Ex9 — IIR

Delivery deadline: **18 November** 2022.

We highly recommend you to collaborate and interact with each other when working on these tasks. Still, the exercises shall be solved independently and must be assessed in the weekly submission sessions. It is expected that everyone is familiar with the approval requirements for exercises. This is described on the course homepage: `http://www.uio.no/studier/emner/matnat/ifi/IN3190/h22/curriculum-related/`

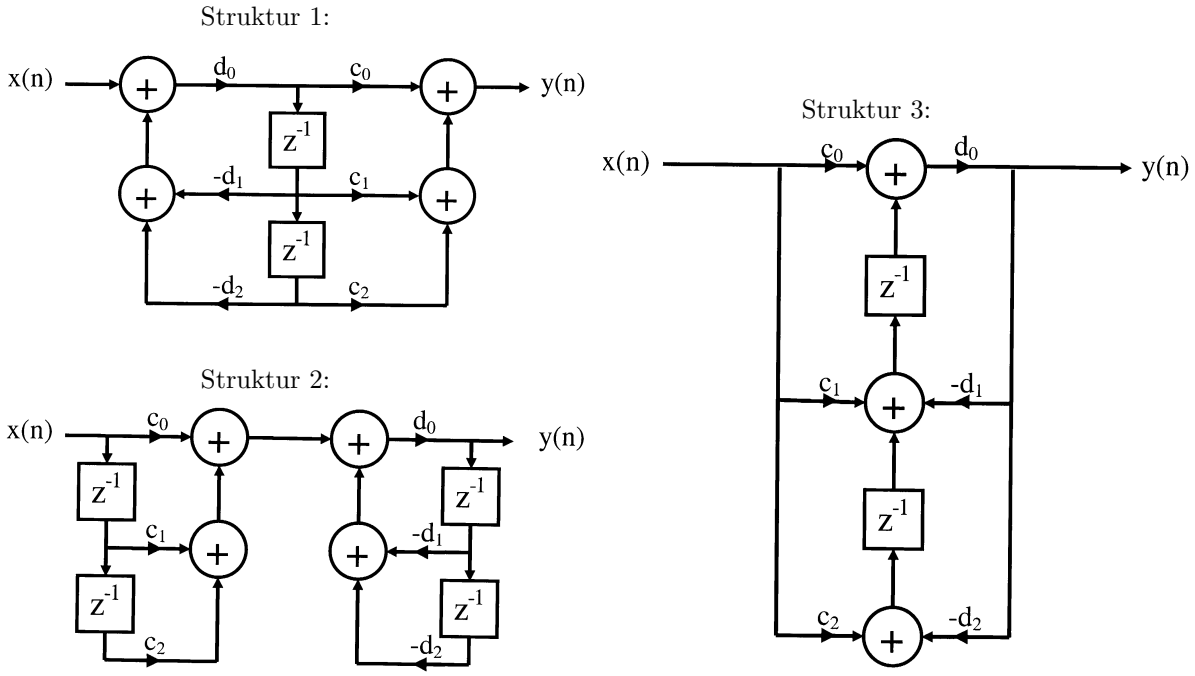
Observe:
The sum of points in total for the tasks distributed the current week is 12, while the maximum score obtainable in one week is 10. The score registered will hence be obtained using the formula `min(your_score_sum, 10)`.

Exercise 1 — Strukturer — 1 Points

Two system functions, $G(z)$ and $H(z)$, are given as:

$$G(z) = \frac{c_0 + c_1 z^{-1} + c_2 z^{-2}}{1/d_0 + d_1 z^{-1} + d_2 z^{-2}}$$

$$\text{and} \quad H(z) = \frac{1/d_0 + d_1 z^{-1} + d_2 z^{-2}}{c_0 + c_1 z^{-1} + c_2 z^{-2}}.$$



Figur 1: Filter structures

Figure 1 shows 3 different filter structurs. For filter structures 1-3, determine if the structure implements a filter described by system function $G(z)$, $H(z)$, or optionally another filter.

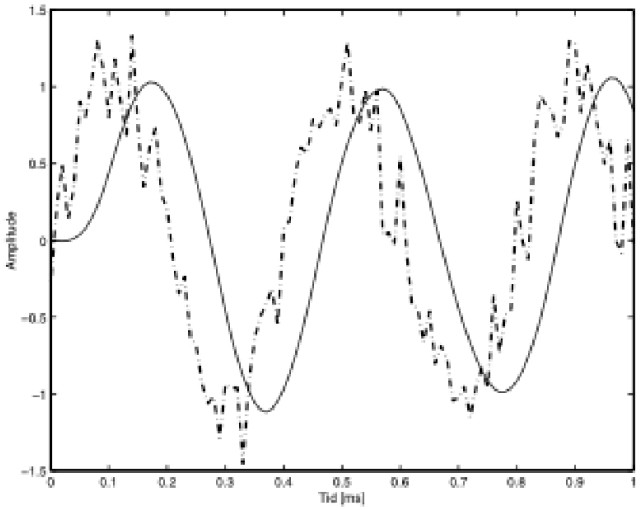
Exercise 2 — Filterdesign — 4 Points

1. Let $y(n) = x(R - n)$, where $x(n)$ is a real sequence. Show that the z-transform of $y(n)$ can be written as

$$Y(z) = z^{-R}\, X(1/z).$$

2. Now, let $R = 2$ and $x(n)$ causal. $X(z)$ has a zero point in $z = -1$ and three poles in $z)\frac{\sqrt{3}}{4}(1 \pm \jmath)$ and $z = \frac{1}{2}$. Specify the ROC for $X(z)$.

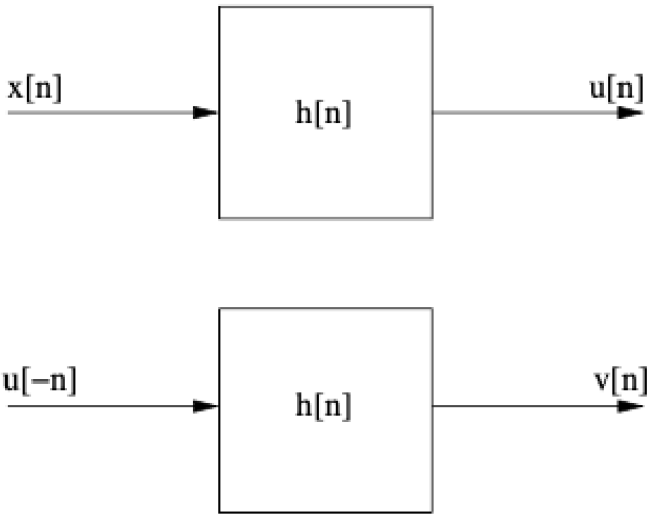
3. Draw the pole zero plot for $Y(z)$. Specify the ROC for $Y(z)$. Is $Y(z)$ stable? Give reasons for you answer!



Figur 2: Noisy signal (dashed line) filtered with a Butterworth filter (solid line).

4. In filter design we most often work with the magnitude of the frequency respons without paying too much attention to the phase of the system.

   In Figure 2 we see a noisy sinusoidal signal (dashed line) with frequency $f_0 = 2.5$ kHz. This signal has been filtered with a 3. order Butterworth filter with cut frequency $f_c = 3$ kHz. We observe that the filtered signal (splid line) is significantly delayed compared to the noisy signal. In some situations, we want the phase to be zero. This is not an option using causal filters. Here, we will study a non-causal approach for obtaining zero phase. To do so, we will use the results perviously obtained in this task.



Figur 3: Zero-phase system.

   Let $h(n)$ be a causal filter, and let $y(n) = v(-n)$, where $v(n)$ is given by the operation shown in the block diagram in Figure 3. What is the combined impulse response $h_{\text{total}}(n)$ for this system, in relation to $h(n)$? Show that $h_{\text{total}}(n)$ has zero phase.

Exercise 3— Tema: IIR filter. Bilinear transformation — 4 Points

a) Consider the following specifications for an analog low pass filter: Passband edge frequency $\Omega_p = 2$ rad/s, Stopband edge frequency $\Omega_s = 4$ rad/s, Passband ripple $A_p = 2$ dB, Stopband attenuation $A_s = 30$ dB. Using the functions `cheb1ord` and `cheby1` in Matlab/Python, determine the filter order N and response $H(s)$ of the continuous (analogue) Chebyshev type I filter satisfying those specifications.

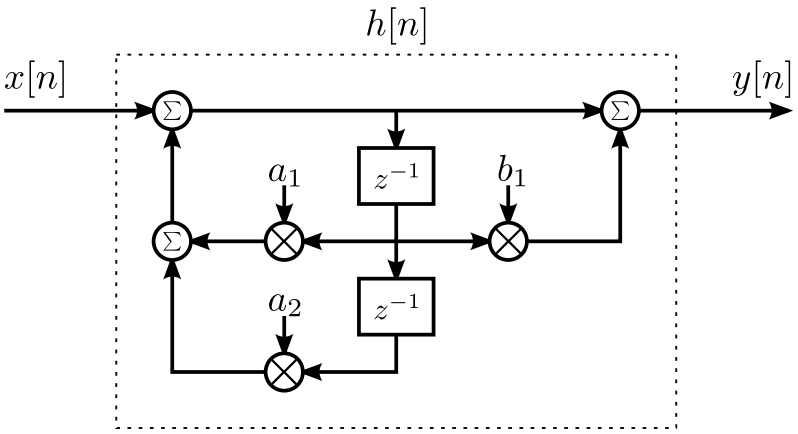
b) Using the `bilinear` function and the previously determined filter response $H(s)$, determine the response $H_d(z)$ of the discrete (digital) implementation of the analogue filter.

c) Setting $s = j\Omega$ and $z = e^{j\omega}$ plot the frequency responses of both continuous and discrete filters. Use the sampling frequency $\Omega_s = 7$ rad/s and plot the response for $0 \leq \Omega \leq 7$. How do the responses compare?

d) Verify your plots using the `freqs` and `freqz` Matlab/Python functions..

Exercise 4 — System analysis 3 Points

A filter is described as:



where Σ indicates sum and ⊗ indicates multiplication.

a) Find the system function $H(z)$.

In the following two exercises, assume that $H(z) = \frac{1+z^{-1}}{1-z^{-1}+0.5z^{-2}}$:

b) Plot poles and zeros, and determine whether this filter is stable and/or causal.

c) Find the expressions for the filter magnitude response $\left|H(e^{j\omega})\right|$ and phase response $\Theta_H(e^{j\omega})$. The final expressions should not contain complex numbers. Avoid spending time trying to simplify the terms.