Ex12 — Multirate

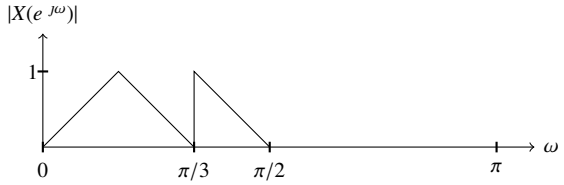
Delivery deadline: **2 December** 2022.

We highly recommend you to collaborate and interact with each other when working on these tasks. Still, the exercises shall be solved independently and must be assessed in the weekly submission sessions. It is expected that everyone is familiar with the approval requirements for exercises. This is described on the course homepage:
`http://www.uio.no/studier/emner/matnat/ifi/IN3190/h22/curriculum-related/`

Observe:
The sum of points in total for the tasks distributed the current week is 12.5, while the maximum score obtainable in one week is 10. The score registered will hence be obtained using the formula `min(your_score_sum, 10)`.

Exercise 1 — Up- and down-sampling 3 Points

A signal $x[n]$ has Fourier transformation $X(e^{j\omega})$ as shown in the figure below.



The signal is used as input to the systems I and II, which are defined in the figure below:

I: $x[n] \rightarrow \boxed{\uparrow 3} \rightarrow w_1[n] \rightarrow \boxed{H_0(z)} \rightarrow z_1[n] \rightarrow \boxed{\downarrow 2} \rightarrow y_1[n]$

II: $x[n] \rightarrow \boxed{\downarrow 2} \rightarrow w_2[n] \rightarrow \boxed{\uparrow 3} \rightarrow z_2[n] \rightarrow \boxed{H_0(z)} \rightarrow y_2[n]$

"$\rightarrow \boxed{\downarrow M} \rightarrow$" means downsampling by a factor M (keep every Mth sample) and "$\rightarrow \boxed{\uparrow N} \rightarrow$" means zero-interpolation by a factor N (insert $N-1$ zeros between each sample). $H_0(z)$ is an ideal low-pass filter with a cut-off frequency $w_c = \pi/3$ and gain equal to 2.

For both systems, sketch the Fourier transformation of the signals $w_1[n]$, $w_2[n]$, $z_1[n]$, $z_2[n]$, $y_1[n]$ og $y_2[n]$. Remember to use correctly marked axes with units for all plots. (1/2 point for each correct plot).

Exercise 2 — MA-filters 4 Points

An MA-filter of order $K-1$ is a causal FIR-filter with K coefficients, and an impulse response given as:

$$h[n] = \frac{1}{K} \sum_{k=0}^{K-1} \delta[n-k] \tag{1}$$

a) Find the frequency response $H(\Omega)$ for the MA-filter of order $K-1$.
b) Find the number of zeroes of an MA-filter of order $K-1$, and which frequencies they affect. Show your work.
c) Find all possible orders of an MA-filter which is to completely cancel a sinusoid with frequency $\Omega = \frac{\pi}{P}$, where P is an integer.
d) Show that an MA-filter of order $K-1$ can be implemented as an FIR-filter with impulse response $h_{FIR}[n]$ placed in cascade with an unstable IIR-filter with impulse response $h_{IIR}[n]$, where the impulse responses are given as:

$$h_{FIR}[n] = \frac{1}{K}(\delta[n] - \delta[n-K]), \quad h_{IIR}[n] = u[n] \tag{2}$$

Exercise 3 — Multiple choise 2.5 Points

In the following 5 subtasks, several possible answers are given, only one of these is correct. You must enter one and only one answer for each subtask. Correct answer gives 1/2 point, wrong answer gives -1/4 point, open answer gives 0 points. Hedgeing (more than one answer to a partial task) gives 0 points.

Task 3-1

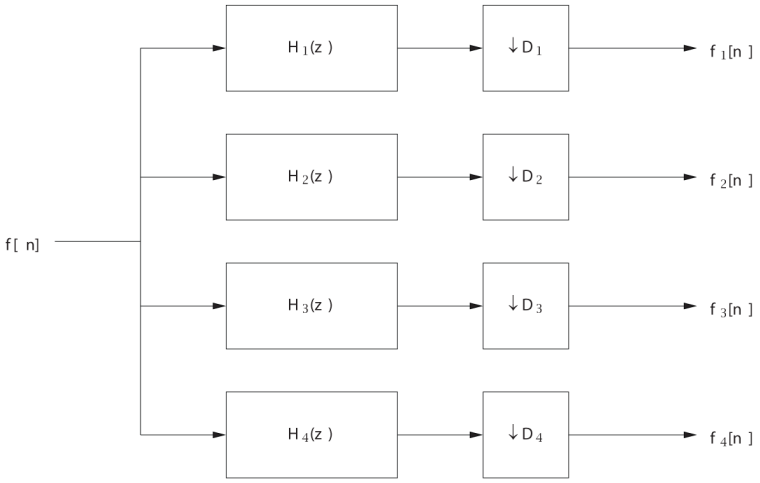
Figure 1 shows an application of multirate signal processing called a *filterbank*. It is used in e.g. MP3 encoding. The filter bank splits an input signal $f[n]$ (as here assumed to be sampled at the lowest frequency that satisfies Shannon's sampling theorem) into 4 sub-signals $f_1[n]$, $f_2[n]$, $f_3[n]$ and $f_4[n]$. The filters $H_1(z), \cdots, H_4(z)$ are ideal bandpass filters with magnitude response as shown in Figure 2. These filters are used to band limit the 4 sub-signals. Assume that the downsampling factors $D_1, \cdots, D_4$ are chosen so that each sub-signal is sampled with the lowest sampling rate which satisfies Shannon's sampling theorem. Decide which statement **a)** to **d)** that is correct when comparing the sum of the number-samples-per-second for the signals $f_1[n], \cdots, f_4[n]$ to the number-of-samples-per-second for the signal $f[n]$: 0.5 p.

a) The sum is larger.

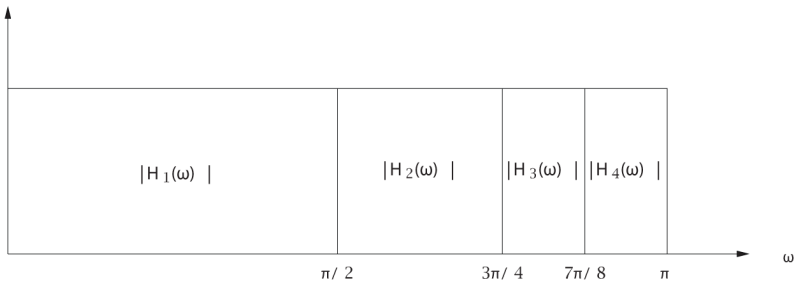
b) The sum is equally large.

c) The sum is less.

d) This is impossible to determine based on the information given in the text above.



Figur 1: Task 3-1



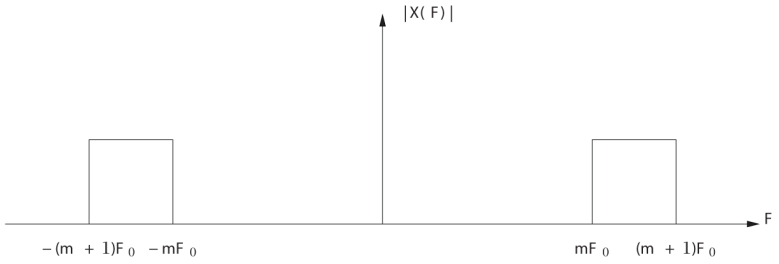
Figur 2: Task 3-1

Task 3-2

Given a signal $x(t)$ in continuous time, with magnitude spectrum $|X(F)|$ as shown in Figure 3. What is the smallest sufficient sampling rate for this signal? 0.5 p.

a) $2(m+1)F_0 - 2mF_0$ **b)** $2mF_0 + 2F_0$

c) $2[(m+1)F_0 + mF_0]$ **d)** $(2m+1)F_0$



Figur 3: Task 3-2

Task 3-3

In multirate signal processing, we can change the sampling rate from f_s to an arbitrary fraction $\frac{I}{D}f_s$ by combining upsampling by a factor I and downsampling by a factor D. If I and D are large non-prime numbers ($I = I_1 I_2 \cdots I_M$, $D = D_1 D_2 \cdots D_N$), then it is possible to implement upsampling and downsampling in M and N steps, respectively. Below is given four different implementations that all change the sampling rate from f_s to $\frac{16}{25}f_s$. $H_m(z)$ is a low-pass filter with cut-off frequency π/m. Which of the implementations has the least loss of information for any given input signals, where f_s corresponds to the Nyquist rate? 0.5 p.

a) $\rightarrow$ [$\uparrow 4$] $\rightarrow$ [$H_5(z)$] $\rightarrow$ [$\downarrow 5$] $\rightarrow$ [$\uparrow 4$] $\rightarrow$ [$H_5(z)$] $\rightarrow$ [$\downarrow 5$] $\rightarrow$

b) $\rightarrow$ [$\uparrow 4$] $\rightarrow$ [$H_4(z)$] $\rightarrow$ [$\uparrow 4$] $\rightarrow$ [$H_5(z)$] $\rightarrow$ [$\downarrow 5$] $\rightarrow$ [$H_5(z)$] $\rightarrow$ [$\downarrow 5$] $\rightarrow$

c) $\rightarrow$ [$H_5(z)$] $\rightarrow$ [$\downarrow 5$] $\rightarrow$ [$\uparrow 4$] $\rightarrow$ [$H_4(z)$] $\rightarrow$ [$\uparrow 4$] $\rightarrow$ [$H_5(z)$] $\rightarrow$ [$\downarrow 5$] $\rightarrow$

d) $\rightarrow$ [$H_5(z)$] $\rightarrow$ [$\downarrow 5$] $\rightarrow$ [$H_5(z)$] $\rightarrow$ [$\downarrow 5$] $\rightarrow$ [$\uparrow 4$] $\rightarrow$ [$H_4(z)$] $\rightarrow$ [$\uparrow 4$] $\rightarrow$ [$H_4(z)$] $\rightarrow$

e) It is impossible to decide without more information about the signal.

Task 3-4

Given a filter $h[n]$ with system function $H(z) = \frac{1-1.6z^{-1}+z^{-2}}{1-1.5z^{-1}+0.8z^{-2}}$. How much does this filter amplify the dc component of a signal (i.e. the component with frequency $f = 0$)? 0.5 p.

a) 0

b) 1.33

c) 1

d) 1.6

Task 3-5

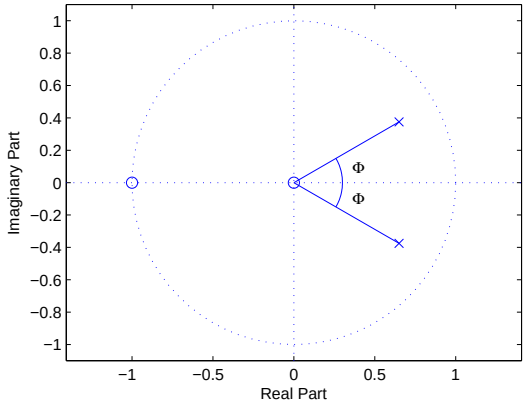
A filter has a pole-zero plot as given in Figure 4. Which of the following differential equations describes this system? 0.5 p.

a) $y[n] = -4.7880y[n-1] - 0.5630y[n-2] + x[n] + x[n-1]$

b) $y[n] = 1.2990y[n-1] - 0.5625y[n-2] + x[n] + x[n-1]$

c) $y[n] = -1.7190y[n-1] - 0.5620y[n-2] + x[n] + x[n-1]$

d) $y[n] = 2.5y[n-1] - 0.5610y[n-2] + x[n] + x[n-1]$



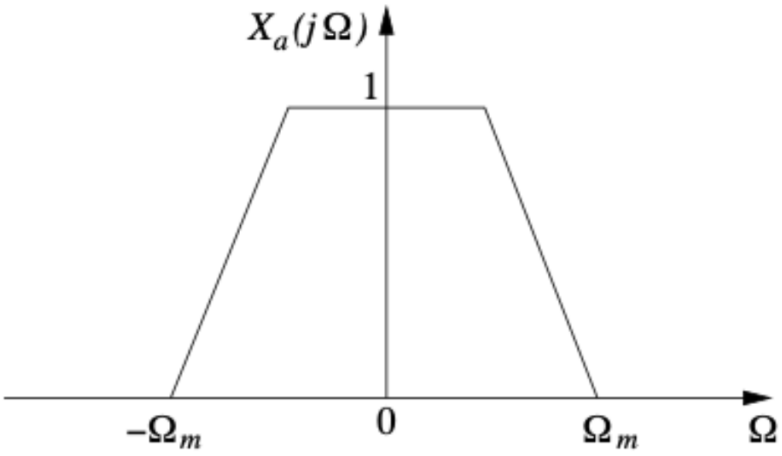
Figur 4: Task 3-5

Exercise 4 — MA-filters 3 Points

We have given a band-limited signal $x_a(t)$ with a continuous-time Fourier transform $X_a(j\Omega)$ which is symmetric about $\Omega = 0$, and given for $\Omega \geq 0$ as

$$X_a(j\Omega) = \begin{cases} 1, & 0 \leq \Omega \leq \Omega_m/2 \\ 2 - \frac{2\Omega}{\Omega_m}, & \frac{\Omega_m}{2} < \Omega \leq \Omega_m \\ 0, & \Omega > \Omega_m. \end{cases}$$

A sketch of $X_a(j\Omega)$ is shown in Figure 5. The signal $x_a(t)$ is then sampled to the sequence $x(n)$ with the sampling period $T = \pi/(2\Omega_m)$.



Figur 5: $x_a(t)$ shown in the frequency domain.

Task a

Sketch $X(e^{j\omega})$ for $0 \leq \omega < 2\pi$. Remember axis and labels. 1 p.

Task b

$x(n)$ is then downsampled by a factor 4 to $x_d(n)$. Explain the relationship between $X(e^{j\omega})$ and $X_d(e^{j\omega})$. Use this to make a sketch of $\left|X_d(e^{j\omega})\right|$ for $0 \leq \omega < 2\pi$. Remember axis and labels. 2 p.