Ex7 — Sampling

Delivery deadline: **28 October** 2022.

We highly recommend you to collaborate and interact with each other when working on these tasks. Still, the exercises shall be solved independently and must be assessed in the weekly submission sessions. It is expected that everyone is familiar with the approval requirements for exercises. This is described on the course homepage: `http://www.uio.no/studier/emner/matnat/ifi/IN3190/h22/curriculum-related/`

Observe:
The sum of points in total for the tasks distributed the current week is 13, while the maximum score obtainable in one week is 10. The score registered will hence be obtained using the formula `min(your_score_sum, 10)`.

Goals and reflections:

- The process of sampling a continuous time series can be represented as a multiplication by a train of impulses (a *Dirac comb*).
- It turns out that the Fourier domain representation of an impulse train is another impulse train.
- Multiplication in the time-domain is equivalent to convolution in the Fourier domain. Hence, sampling — which is a multiplication by a Dirac comb in the time domain — corresponds to *convolution* with a Dirac comb in the frequency domain.
- This may be seen as the fundamental reason why *samplig in the time-domain generates a periodization.*
- Moreover, the distance between the Diracs of the Fourier-domain impulse train is inversely proportional to the temporal sampling interval T.
- In addition, it turns out that this framework also explains why you need to sample at a sampling rate which is twice the maximum frequency where there are contents in the continuous time-series that we're sampling from.

Exercise 1— Fundamentals $\mathbf{2 + 2 + 2 = 6}$ points

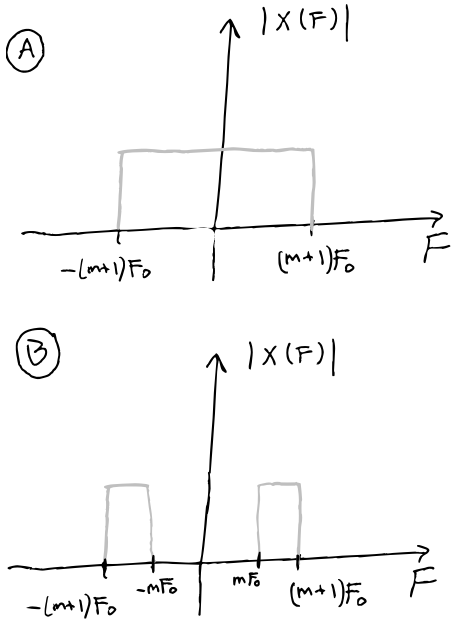


Figure 1: A / top panel: the magnitude spectrum of the signal $x_1(t)$. B / bottom panel: the magnitude spectrum of the signal $x_2(t)$.

A) Given a *band-limited signal* $x_1(t)$ in continuous time, with the magnitude spectrum $|X(F)|$, as displayed in Fig. **??** A.

What is the smallest sampling rate you can apply to this signal, which without introducing aliasing artifacts?

a) $2(m+1)F_0 - 2mF_0 = 2F_0$

b) $2mF_0 + 2F_0 = 2F_0(m+1)$

c) $2[(m+1)F_0 + mF_0] = 2F_0(2m+1)$

d) $(2m+1)F_0$

B) Given a *band-pass signal* $x_2(t)$ in continuous time, with the magnitude spectrum $|X(F)|$, as displayed in Fig. **??** B.

What is the smallest sampling rate you can apply to this signal, which without introducing aliasing artifacts?

a) $2(m+1)F_0 - 2mF_0 = 2F_0$

b) $2mF_0 + 2F_0 = 2F_0(m+1)$

c) $2[(m+1)F_0 + mF_0] = 2F_0(2m+1)$

d) $(2m+1)F_0$

C) You probe the light from a neutron star and the physical signal $x_a(t)$ registered by your sensor is a sinusoidal radio wave at 400 Hz.

Your system samples this signal using the sampling rate F_T Hz, to yield the digital signal $x(n)$ which is stored on disk. Later, you want to use $x(n)$ to reconstruct the original continuous-time signal. We denote the reconstructed signal $x_r(t)$ and this is what you later use for your analysis and interpretation.

a) What is the apparent frequency of the sinusoid of the reconstructed signal $x_r(t)$ if the sampling rate $F_T = 2000$ Hz?

b) What is the apparent frequency of the sinusoid of the reconstructed signal $x_r(t)$ if the sampling rate $F_T = 1500$ Hz?

c) What is the apparent frequency of the sinusoid of the reconstructed signal $x_r(t)$ if the sampling rate $F_T = 500$ Hz?

d) What is the apparent frequency of the sinusoid of the reconstructed signal $x_r(t)$ if the sampling rate $F_T = 450$ Hz?

e) What is the apparent frequency of the sinusoid of the reconstructed signal $x_r(t)$ if the sampling rate $F_T = 300$ Hz?

Exercise 2— Sampling and aliasing
Parts of a previous written exam — 7 Points

a) You use a sensor which samples a physical signal at 2000 Hz.

- What is the folding frequency, and what does this mean? 1 p.
- You intend to analyze a signal which is a 500 Hz cosine, $x_c(t)$. What is the period T of this signal According to the Shannon-criterion, what is the smallest appropriate sampling frequency of this signal? 1 p.
- You sample this signal using your sensor system. What is the expression for for your sampled signal $x(n)$? You may assume there are no quantization errors and that the phase of the signal is $\phi = 0$ at time $t = 0$. Is $x(n)$ periodic? Why? 1 p.

b)

- Sketch the frequency response $X_c(e^{j2\pi F})$ corresponding to the continuous signal $x_c(t)$ given in a). Make sure to include clear x and y axis labels. Let the x axis cover the frequency regime $[-6000, 6000]$ Hz. 1 p.
- In another plot, display the frequency response of your sampled signal $x(n)$. Use the same axes regimes as in a). 2 p.
- In what way do these plots differ, and why? 1 p.