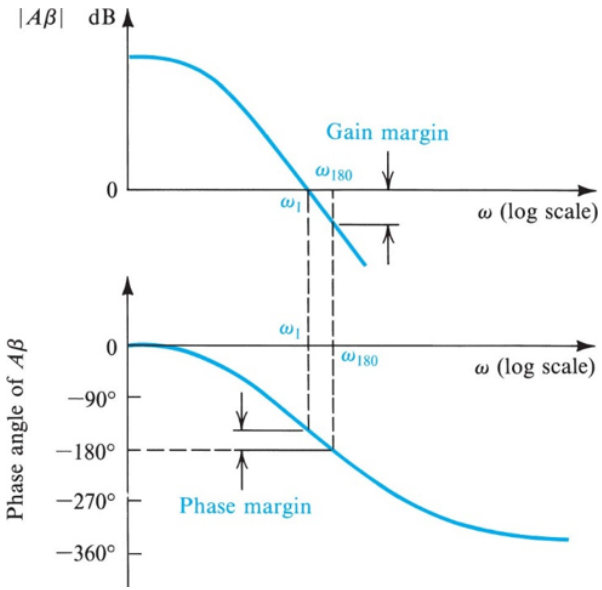
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Why the Resonance?

Pure feedforward systems are stable and do not resonate. Positive feedback can lead to instability/oscillations.



Negative feedback should not increase amplification nor destabilize. The crux: a phase shift $\phi = -180^o$ turns negative feedback into positive feedback. If the loop gain $A\beta > 1$ at this point, the circuit has infinite gain, i.e. is unstable.

In a low pass circuit the difference of the phase from -180^o, i.e. $\phi + 180^o$ where the loop gain becomes unity ($A\beta = 1$ or $A = \frac{1}{\beta}$) is the *phase margin* (PM). A high PM indicates no or little resonance. A negative phase margin indicates an unstable circuit.

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Q-factor if A has two real poles (1/2)

Important: here ω_{p1} and ω_{p2} are the poles of the ***open loop*** transfer function! Also: a pure feed forward 2nd order filter will only have real poles. Solving equation to get the ***closed loop*** poles: $1 + A(s)\beta = 0 \Rightarrow$

$$0 = s^2 + s(\omega_{p1} + \omega_{p2}) + (1 + A_0\beta)\omega_{p1}\omega_{p2} \quad (10.67)$$

$$s = -\frac{1}{2}(\omega_{p1} + \omega_{p2}) \pm \frac{1}{2}\sqrt{(\omega_{p1} + \omega_{p2})^2 - 4(1 + A_0\beta)\omega_{p1}\omega_{p2}} \quad (10.68)$$

Generally more separation between ω_{p1} and ω_{p2} (i.e. a dominant pole) helps to keep the poles of A_f remain real values and to keep A_f stable at higher loop gains.

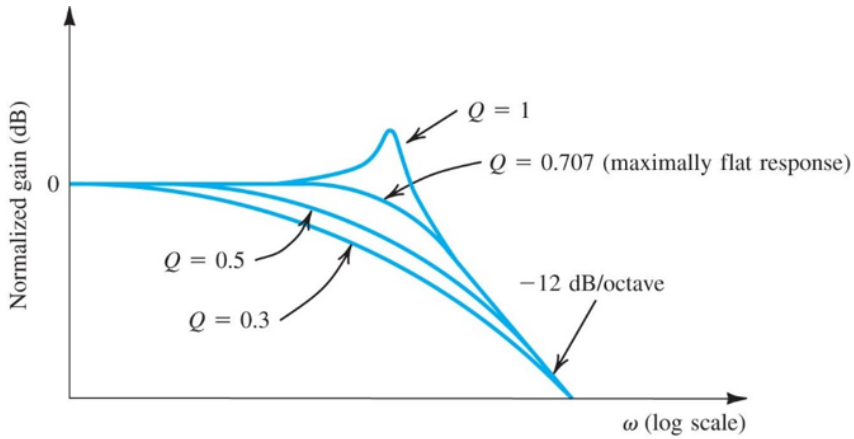
A Has two real poles (2/2)

Rewriting the same:

$$
\begin{aligned}
0 &= s^2 + s\frac{\omega_0}{Q} + \omega_0^2 \quad (10.69) \\
s &= \frac{-\frac{\omega_0}{Q} \pm \sqrt{\frac{\omega_0^2}{Q^2} - 4\omega_0^2}}{2} \\
Q &= \frac{\sqrt{(1 + A_0\beta)\omega_{p1}\omega_{p2}}}{\omega_{p1} + \omega_{p2}} \quad (10.70) \\
\omega_0 &= \sqrt{(1 + A_0\beta)\omega_{p1}\omega_{p2}}
\end{aligned}
$$

If $Q > 0.5$ the poles become complex conjugate and we see resonance.

Frequency Response dependency on Q-factor graph



(b)

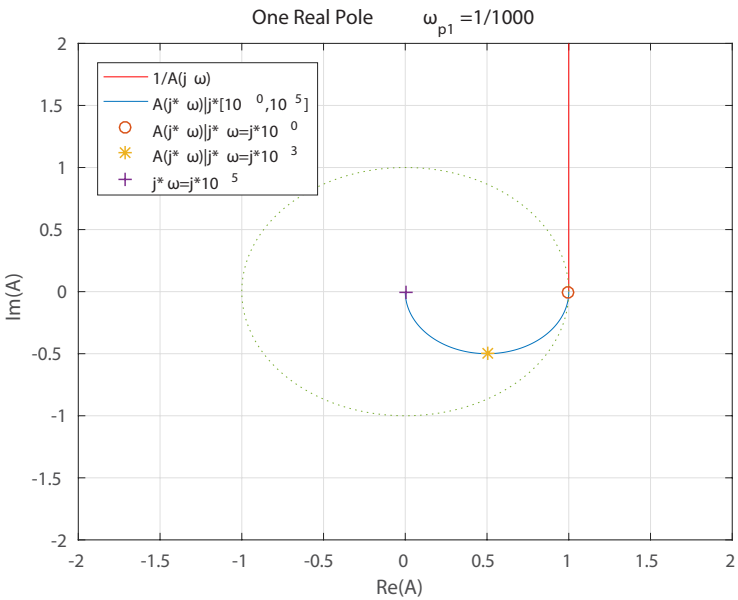
Where the poles of the ***closed loop*** transfer function are:

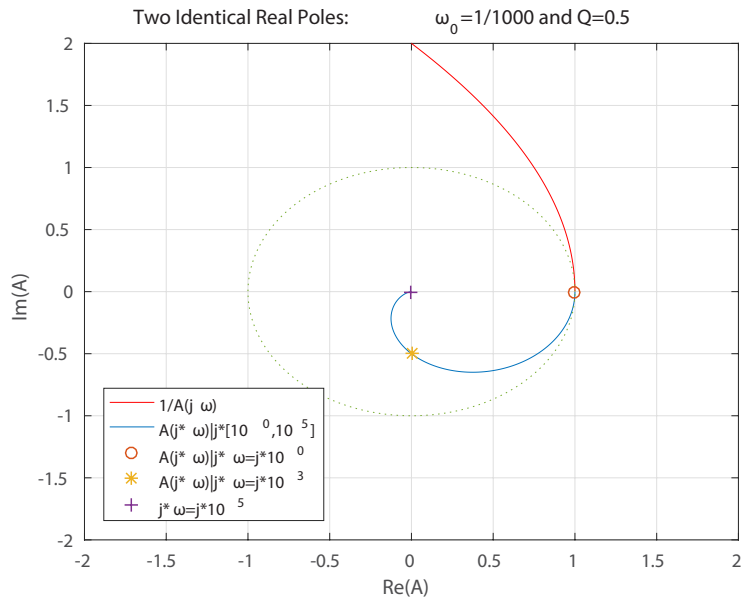
$$\omega_{pf1,pf2} = \frac{-\frac{\omega_0}{Q} \pm \sqrt{\frac{\omega_0^2}{Q^2} - 4\omega_0^2}}{2}$$

Resonance dependency on Q-factor (1/2)

$$\frac{1}{A_f(j\omega)} = \frac{1}{A_{f0}}\left[j^2\omega^2\frac{1}{\omega_{p1}\omega_{p2}} + j\omega\left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right) + 1\right]$$

Poles are either real or a conjugate complex pair

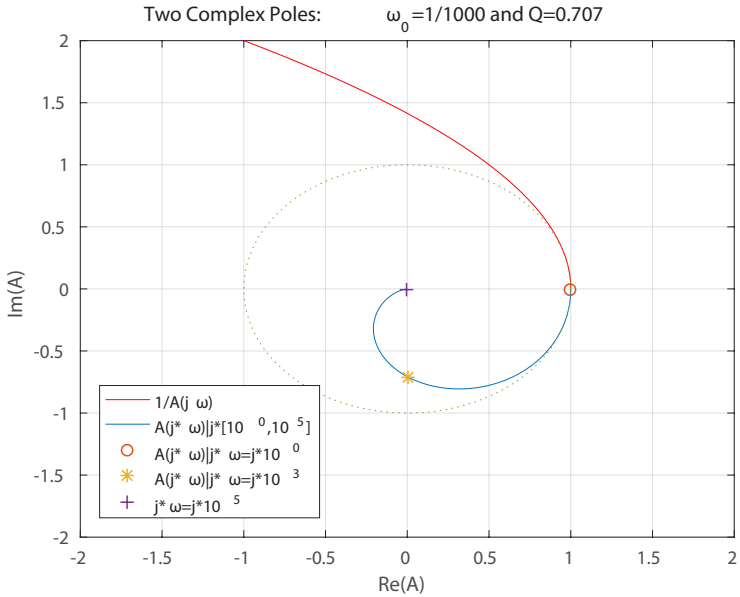


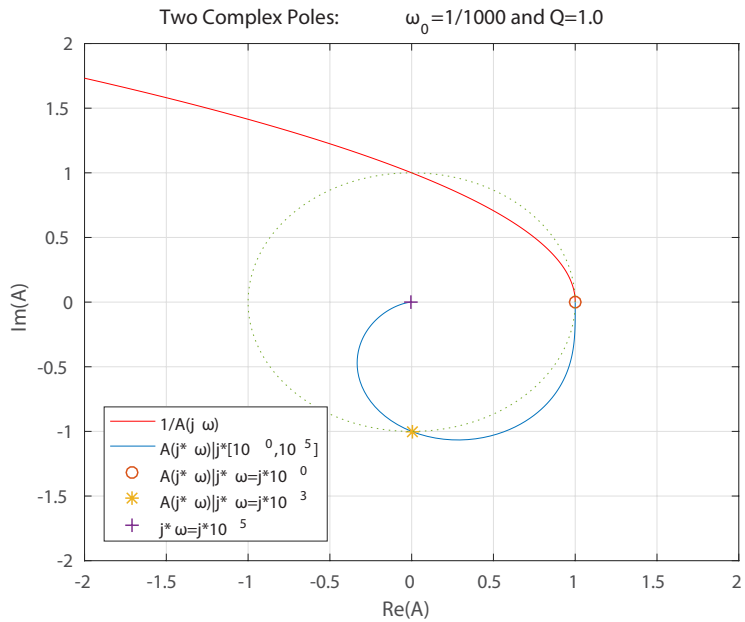


Resonance dependency on Q-factor (2/2)

$$\frac{1}{A_f(j\omega)} = \frac{1}{A_{f0}}\left[j^2\omega^2\frac{1}{\omega_{p1}\omega_{p2}} + j\omega\left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right) + 1\right]$$

Poles are either real or a conjugate complex pair





Relation Q and PM

Table 5.1 The relationship between PM, ω_t/ω_{eq}, Q factor, and percentage overshoot

PM (Phase margin)	ω_t/ω_{eq}	Q factor	Percentage overshoot for a step input
55°	0.700	0.925	13.3%
60°	0.580	0.817	8.7%
65°	0.470	0.717	4.7%
70°	0.360	0.622	1.4%
75°	0.270	0.527	0.008%
80°	0.175	0.421	-
85°	0.087	0.296	-

Table 5.1
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From Carusone, Johns and Martin book. ω_{eq} here is an approximation of the second order pole from all zeros and poles of higher order than one, i.e. the entire transfer function is approximated as a second order transfer function to derive a Q.

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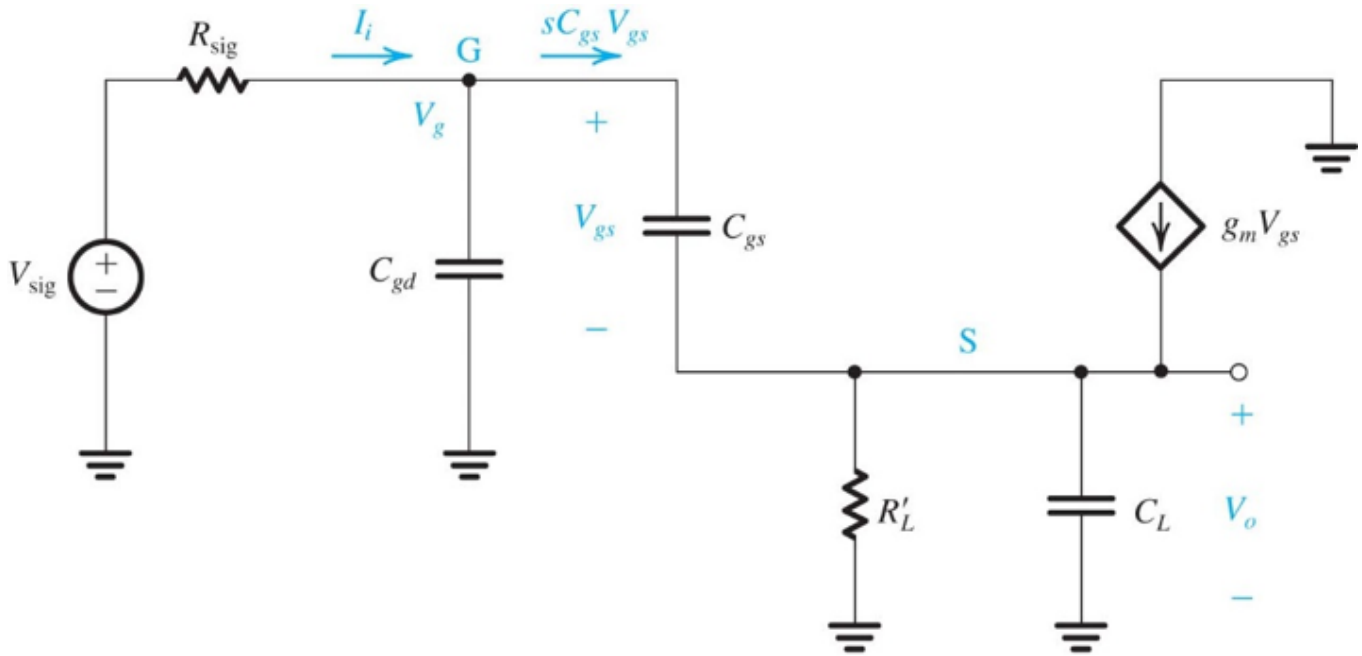
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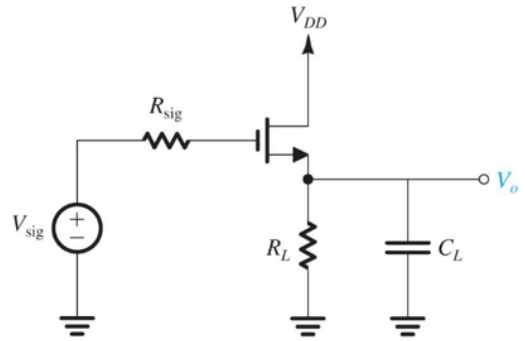
- Phase margin
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Intuition for Resonance/Instability



(c)

Source Follower HF Response



(a)

$$A(s) = A_M \frac{1 + \left(\frac{s}{\omega_z}\right)}{1 + b_1 s + b_2 s^2} = A_M \frac{1 + \left(\frac{s}{\omega_z}\right)}{1 + \frac{1}{Q}\frac{s}{\omega_0} + \frac{s^2}{\omega_0^2}} \quad (9.117, 9.126)$$

$$\omega_z = \frac{g_m}{C_{gs}} \quad (9.119)$$

$$b_1 = \left(C_{gd} + \frac{C_{gs}}{g_m R_L' + 1}\right) R_{sig} + \left(\frac{C_{gs} + C_L'}{g_m R_L' + 1}\right) R_L' \quad (9.120)$$

$$b_2 = \frac{(C_{gs} + C_{gd}) C_L + C_{gs} C_{gd}}{g_m R_L' + 1} R_{sig} R_L' \quad (9.121)$$

$$Q = \frac{\sqrt{b_2}}{b_1} (9.128)$$

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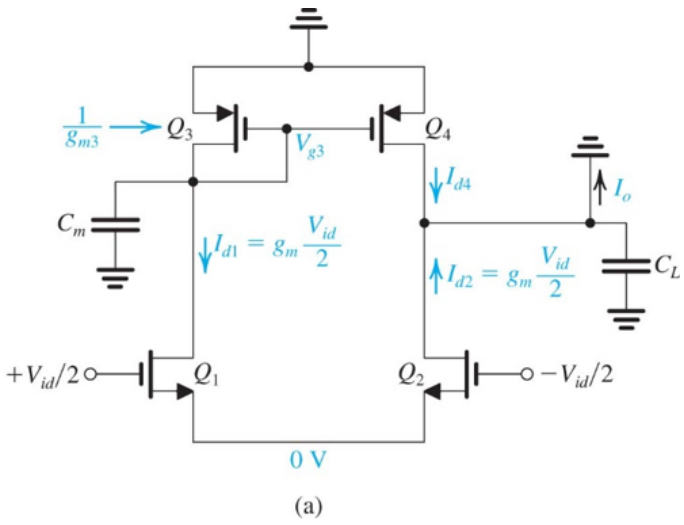
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HF Analysis of Current-Mirror-Loaded CMOS Amp (1/3)



(a)

Neglecting r_o in current mirror:

$$v_{g3}(sC_m + g_{m3}) + \frac{v_{id}}{2} g_m = 0$$

$$i_o = -v_{g3} g_{m3} + \frac{v_{id}}{2} g_m$$

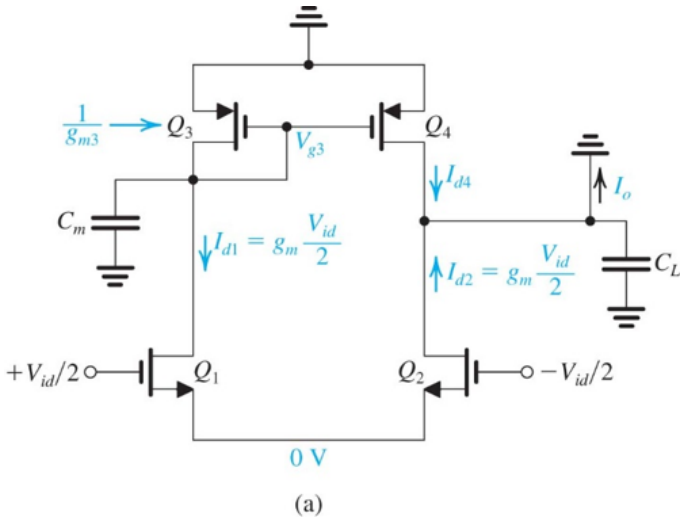
$$v_{g3} = -\frac{\frac{v_{id}}{2} g_m}{sC_m + g_{m3}}$$

$$i_o = \frac{v_{id}}{2} g_m \left(1 + \frac{g_{m3}}{sC_m + g_{m3}}\right)$$

$$i_o = \frac{v_{id}}{2} g_m \left(\frac{sC_m + 2g_{m3}}{sC_m + g_{m3}}\right)$$

$$\frac{i_o}{v_{id}} = g_m \frac{s\frac{C_m}{2g_{m3}} + 1}{s\frac{C_m}{g_{m3}} + 1}$$

HF Analysis of Current-Mirror-Loaded CMOS Amp (2/3)



(a)

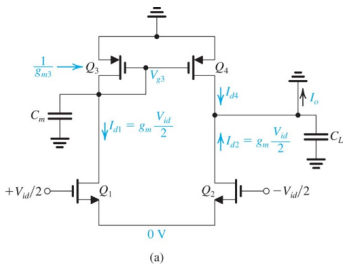
Neglecting r_o in current mirror:

$$G_M = g_m \frac{1 + s\frac{C_m}{2g_{m3}}}{1 + s\frac{C_m}{g_{m3}}}$$

$$\omega_{p2} = \frac{g_{m3}}{C_m}$$

$$\omega_z = \frac{2g_{m3}}{C_m}$$

HF Analysis of Current-Mirror-Loaded CMOS Amp (3/3)



$$\frac{v_o}{v_{id}} = g_m R_o \left(\frac{1 + s\frac{C_m}{2g_{m3}}}{1 + s\frac{C_m}{g_{m3}}} \right) \left(\frac{1}{1 + \frac{1}{sC_L R_o}} \right) \quad (9.144)$$

$$\omega_{p1} = \frac{1}{C_L R_o} \quad (9.145)$$

And ω_{p1} is usually clearly dominant.