EX01 — TRIGONOMETRY, COMPLEX NUMBERS, AND GEOMETRIC SERIES

Delivery deadline: **Friday 2 September** 2022.

We highly recommend you to collaborate and interact with each other when working on these tasks. Still, the exercises shall be solved independently and must be assessed in the practice lessons. It is expected that everyone is familiar with the approval requirements for exercises. This is described on the course homepage:
`http://www.uio.no/studier/emner/matnat/ifi/IN3190/h22/curriculum-related/`

Observe: The sum of points in total is 13, but the maximum score obtainable in one week is 10.

Goal: The course IN3190/4190 requires a certain degree of knowledge of mathematics, both in the form of knowledge of theory and experience with use and practical calculation. The tasks here summarize some of the most important points you should know.

Exercise 1 Trigonometric functions 3 Points

a) Plot the following trigonometric functions either below each other (with parallel t-axes) or at the same axes, for the interval $-1 \leq t \leq 2.5$, so that you can see how they relate to each other in terms of frequency and phase shift. *1/2p per subtask*

1. $\cos(2\pi t)$
2. $\cos(2\pi t + \pi)$
3. $\cos(8\pi t)$
4. $\cos(4\pi t - \pi/3)$

b) Find the frequency, phase shift, and amplitude for the cosine functions in Figure 1. *1/4p per subtask*

Exercise 2 Discrete trigonometric functioner 2 Points

a) Which of the following *discrete* functions are periodic, and what are their fundamental periods (ie N)? *1/3p per subtask*

1. $\cos(0.5n + \pi/2)$
2. $\cos(\pi n + \pi/2)$
3. $\cos\left(\frac{\sqrt{2}}{2}\pi n\right)$

b) Which cosine function in discrete time is obtain by taking 5 samples per half period of a cosine in continuous time? *1/2p*

c) Which cosine function in discrete time is obtain by taking samples spaced 1 seconds of a cosine in continuous time with angular frequency 1? *1/2p*

Exercise 3 Complex numbers 3 Points

a) Perform the calculation in exponential/polar ($z = re^{j\phi}$) and/or cartesian ($z = a + jb$) form as given. Include intermediate calculations, especially when the answer is given. *1/4p per subtask*

1. z^* in exponential form
2. zz^* in exponential and cartesian form (what does this correspond to?)
3. z^k in exponential form
4. $z + z^*$ in exponential and cartesian form $\boxed{2r\cos\phi}$
5. $z - z^*$ in exponential and cartesian form
6. z^{-1} in exponential and cartesian form (**Observe:** the task is to find c and d such that $c + jd = \frac{1}{a+jb}$, and s and ϕ such that $se^{j\phi} = \frac{1}{re^{j\phi}}$.) $\boxed{\frac{a-jb}{a^2+b^2}, \frac{1}{r}e^{-j\phi}}$

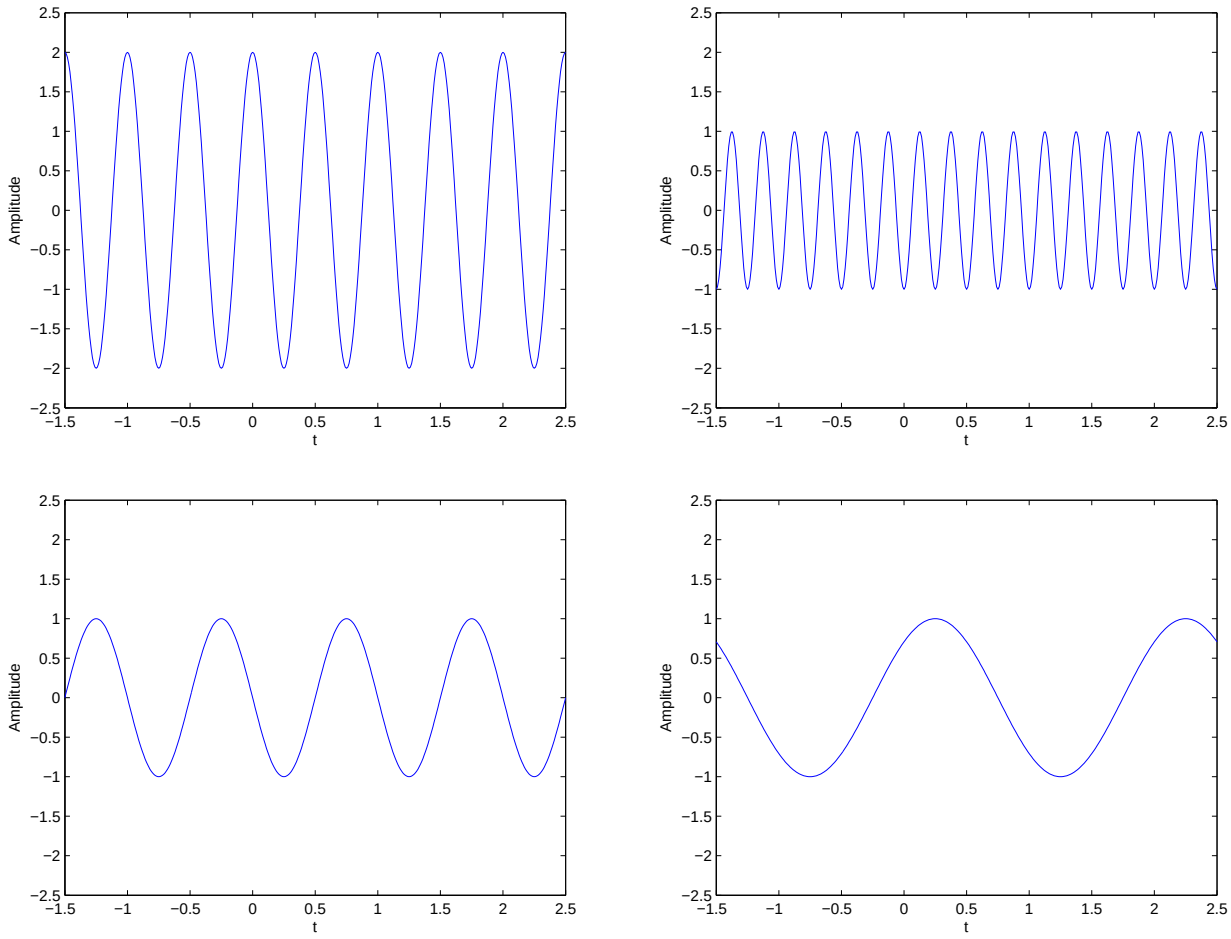


Figure 1: Find the frequency, phase shift and amplitude for the cosine functions shown.

7. Use 4. and 5. to find expressions for $\cos(\phi)$ and $\sin(\phi)$ using complex exponentials (Euler identities).
8. What is the difference between z^{-1} and z^*? Express z^{-1} from $|z|^2$ and z^*.

b) Express the following complex numbers in exponential form (where k us a random interger).
As an example, the numver 1 can be written as $1e^{j\cdot 2\pi k}$. *1/3p per subtask*

1. -1
2. $(-1)^k$
3. j^k

Exercise 4 Complex numbers 2 Points

a) Do the following calculations. If the answer is given, intermediate calculations must be given *1/4p per subtask*

1. $|3+j4|$ = ?
2. $\frac{1}{3+j4}$ in cartesian form = ? $\boxed{\frac{3}{25}-j\frac{4}{25}}$
3. $\frac{1+j2}{1+e^{j\pi/2}}$ in cartesian form = ?
4. $(-1)^n+e^{j\pi n}$ = ?, where n is an interger $\boxed{2\cdot(-1)^n}$

b) Show that *1p*

$$(\cos(\theta)+j\sin(\theta))^n=(\cos(\theta n)+j\sin(\theta n)) \qquad \boxed{\text{Ref. to de Moivre's formel}}$$

Exercise 5 Geometric series 3 Points

a) Calculate the calue of the following finite geometric series: *1/3p per subtask*

1. $\sum_{k=0}^{100} 23^k$ = ?
2. $\sum_{k=5}^{19} (4.5)^k$ = ? (Tips: devide the sum to change the summation limits).

b) Which of the following infinite geometric series converges and what are their value? *1/3p pr deloppg*

1. $\sum_{k=0}^{\infty} 1^k$
2. $\sum_{k=0}^{\infty} \left(\frac{3}{a}\right)^k, a>4$
3. $\sum_{k=-\infty}^{\infty} 2^{-k}$
4. $\sum_{k=-\infty}^{\infty} 2^{-|k|}$

c) Find the region of convergenve og the following infinite geometric series
If the answer is given, show the intermediate calculations. *1/3p per subtask*

1. $\sum_{k=0}^{\infty} \left(\frac{x}{2}\right)^k, x\in\mathbb{R}$
2. $\sum_{k=0}^{\infty} (x^{-1})^k, x\in\mathbb{R}$
3. $\sum_{k=0}^{\infty} 2^k z^{-k}, z\in\mathbb{C}$ $\boxed{|z|>2}$