Ex11 — FIR

Delivery deadline: **25 November** 2022.

We highly recommend you to collaborate and interact with each other when working on these tasks. Still, the exercises shall be solved independently and must be assessed in the weekly submission sessions. It is expected that everyone is familiar with the approval requirements for exercises. This is described on the course homepage: `http://www.uio.no/studier/emner/matnat/ifi/IN3190/h22/curriculum-related/`

Observe:
The sum of points in total for the tasks distributed the current week is 14, while the maximum score obtainable in one week is 10. The score registered will hence be obtained using the formula `min(your_score_sum, 10)`.

Exercise 1— FIR filter.
Exercise 10.28 from Manolakis & Ingle: 2 Points

a)

28. Design a lowpass FIR filter to satisfy the specifications: $\omega_p = 0.3\pi$, $A_p = 0.5$ dB, $\omega_s = 0.5\pi$, and $A_s = 50$ dB.
    (a) Use an appropriate fixed window to obtain a minimum length linear-phase filter. Provide a plot similar to Figure 10.12.
    (b) Repeat (a) using the Kaiser window and compare the lengths of the resulting filters.

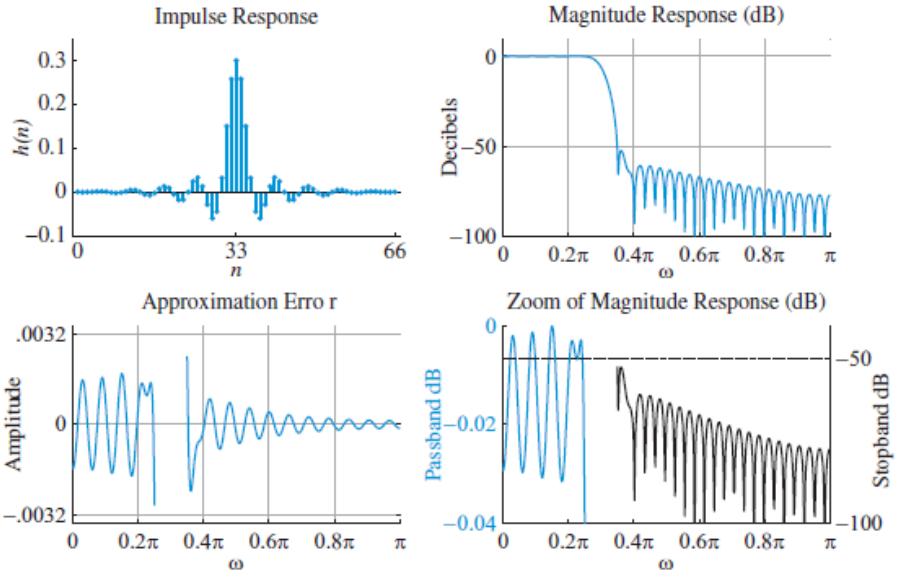


Figure 10.12 Impulse, approximation error, and magnitude response plots of the filter designed in Example 10.2 using a Hamming window to satisfy specifications: $\omega_p = 0.25\pi$, $\omega_s = 0.35\pi$, $A_p = 0.1$ dB, and $A_s = 50$ dB.

Hint a): Use 'Table 10.3' in this weeks lecture to determine which window and length to use.
Hint b): Use Matlab or Python to determine the filter order (function: 'kaiserord').

Exercise 2– Filter design Points: 5

Assume we want to find an approximation to an ideal lowpass filter with the following specifications:

$$|H_d(e^{j\omega})| = \begin{cases} 1, & |\omega| < \pi/2 \\ 0, & \pi/2 \leq |\omega| \leq \pi \end{cases}$$

a) The filter must be a causal FIR filter of length N (even). For which indexes of n must then $h(n)$ exist?

b) Assume the filter should have linear phase. Which type(s) of filter(s) can then be used, Type I, II, III, or IV? Justify your answer.

c) Assume the filter is designed by

$$h(n) = \mathrm{DFT}^{-1}[H_d(k)]$$

Find the values of $H_d(k)$ for $k = 0, 1, \ldots, N-1$, when the filter should be as specified above, i.e. length N (even), causal, and with linear phase.

d) Make a sketch of the frequency response of the filter $H(e^{j\omega})$, i.e. both phase and amplitude. Indicate at which frequencies the deviation in phase and amplitude will be larges compared to $H_d(e^{j\omega})$.

e) Assume then that you find the impulse response using the approach given in c). Which properties do you expect $h(n)$ to have? Find $h(n)$.

Exercise 3— Designing FIR filters (Matlab/Python) 4 Points

We want to create a low-pass filter with the following specifications:

$$0.99 \leq |H(e^{j\omega})| \leq 1.01, \quad 0 \leq \omega \leq 0.40\pi \tag{1}$$

$$|H(e^{j\omega})| \leq 0.001, \quad 0.42\pi \leq \omega \leq \pi \tag{2}$$

An FIR equiripple filter of order 255 can be used to create a filter that satisfies these specifications.

a) Estimate the order of a corresponding Kaiser filter using Matlab/Python. A function you could use is called 'kaiserord'.

b) Estimate the order of a corresponding elliptic filter using Matlab/Python. A function you could use is called 'ellipord'.

c) Compare the number of delays and multiplications needed to implement the three filters, the FIR equiripple, the filter from a), and b).

d) Plot the three filters using matlab/python (e.g. with fvtool or freqz) and discuss similarities and differences in terms of amplitude and phase for passband, stopband, transition band.
Hint: The filters can be constructed using the functions 'firpm' (matlab) / 'remez' (Python), 'fir1' (matlab) / 'firwin' (python) and 'ellip'.

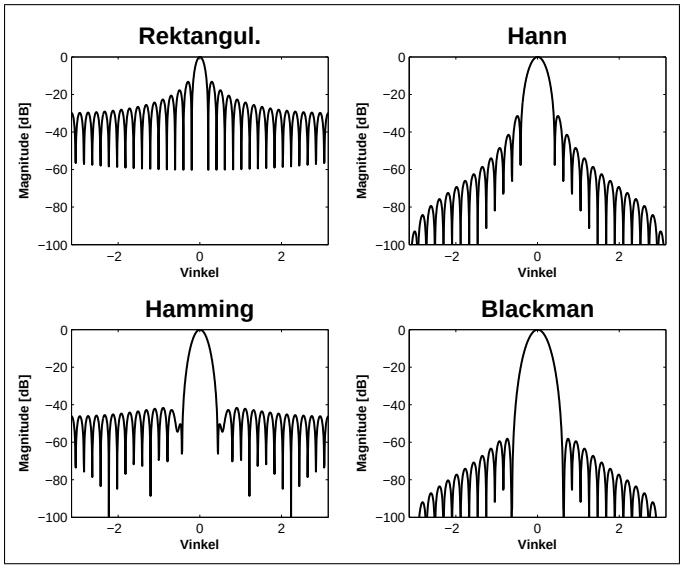
Exercise 4 Points: 3

a) Given the frequency response $H_{LP}(e^{j\omega})$ of an ideal low-pass filter with cutoff frequency ω_c, show that

$$h_{LP}[n] = \frac{\sin[\omega_c n]}{\pi n}, \quad -\infty < n < \infty$$

b) To obtain an FIR approximation, the ideal low-pass filter can be multiplied with a window $w[n]$ of length $2M+1$. Derive the DTFT $W_{Rect}(e^{j\omega})$ for a rectangular window of length $2M+1$, centered at $n = 0$.

c) The figure below shows the DTFT of rectangular, von hann (hanning), hamming and blackman windows for M=15. Based on the modulation theorem (product in time $\leftrightarrow$ periodic convolution in frequency), give a short justification for the advantages and disadvantages of using them different windows.



Make two sketches that highlight the differences in the DTFT of the truncated ideal low-pass filter response for the two window choices rectangular and blackman window.