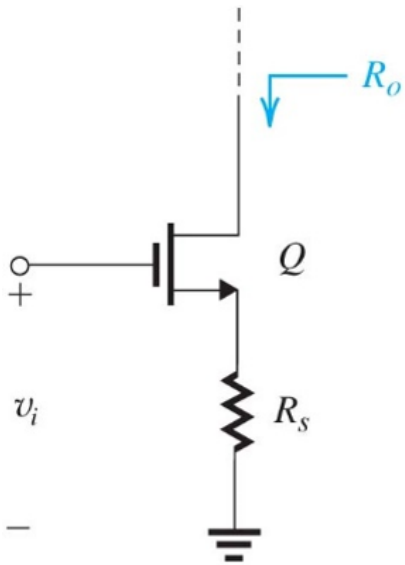
CS with source degeneration no R_L

Good as current source, e.g in current mirror, but not so good as amplifier.



$$R_o = R_s + r_o + g_m r_o R_s$$
$$R_o \simeq (1 + g_m R_s)\, r_o$$

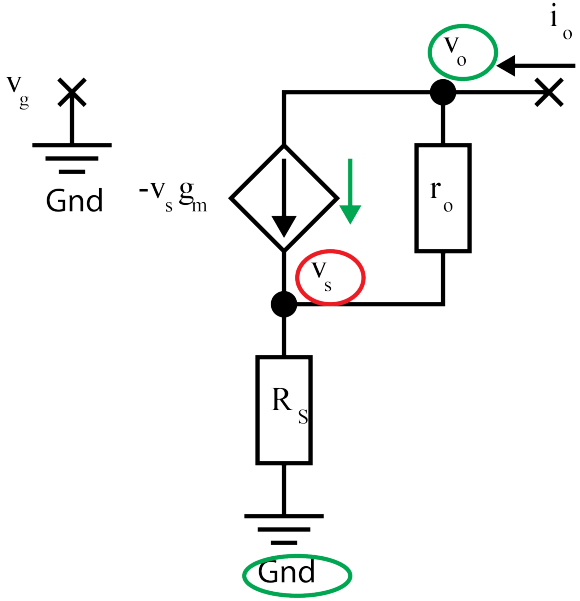
$$R_O \approx (1 + g_m R_S) r_o$$

$$g_m \rightarrow G_m = \frac{g_m}{1 + g_m R_s}$$

Much higher output resistance. Still no net-increase in gain!

Derivation of R_O (1/3)

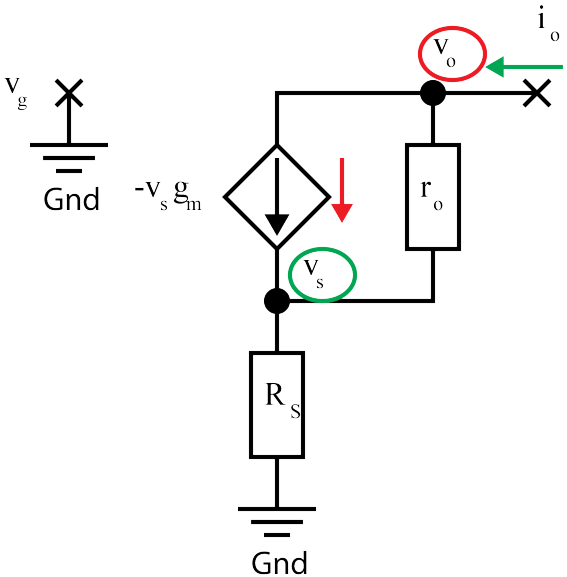
Kirchoff for node v_s:



$$v_s\left(\frac{1}{R_S}+\frac{1}{r_o}\right)=(0-v_s)g_m+0\frac{1}{R_S}+v_o\frac{1}{r_o}$$

Derivation of R_O (2/3)

Kirchoff for node v_o:



$$v_o \frac{1}{r_o} + (0 - v_s) g_m = v_s \frac{1}{r_o} + i_o$$

Derivation of R_O (3/3)

From node v_s:

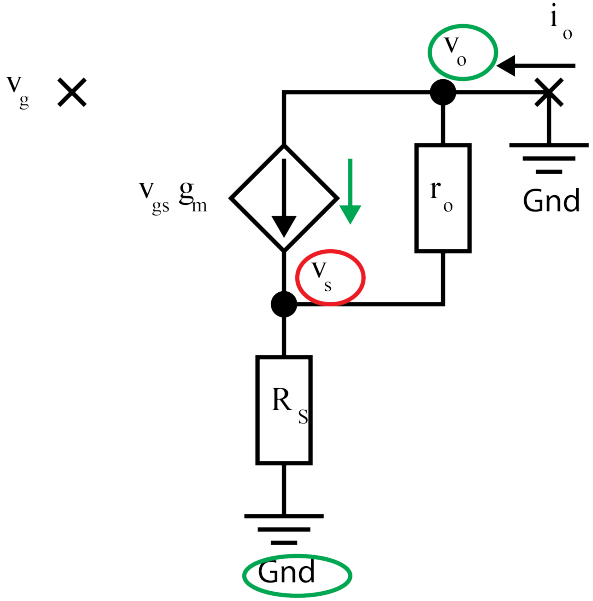
$$v_s = v_o \frac{\frac{1}{r_o}}{\frac{1}{R_S} + \frac{1}{r_o} + g_m}$$

Substitute into equation for node v_o:

$$\begin{aligned}
v_o \frac{1}{r_o} - v_s g_m &= v_s \frac{1}{r_o} + i_o \\
i_o &= v_o \frac{1}{r_o} - v_s (g_m + \frac{1}{r_o}) \\
\frac{i_o}{v_o} = \frac{1}{R_o} &= \frac{1}{r_o} - \left(\frac{\frac{1}{r_o}}{\frac{1}{R_S} + \frac{1}{r_o} + g_m} \right) (g_m + \frac{1}{r_o}) \\
&= \frac{\frac{1}{r_o}\left(\frac{1}{R_S} + \frac{1}{r_o} + g_m\right) - \frac{1}{r_o}(g_m + \frac{1}{r_o})}{\left(\frac{1}{R_S} + \frac{1}{r_o} + g_m\right)} \\
&= \frac{1}{(r_o + R_S + r_o R_S g_m)} \overset{R_S << r_o}{\approx} \frac{1}{r_o(1 + R_S g_m)}
\end{aligned}$$

Derivation of G_m (1/3)

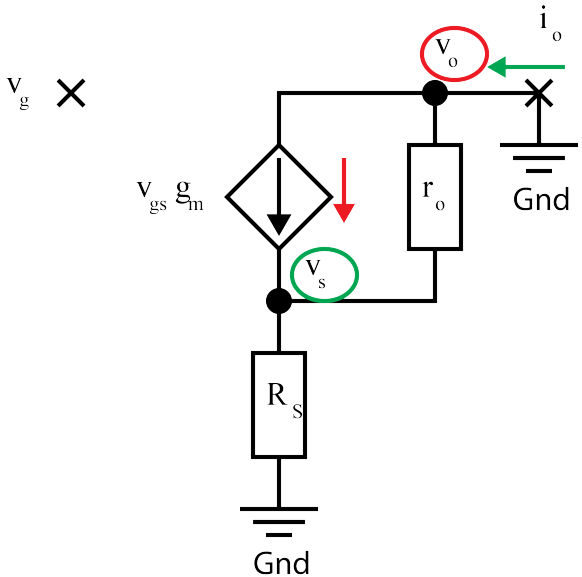
Kirchoff for node v_s:



$$v_s\left(\frac{1}{R_S}+\frac{1}{r_o}\right)=(v_g-v_s)g_m+0\frac{1}{R_S}+0\frac{1}{r_o}$$

Derivation of G_m (2/3)

Kirchoff for node v_o:



$$0\frac{1}{r_o} + (v_g - v_s)g_m = v_s\frac{1}{r_o} + i_o$$

Derivation of G_m (3/3)

From node v_s:

$$v_s = v_g \frac{g_m}{\left(\frac{1}{R_S} + \frac{1}{r_o} + g_m\right)}$$

Substitute into equation for node v_o:

$$\begin{aligned}
i_o &= v_g g_m - v_s\left(g_m + \frac{1}{r_o}\right) \\
G_m &= \frac{i_o}{v_g} = g_m - \frac{g_m}{\left(\frac{1}{R_S} + \frac{1}{r_o} + g_m\right)}\left(g_m + \frac{1}{r_o}\right) \\
&= g_m \frac{\left(\frac{1}{R_S} + \frac{1}{r_o} + g_m\right) - \left(g_m + \frac{1}{r_o}\right)}{\left(\frac{1}{R_S} + \frac{1}{r_o} + g_m\right)} \\
&= g_m \frac{1}{\left(1 + \frac{R_S}{r_o} + R_S g_m\right)}
\end{aligned}$$

CS with source degeneration summary

R_I	∞
R_o	$r_o + R_S(g_m r_o + 1) \approx r_o(1 + R_S g_m)$
G_M	$g_m \frac{1}{1+R_S\left(g_m+\frac{1}{r_o}\right)} \approx g_m \frac{1}{1+R_S g_m}$
A	$G_M(R_o \| \| R_L) = g_m r_o \frac{R_L}{R_L+R_o}$