Ex05 — Invers Z-transform & Transform Analysis

Delivery deadline: **30 September** 2022.

We highly recommend you to collaborate and interact with each other when working on these tasks. Still, the exercises shall be solved independently and must be assessed in the weekly submission sessions. It is expected that everyone is familiar with the approval requirements for exercises. This is described on the course homepage: `http://www.uio.no/studier/emner/matnat/ifi/IN3190/h22/curriculum-related/`

Observe: The sum of points in total is 14.5, while the maximum score obtainable in one week is 10.

Exercise 1 — Oppg 4.21, Ambardar: Poler og nullpunkter 2.5 Poeng

4.21 (Poles and Zeros) Make a rough sketch of the pole and zero locations of the z-transform of each of the signals shown in Figure P4.21.

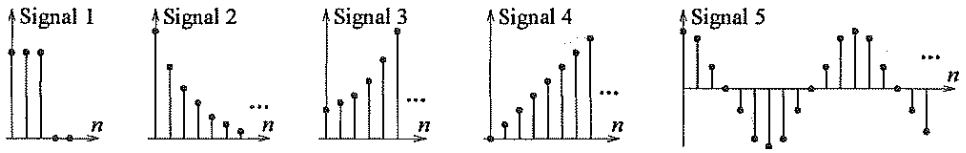


Figure P4.21 Figure for Problem 4.21

[**Hints and Suggestions:** Signal 1 has only three samples. Signals 2 and 3 appear to be exponentials. Signal 4 is a ramp. Signal 5 appears to be a sinusoid.]

Hint: Find $x(n)$ in each example, calculate $X(z)$ and find the poles/zeros.

Exercise 2 4 Poeng

Equations S_1 to S_7 describes 7 systems. Figure 1 shows 6 frequency responses, 4 pole-zero plots and 2 phase plots. Determine which of the 6 systems that corresponds to the 6 frequency responses, which 4 systems that corresponds to the 4 pole-zero plots, and which 2 systems that corresponds to the 2 phase plots. *1/3 p per correct match*

$$
\begin{aligned}
S_1 &: \; y[n] = 0.77y[n-1] + x[n] + x[n-1] \\
S_2 &: \; y[n] = 0.77y[n-1] + 0.77x[n] - x[n-1] \\
S_3 &: \; H(z) = \frac{1 - z^{-1}}{1 + 0.77z^{-1}} \\
S_4 &: \; H(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} \\
S_5 &: \; H(z) = 3 - 3z^{-1} \\
S_6 &: \; y[n] = \sum_{k=0}^{7} x[n-k] \\
S_7 &: \; y[n] = x[n] - x[n-1] + x[n-2] - x[n-3] + x[n-4] - x[n-5]
\end{aligned}
$$

Hint: Rewrite to $H(z)$ if possible. Count poles and zeros from polynomial order. Calcuate the response for some values of w. Observe that "1" on the w-axis in these plots means 1π. The frequency $w = 0$ corresponds to $z = 1$ in the corresponding pole-zero plot, and $w = \pi$ corresponds to $z = -1$.

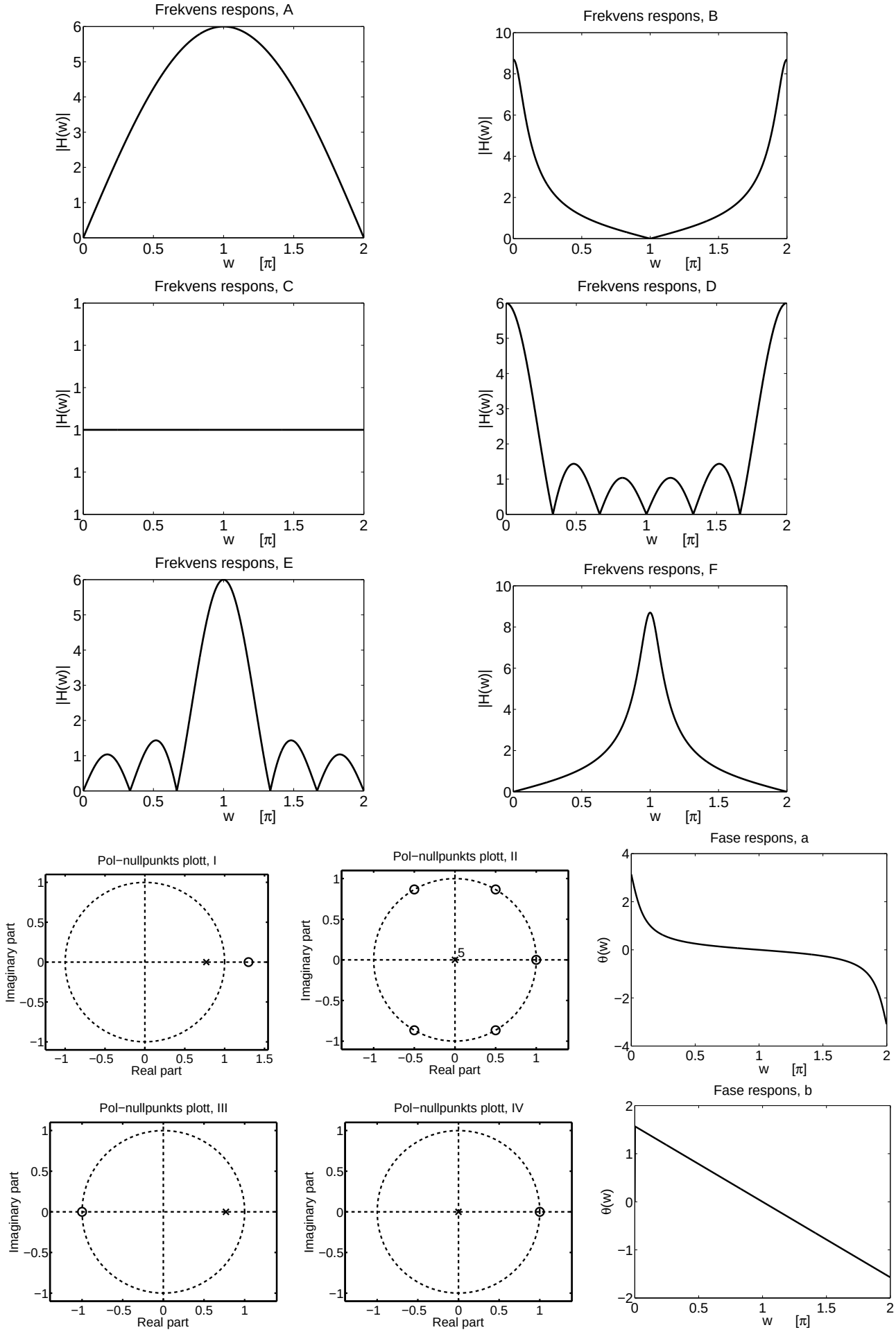


Figure 1: Merk at w i aksene for frekvensresponsene her er oppgitt fra $0-2$, hvor det menes $0-2\pi$.

Exercise 3 — Oppgave 4.35 fra Ambardar: Invers transform 2 Poeng

4.35 (Inverse Transforms) For each $X(z)$, find the signal $x[n]$ for each valid ROC.

$$\textbf{(a)}\ X(z) = \frac{z}{(z+0.4)(z-0.6)} \qquad \textbf{(b)}\ X(z) = \frac{3z^2}{z^2 - 1.5z + 0.5}$$

Bruk lineæritet og geometriske rekker.

a) $\boxed{\frac{X(z)}{z} = \frac{1}{z-0.6} - \frac{1}{z+0.4}}$ b) $\boxed{\frac{X(z)}{z} = \frac{6}{z-1} - \frac{3}{z-0.5}}$

Exercise 4— 3.4 from Manolakis 2 Poeng

Use the method of partial fraction expansion to determine the sequences corresponding to the following z-transforms:

(a) $X(z) = \frac{1-\frac{1}{3}z^{-1}}{(1-z^{-1})(1+2z^{-1})}$, all possible ROCs.

(b) $X(z) = \frac{1-z^{-1}}{1-\frac{1}{4}z^{-1}}$, $x[n]$ is causal. Hint: use Tabel 3.1 og partial fraction expansion

(c) BONUS: $X(z) = \frac{1}{(1-0.5z^{-1})(1-0.25z^{-1})}$, $x[n]$ is absolutely summable.

Table 3.1 Some common z-transform pairs

	Sequence $x[n]$	z-Transform $X(z)$	ROC
1.	$\delta[n]$	1	All z
2.	$u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
3.	$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
4.	$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
5.	$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
6.	$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
7.	$(\cos\omega_0 n)u[n]$	$\frac{1-(\cos\omega_0)z^{-1}}{1-2(\cos\omega_0)z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
8.	$(\sin\omega_0 n)u[n]$	$\frac{(\sin\omega_0)z^{-1}}{1-2(\cos\omega_0)z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
9.	$(r^n\cos\omega_0 n)u[n]$	$\frac{1-(r\cos\omega_0)z^{-1}}{1-2(r\cos\omega_0)z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$
10.	$(r^n\sin\omega_0 n)u[n]$	$\frac{(\sin\omega_0)z^{-1}}{1-2(r\cos\omega_0)z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$

Exercise 5— Fra Manolakis. : 4 Poeng

Determine the system function, magnitude response, and phase response of the following systems. Plot using Matlab/Python the magnitude and phase response. Use the pole-zero pattern to explain the shape of their magnitude response:

.5p for each magnitude and phase resp. 1p for each "plot and explanation"

(a) $y[n] = \frac{1}{2}(x[n] - x[n-1])$, $\boxed{|H(e^{j\omega})| = |sin(\omega/2)|,\ \angle H(e^{j\omega}) = \pi/2 - \omega/2}$

(b) $y[n] = \frac{1}{2}(x[n] - x[n-2])$, Hint: Reuse result from (a). $|H(e^{j\omega})| = |sin(\omega)|$, $\angle H(e^{j\omega}) = \pi/2 - \omega$

Hint: If you chose to express the respons as "cos(w) + j sin(w)", then use *Trigonometric identities*. If you keep it as complex exponentials, have a look at the solution and try to rearrange the result to give something similar.