

ØV4 — z-transformen

Innleveringsfrist: **11. september** 2020.

Ukeoppgavene skal løses selvstendig og vurderes i øvingstimene. Det forventes at alle har satt seg inn i fagets øvingsopplegg og godkjenningskrav for øvinger. Dette er beskrevet på hjemmesiden til IN3190: <http://www.uio.no/studier/emner/matnat/ifi/IN3190/h20/informasjon-om-ovingsopplegget/>

Oppgave 1 — Oppgave 4.11 fra Ambardar: Z-transform og ROC 2 Poeng

In case you happen to look up this task in the Ambardar book, please note that there is a misprint: the $|$ signs are lacking around α

The causal signal $x(n) = \alpha^n u(n)$ has the transform $X(z)$ whose ROC is $|z| > |\alpha|$.

Find the ROC of the Z-transform of the following:

a) $y(n) = x(n - 5)$

b) $p(n) = x(n + 5)$

c) $g(n) = x(-n)$

d) $h(n) = (-1)^n x(n)$

e) $p(n) = \alpha^n x(n)$

a) $|z| > |\alpha|$, b) $|z| > |\alpha|$, c) $|z| < 1/|\alpha|$, d) $|z| > |\alpha|$, e) $|z| > |\alpha^2|$.

Oppgave 2 — Oppgave 4.18 fra Ambardar: Egenskaper, z-transf. 2 Poeng

The Z-transform of the signal $x(n) = 2^n u(n)$ is $X(z)$.

Use properties to find the time signal corresponding to the following.

(a) $F(z) = X(-z)$ (b) $G(z) = X(1/z)$ (c) $H(z) = zX'(-z)$

(a) $f[n] = (-2)^n u[n]$, (b) $g[n] = 2^{-n} u[-n]$, (c) $h[n] = -n(-2)^n u[n]$

Oppgave 3 — Oppg. 3.1 fra Manolakis

2 Poeng

Determining the z-transform and sketch the pole-zero plot with the ROC for each of the following sequences

(a) $x[n] = \left(\frac{1}{2}\right)^n (u[n] - u[n - 10])$,

(b) $x[n] = \left(\frac{1}{2}\right)^{|n|}$,

(c) $x[n] = 5^{|n|}$,

(d) $x[n] = \left(\frac{1}{2}\right)^n \cos(\pi n/3) u[n]$.

Oppgave 4— Tidligere eksamensoppgave

2 Poeng

a)

z -transformen er kjent for å ha følgende egenskap ("tidsskift"):

Hvis

$$x(n) \xrightarrow{z} X(z)$$

så er

$$x(n-k) \xrightarrow{z} z^{-k}X(z).$$

Vis denne egenskapen.

b)

Bestem z -transformen til signalet

$$x_1(n) = \alpha^n u(n) = \begin{cases} \alpha^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

Er $x_1(n)$ et effekt-signal (power-signal) eller et energi-signal? Begrunn svaret!

c)

Bestem z -transformen til signalet

$$x_2(n) = u(n) - u(n-N).$$

Er $x_2(n)$ et effekt-signal (power-signal) eller et energi-signal? Begrunn svaret!

Oppgave 5— Exam task in 2012: Z -transform and region of convergence (ROC)

2 poeng

a) Find the Z -transform and ROC to the data sequence

$$x(n) = \begin{cases} \frac{1}{n} & \text{for } n \in [-2, 2], \quad n \in \mathbb{Z} \\ 0 & \text{for } n = 0, \\ 0 & \text{otherwise,} \end{cases}$$

where \mathbb{Z} represents the room of integer numbers.

b) Find the Z -transform and ROC to the function

$$x(n) = n2^{n-1}u(n-1)$$

Hint: You can apply some properties of the Z -transform to simplify the task – or you can go directly into the Z -transform definition and apply an appropriate variable substitution.

c) Consider two finite data sequences $x(n)$ and $h(n)$. Show that this rule for convolution holds:

$$x(n) * h(n) \xrightarrow{Z} X(z)H(z),$$

where $*$ denotes the convolution operator. Briefly explain why this property can be useful.