SIMD and AVX512

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Outline

• What is SIMD
• History of SIMD with x86 hardware
• Matrix multiplication example with AVX512
  – Intrinsics
  – FMA
  – The Vector Class Library
  – Peak flops
• SIMD vertical operators
• SIMD horizontal operators
  – Reduction example with AVX512
  – 4x4 matrix transpose
• Gather/scatter
### Single Instruction Multiple Data

#### Scalar Operation

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>1</td>
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<td>9</td>
<td>=</td>
</tr>
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<td>+</td>
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</tr>
<tr>
<td>8</td>
<td>+</td>
<td>16</td>
<td>=</td>
</tr>
</tbody>
</table>

```java
for(int i=0; i<8; i++) {
    c[i] = a[i] + b[i];
}
```

1. Read 8 values from a
2. Read 8 values from b
3. Add 8 values
4. Write 8 values

24+ operations

#### SIMD Operation

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>1</td>
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<td>=</td>
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<tr>
<td>2</td>
<td></td>
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<td>=</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>16</td>
<td>=</td>
</tr>
</tbody>
</table>

Vec8d av = Vec8d().load(a);
Vec8d bv = Vec8d().load(b);
Vec8d cv = av + bv;
cv.store(c);

1. Read 8 values from a in one operation
2. Read 8 values from b in one operation
3. Add 8 values in one operation
4. Store 8 values in one operation

4 operations
History of Intel x86/x87 SIMD

Intel SIMD ISA Evolution

SIMD extensions on top of x86/x87

64b SIMD

128b SIMD

256b SIMD

512b SIMD

31 instructions

12 instructions

7 instructions

47 instructions

16 instructions

13 instructions

144 instructions

70 instructions

History AMD SIMD with x86/x87

64-bit wide, X87 register based
- 3DNow! 1998 single floating point (first for SIMD), AMD only, obsolete

128-bit wide
- SSE2/SSE3 2003
- SSE4a 2007
- XOP 2011 Many nice features of AVX512, obsolete (in theory)
- FMA4 2011 First x86 hardware with FMA, obsolete (in theory)
- SSSE3 2011 Intel had SSSE3 by 2004
- SSE4.1/4.2 2011 Intel had SSE4 by 2008
- AVX 2011 256-bit registers but 128-bit ports
- AVX2 2015 256-bit registers but 128-bit ports

256-bit wide
- AVX2 2019 256-bit ports with Zen2
## SIMD popularity

All 64-bit processors have SSE2

Steam (Valve gaming platform) Survey January 2019

<table>
<thead>
<tr>
<th>SIMD</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSE2</td>
<td>100%</td>
</tr>
<tr>
<td>SSE3</td>
<td>100%</td>
</tr>
<tr>
<td>SSSE3</td>
<td>97%</td>
</tr>
<tr>
<td>SSE4.1</td>
<td>95%</td>
</tr>
<tr>
<td>SSE4.2</td>
<td>94%</td>
</tr>
<tr>
<td>AVX</td>
<td>87%</td>
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<tr>
<td>AVX2</td>
<td>NA</td>
</tr>
<tr>
<td>AVX512</td>
<td>NA</td>
</tr>
<tr>
<td>AVX512</td>
<td>NA</td>
</tr>
<tr>
<td>SSE4a</td>
<td>17%</td>
</tr>
</tbody>
</table>

The fact that AMD did not offer: SSSE3, SSE4, and AVX until 2011 is likely the reason these technologies are less popular.

https://store.steampowered.com/hwsurvey
## SIMD Elements per Data Type

<table>
<thead>
<tr>
<th></th>
<th>char</th>
<th>short</th>
<th>int</th>
<th>int64_t</th>
<th>float</th>
<th>double</th>
</tr>
</thead>
<tbody>
<tr>
<td>MMX</td>
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<td>4</td>
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<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>SSE</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>0</td>
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<tr>
<td>SSE2</td>
<td>16</td>
<td>8</td>
<td>4</td>
<td>2</td>
<td>4</td>
<td>2</td>
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<tr>
<td>AVX</td>
<td>16</td>
<td>8</td>
<td>4</td>
<td>2</td>
<td>8</td>
<td>4</td>
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<td>AVX2</td>
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<td>8</td>
<td>4</td>
<td>8</td>
<td>4</td>
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<td>AVX512F</td>
<td>32</td>
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<td>16</td>
<td>8</td>
<td>16</td>
<td>8</td>
</tr>
<tr>
<td>AVX512BW</td>
<td>64</td>
<td>32</td>
<td>16</td>
<td>8</td>
<td>16</td>
<td>8</td>
</tr>
</tbody>
</table>
Dense Matrix Multiplication

The goal is to calculate: $C = A*B$

$A$, $B$, and $C$ are $n \times n$ matrices

Each element of $C$ can be written:

$$C_{i,j} = \sum (A_{i,k} \cdot B_{k,j}) \text{ for } k = 1 \text{ to } n$$
Matrix multiplication in C

```c
void gemm(double *a, double *b, double *c, int n) {
    for(int i=0; i<n; i++) {
        for(int j=0; j<n; j++) {  // row(k,C)
            for(int k=0; k<n; k++) {
                c[i*n + j] = a[i*n + k]*b[k*n + j] + c[i*n + j];
            }
        }
    }
}
```

gemm (general matrix multiplication)

\[ M_{i,j} = M[i*n + k] \]

In C, arrays start at 0 not 1 (unlike FORTRAN).

The inner loop over \( k \) is cache unfriend as every iteration has to reads with a stride \( k*n \) (\( b[k*n + j] \)).

For a 1024x1024 matrix this code gets 0.3 % of the peak FLOPS.
Dense Matrix Multiplication

The i-th row of \((C = A \times B)\) can be written as

\[
\text{row}(i, C) = \sum (A_{i,k} \times \text{row}(k, B)) \text{ for } k = 1 \text{ to } n
\]
2x2 matrix multiplication example

\[
\begin{array}{ccc}
\text{A} & \text{B} \\
\text{row(1,A)} = 1 & 2 & \text{row(1,B)} = 5 & 6 \\
\text{row(2,A)} = 3 & 4 & \text{row(2,B)} = 7 & 8 \\
\end{array}
\]

\[
\begin{align*}
\text{row(1,C)} &= A_{1,1} \times \text{row(1,B)} + A_{1,2} \times \text{row(2,B)} \\
&= 1 \times (5 \ 6) + 2 \times (7 \ 8) \\
&= (1 \ 1) \times (5 \ 6) + (2 \ 2) \times (7 \ 8) \\
&= (5 \ 6) + (14 \ 16) = (19 \ 22) \\
\text{row(2,C)} &= A_{2,1} \times \text{row(1,B)} + A_{2,2} \times \text{row(2,B)} \\
&= 3 \times (5 \ 6) + 4 \times (7 \ 8) \\
&= (3 \ 3) \times (5 \ 6) + (4 \ 4) \times (7 \ 8) \\
&= (15 \ 18) + (28 \ 32) = (43 \ 50)
\end{align*}
\]
Swapping the j and k loop

```c
void gemm2(double *a, double *b, double *c, int n) {
    for(int i=0; i<n; i++) {
        for(int k=0; k<n; k++) {  // row(k,C)
            for(int j=0; j<n; j++) {
                c[i*n + j] = a[i*n + k]*b[k*n + j] + c[i*n + j];
            }
        }
    }
}
```

The function gemm2 swaps the k and j loops.
The loop over j calculates: $A_{i,k} \times \text{row}(k,B)$
The loop over k calculates: $\text{row}(i,C) = \text{Sum } A_{i,k} \times \text{row}(k,B)$
The loop over i calculates: $C = A \times B$

gemm2 is more than six times faster than gemm
Matrix multiplication

Recall \( \text{row}(i,C) = \sum A_{i,k} \cdot \text{row}(k,B) \) for \( k = 1 \) to \( n \)

```c
void gemm2(double *a, double *b, double *c, int n) {
    for(int i=0; i<n; i++) {
        for(int k=0; k<n; k++) {  // Sum \( A_{i,k} \cdot \text{row}(k,B) \)
            foo(a[i*n + k], &b[k*n], &c[i*n], n);
        }
    }
}
```

```c
void foo(double s, double *b, double *c, int n) {
    for(int i=0; i<n; i++) {
        c[i] = s*b[i] + c[i];
    }
}
```

- \text{foo} calculates \( A_{i,k} \cdot \text{row}(k,B) \)
- \text{foo} is the function we will optimize with SIMD
AVX512 with intrinsics

The preferred method for low programming is using *intrinsics* instead of assembly. This is because intrinsics are much more convenient (except for their names).

```c
void foo(double s, double *b, double *c, int n) {
   __m512d sv = _mm512_set1_pd(s);           // broadcast s to all 8 values
   for(int i=0; i<n/8; i++) {
      __m512d cv = _mm512_loadu_pd(&c[8*i]);  // load 8 double from c
      __m512d bv = _mm512_loadu_pd(&b[8*i]);  // load 8 double from b
      __m512d t1 = _mm512_mul_pd(bv, sv);     // sv*bv
      cv = _mm512_add_pd(t1, cv);            // cv = sv*av + cv
      _mm512_storeu_pd(&c[8*i],cv);          // write cv to c array
   }
}
```

Notice that the loop goes over \( n/8 \) elements. In principle this loop can be eight times faster than the scalar loop in the function foo.

gemm3 is more than 10 times faster than gemm
Intrinsics and assembly

Use GodBolt (https://godbolt.org/) to see the assembly for GCC, ICC, Clang, and MSVC

```c
#include <x86intrin.h>

void foo(double s, double *b, double *c, int n) {
    __m512d sv = _mm512_set1_pd(s); //
    for(int i=0; i<n; i++) { //
        __m512d cv = _mm512_loadu_pd(&c[8*i]); //
        __m512d bv = _mm512_loadu_pd(&b[8*i]); //
        __m512d t1 = _mm512_mul_pd(bv, sv); //
        cv = _mm512_add_pd(t1, cv); //
        _mm512_storeu_pd(&c[8*i],cv); //
    }
}
```

Assembly:
```
xor eax, eax
vbroadcastsd zmm16, xmm0
test edx, edx
jle ..B1.5 # Prob 10%
    # Preds ..B1.1 ..B1.3
lea ecx, DWORD PTR [rax*8]
inc eax
movsxd rcx, ecx
vmulpd zmm17, zmm16, ZMMWORD PTR [rdi+rcx*8]
vaddpd zmm18, zmm17, ZMMWORD PTR [rsi+rcx*8]
vmovups ZMMWORD PTR [rsi+rcx*8], zmm18
cmp eax, edx
jl ..B1.3 # Prob 82%
    # Preds ..B1.3 ..B1.1
vzeroupper
ret
```
# Intrinsic

**Intrinsics**

- `#include <x86intrin.h>` for GCC, ICC, and Clang. For MSVC see [https://stackoverflow.com/questions/11228855/headers-files-for-x86-simd-intrinsics](https://stackoverflow.com/questions/11228855/headers-files-for-x86-simd-intrinsics)

<table>
<thead>
<tr>
<th></th>
<th>Data type</th>
<th>Intrinsic prefix</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SSE</strong></td>
<td>__m128 (float)</td>
<td><em>mm</em></td>
</tr>
<tr>
<td></td>
<td>__m128d (double)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>__m128i (int)</td>
<td></td>
</tr>
<tr>
<td><strong>AVX</strong></td>
<td>__m256 (float)</td>
<td><em>mm256</em></td>
</tr>
<tr>
<td></td>
<td>__m256d (double)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>__m256i (int)</td>
<td></td>
</tr>
<tr>
<td><strong>AVX512</strong></td>
<td>__m512 (float)</td>
<td><em>mm512</em></td>
</tr>
<tr>
<td></td>
<td>__m512d (double)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>__m512i (int)</td>
<td></td>
</tr>
</tbody>
</table>
Advantages of Intrinsics

Advantages of intrinsics over assembly:

- You don't have to worry about 32-bit and 64-bit mode.
  - 32-bit and 64-bit assembly syntax are incompatible.

- You don't need to worry about registers and register spilling.
  - There are a finite number of logical registers, in assembly if you use them all, and need more, then you have to manage this manually.

- No need to worry about AT&T and Intel assembly syntax.
  - There are two competing dialects of assembly (like bokmål and nynorsk).

- No need to worry about function calling conversions.
  - 32-bit and 64-bit and Windows, MacOS, and Linux all use different function calling conventions which you have to adhere to in assembly.

- The compiler can optimize intrinsics further which it won't do with inline assembly.

- Intrinsics are compatible (mostly) with the four most popular C/C++ compilers: GCC, ICC, Clang, MSVC.
Disadvantages of Intrinsics

The main disadvantage of intrinsics are:

- Complicated Names: e.g. \texttt{__m512d v1 = __mm512_fmadd_pd(sv, bv, cv)}
- Only compatible with x86 hardware.
- If you want exact control you still need assembly.
- Some hardware features can only be accessed with assembly.
Fused Multiply Add (FMA)

\[ \text{fma}(a,b,c) \text{ does } a \cdot b + c \text{ in one operation} \]
This can both improve performance and accuracy.

Without FMA
\[ p = \text{round}(a \cdot b) \]
\[ s = \text{round}(p + c) : \text{two operations, round twice} \]

With FMA
\[ s = \text{round}(a \cdot b + c) : \text{one operation, round once} \]
void foo_intrin_fma(double s, double *b, double *c, int n) {
    __m512d sv = _mm512_set1_pd(s);             // broadcast s to all 8 values
    for(int i=0; i<n/8; i++) {
        __m512d cv = _mm512_loadu_pd(&c[8*i]);    // load 8 double from c
        __m512d bv = _mm512_loadu_pd(&b[8*i]);    // load 8 double from b
        cv = _mm512_fma_pd(sv, bv, cv); // cv = sv*bv + cv
        _mm512_storeu_pd(&c[8*i], cv);            // write cv to c array
    }
}

Modern compilers will likely convert

    t1 = _mm512_mul_pd(bv, sv);       // sv*bv
    cv = _mm512_add_pd(t1, cv);      // cv = sv*av + cv

to

    cv = _mm512_fmaadd_pd(sv, bv, cv); // cv = sv*bv + cv

FMA4: four operands e.g. d = a*b + c
FMA3: three operands e.g. c = a*b + c

Intel proposed FMA4, AMD implemented it first, Intel came out with FMA3 instead.
The Vector Class Library (VCL)

C++ library for SIMD operations

https://www.agner.org/optimize/#vectorclass

```c
void foo_VCL(double s, double *b, double *c, int n) {
    Vec8d sv = s;                       // broadcast s to all 8 values
    for(int i=0; i<n/8; i++) {
        Vec8d cv = Vec8d().load(&c[8*i]); // load 8 double from c
        Vec8d bv = Vec8d().load(&b[8*i]); // load 8 double from a
        cv = sv*bv + cv;
        cv.store(&c[8*i]);                // write cv to c array
    }
}
```

In most cases you will get the same performance as using intrinsics.
Micro-operations go to up to 8 ports
Arithmetic operations go to ports 0, 1, and 5
Fused-AVX512 uses 256-bit ports 0 and 1 to emulate a 512-bit operation
Higher-end AVX512 hardware has a dedicated 512-bit port (port 5)
Hardware without (with) a 512-bit port is called single issue (dual issue).

https://en.wikichip.org/wiki/intel/microarchitectures/skylake_(server)
## Peak FLOPS for GEMM

<table>
<thead>
<tr>
<th>Instruction SET</th>
<th>FLOP/cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intel SSE2 Core2 (2006)</td>
<td>(2-wide MUL)+(2-wide ADD)</td>
</tr>
<tr>
<td>AMD AVX Bulldozer (2011)</td>
<td>(2 FLOP/FMA)*(4-wide FMA)</td>
</tr>
<tr>
<td>Intel AVX (2011)</td>
<td>(4-wide MUL)+(4-wide ADD)</td>
</tr>
<tr>
<td>AMD AVX ZEN (2017)</td>
<td>(2 FLOP/FMA)*(4-wide FMA)</td>
</tr>
<tr>
<td>Intel AVX2 (2013)</td>
<td>(2 FLOP/FMA)((4-wide FMA)+(4-wide FMA))</td>
</tr>
<tr>
<td>AMD AVX2 ZEN2 (2019?)</td>
<td>(2 FLOP/FMA)((4-wide FMA)+(4-wide FMA))</td>
</tr>
<tr>
<td>AVX512 (single issue)</td>
<td>(2 FLOP/FMA)*(8-wide FMA)</td>
</tr>
<tr>
<td>AVX512 (dual issue)</td>
<td>(2 FLOP/FMA)((8-wide FMA)+(8-wide FMA))</td>
</tr>
</tbody>
</table>

Peak double floating point flops, for single floating point double the FLOP/cycle
Performance tests for a 1024x1024

1024x1024 matrix: floating point operations = 2*n³ = 2.15 GFLOP

Intel Xeon 6142 CPU (duel issue AVX512) @ 3.5 GHz and AVX512
Peak FLOPS for a single core = 32*3.5GHZ = 112 GFLOPS

<table>
<thead>
<tr>
<th></th>
<th>Time</th>
<th>GFLOPS</th>
<th>GFLOPS/peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>gemm1</td>
<td>4.0 s</td>
<td>0.5</td>
<td>0.3 %</td>
</tr>
<tr>
<td>gemm2</td>
<td>0.7 s</td>
<td>3.0</td>
<td>2.7 %</td>
</tr>
<tr>
<td>gemm2 with foo_intrinsic</td>
<td>0.35 s</td>
<td>6.0</td>
<td>5.3 %</td>
</tr>
<tr>
<td>gemm2 with foo_intrinsic_fma</td>
<td>0.35 s</td>
<td>6.0</td>
<td>5.3 %</td>
</tr>
</tbody>
</table>

The code is memory bandwidth bound. To get closer to the peak flops (I have achieved over 70% with more advanced code) you have you use loop tiling.
Frequency scaling and dark silicon

Intel Xeon 6142 CPU 16 cores
One of the big problems with AVX and especially AVX512 is that they use so much power that the processor can't operate at its thermal design power (TDP).

<table>
<thead>
<tr>
<th>Instruction Set</th>
<th>1 or 2 cores</th>
<th>16 cores</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSE</td>
<td>3.7 GHz</td>
<td>3.3 GHz</td>
</tr>
<tr>
<td>AVX</td>
<td>3.6 GHz</td>
<td>2.9 GHz</td>
</tr>
<tr>
<td>AVX512</td>
<td>3.5 GHz</td>
<td>2.2 GHz</td>
</tr>
</tbody>
</table>

It's not possible to use all the cores at max frequency with AVX or AVX2. Some system administrators disable AVX512 on servers because code with AVX512 from a single user can greatly effect the performance of another user’s code without AVX512.

Dark silicon means that not all the logic (silicon) can be enabled at the TDP and so must be disabled or clocked down.
Operators in C

• Arithmetic
  \[ a + b, a - b, a \times b, a/b, a\%b \]
• Bitwise
  \[ a | b, a & b, a ^ b, \sim a \]
• Bit shift
  \[ a \ll b, a \gg b \text{ (signed)}, a \gg b \text{ (unsigned)} \]
• Logical operators
  \[ a && b, a || b, \sim a \]
• Comparison operators
  \[ a == b, a != b, a < b, a \leq b, a > b, a \geq b \]
• Tertiary operator
  \[ x = a ? b : c \]
• Special functions:
  \[ \text{sqrt}(x), \text{abs}(x), \text{fma}(a,b,c), \text{ceil}(x), \text{floor}(x) \]
### Arithmetic operators

<table>
<thead>
<tr>
<th>Operator</th>
<th>char</th>
<th>short</th>
<th>int</th>
<th>int64_t</th>
<th>float</th>
<th>double</th>
</tr>
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<tbody>
<tr>
<td>+ (-)</td>
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<td>SSE2</td>
<td>SSE2</td>
<td>SSE2</td>
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<td>AVX512DQ</td>
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<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>SSE</td>
</tr>
<tr>
<td>%</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
</tbody>
</table>

- No 8-bit multiplication (can be emulated with 16-bit multiplication)
- No integer division with x86 SIMD
- Floating point division is slow and only uses one port (internally it may only be 128-bit or 256-bit wide as well).
- The AVX512DQ 64-bit * 64-bit to 64-bit instruction is still slow.
Fast division for constant divisors

Calculate \( r = a/b \) where \( b \) is a constant

With floating point we precompute (at compile time or outside of the main loop) the inverse \( ib = 1.0/b \).

\[
    r = ib*a
\]

Floating point division with constant divisors becomes multiplication

With integers the inverse is more complicated

\[
    ib,n = \text{get_magic_numbers}(b);
\]

\[
    r = ib*a >> n
\]

Integer division with constant divisors becomes multiplication and a bit-shift
Fast Division Examples

Dividing integers by a power of two can be done with a bit shift which is very fast.

- \( \frac{x}{3} = x \times 1431655766/2^{32} \)
  \[ 27 \times 1431655766/2^{32} = 3 \]

- \( \frac{x}{1000} = x \times 274877907/2^{38} \)
  \[ 10000 \times 274877907/2^{32} = 10 \]

- \( \frac{x}{314159} = x \times 895963435/2^{48} \)
  \[ 7 \times 314159 \times 895963435/2^{48} = 7 \]
# Bitwise operators

<table>
<thead>
<tr>
<th></th>
<th>char</th>
<th>short</th>
<th>int</th>
<th>int64_t</th>
<th>float</th>
<th>double</th>
</tr>
</thead>
<tbody>
<tr>
<td>^ (XOR)</td>
<td>SSE2</td>
<td>SSE2</td>
<td>SSE2</td>
<td>SSE2</td>
<td>SSE</td>
<td>SSE2</td>
</tr>
<tr>
<td></td>
<td>(OR) &amp; (AND)</td>
<td>SSE2</td>
<td>SSE2</td>
<td>SSE2</td>
<td>SSE2</td>
<td>SSE</td>
</tr>
<tr>
<td>&gt;&gt; unsigned &gt;&gt;</td>
<td>NA</td>
<td>SSE2</td>
<td>SSE2</td>
<td>AVX512</td>
<td>-</td>
<td>SSE2</td>
</tr>
<tr>
<td>&lt;&lt;</td>
<td></td>
<td>SSE2</td>
<td>SSE2</td>
<td></td>
<td>SSE2</td>
<td></td>
</tr>
<tr>
<td>~ (NOT)</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
</tbody>
</table>

- Signed integers use arithmetic shift which shifts in the sign bit for `>>`.
- Signed and unsigned integers use logical shift which shifts in zeros.
- $\sim x = (x ^ -1)$
## Comparison operators

<table>
<thead>
<tr>
<th></th>
<th>char</th>
<th>short</th>
<th>int</th>
<th>int64_t</th>
<th>float</th>
<th>double</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>==</code></td>
<td>SSE2</td>
<td>SSE2</td>
<td>SSE2</td>
<td>SSE4.1</td>
<td>SSE</td>
<td>SSE2</td>
</tr>
<tr>
<td><code>!=</code></td>
<td>AVX512BW, XOP</td>
<td>AVX512BW, XOP</td>
<td>AVX512F, XOP</td>
<td>AVX512F, XOP</td>
<td>SSE</td>
<td>SSE2</td>
</tr>
<tr>
<td><code>&lt; (&gt;)</code></td>
<td>NA</td>
<td>SSE2</td>
<td>SSE2</td>
<td>SSE4.2</td>
<td>SSE</td>
<td>SSE2</td>
</tr>
<tr>
<td><code>&lt;= (&gt;=)</code></td>
<td>AVX512BW or XOP</td>
<td>AVX512BW or XOP</td>
<td>AVX512F, XOP</td>
<td>AVX512F, XOP</td>
<td>SSE</td>
<td>SSE2</td>
</tr>
</tbody>
</table>

- `(a != b) = ~(a == b) = (a == b) ^ -1`
## Other Vertical Operators

<table>
<thead>
<tr>
<th></th>
<th>char</th>
<th>short</th>
<th>int</th>
<th>int64_t</th>
<th>float</th>
<th>double</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>?:</code></td>
<td>SSE4.1</td>
<td>SSE4.1</td>
<td>SSE4.1</td>
<td>SSE4.1</td>
<td>SSE4.1</td>
<td>SSE4.1</td>
</tr>
<tr>
<td>fma3, fma4</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA *</td>
<td>AVX2 XOP</td>
<td>AVX2 XOP</td>
</tr>
<tr>
<td>min (max)</td>
<td>SSE4.1</td>
<td>SSE2</td>
<td>SSE4.1</td>
<td>AVX512F</td>
<td>SSE</td>
<td>SSE2</td>
</tr>
<tr>
<td>sqrt</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>SSE</td>
<td>SSE2</td>
</tr>
<tr>
<td>abs</td>
<td>SSSE3</td>
<td>SSSE3</td>
<td>SSSE3</td>
<td>AVX512F</td>
<td>AVX512F</td>
<td>AVX512F</td>
</tr>
<tr>
<td>floor/ceil/round</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>SSE4.1</td>
<td>SSE4.1</td>
</tr>
</tbody>
</table>

* AVX512IFMA52 has FMA for 52-bit integers
Horizontal Operators

Shuffle/permute/swizzle elements within a vector
• e.g. v(1, 2, 3, 4) to v(4, 3, 2, 1)
Add elements within a vector
• e.g. v(1, 2, 3, 4) to v(1 + 2, 3 + 4, 0, 0)
There are a massive number of ways to permute elements within vectors e.g.
• permute(v1, int): where int is a compile time integer
• permute(v1, var): where var is a runtime time var
• permute(v1, v2, int), permute elements within vectors with a compile time integer

<table>
<thead>
<tr>
<th></th>
<th>char</th>
<th>short</th>
<th>int</th>
<th>int64_t</th>
<th>float</th>
<th>double</th>
</tr>
</thead>
<tbody>
<tr>
<td>hadd (hsub)</td>
<td>SSE4.1</td>
<td>SSE4.1</td>
<td>SSE4.1</td>
<td>SSE4.1</td>
<td>SSE3</td>
<td>SSE3</td>
</tr>
<tr>
<td>permute(v1, var)</td>
<td>SSSE3</td>
<td>SSSE3</td>
<td>SSSE3 or AVX512F</td>
<td>SSSE3</td>
<td>AVX</td>
<td>AVX</td>
</tr>
</tbody>
</table>
A reduction adds each element of an array and returns the sum

```c
double reduction(double *a, int n) {
    double s = 0;
    for(int i=0; i<n; i++) {
        s += a[i];
    }
    return s;
}
```

```c
double reduction_VCL(double *a, int n) {
    Vec8d s1 = 0;
    for(int i=0; i<n/8; i++) {
        s1 += Vec8d().load(&a[8*i]);   //eight partial sums
    }
    return horizontal_add(s1);       //add the eight partial sums
}
```
double reduction_avx512(double *a, int n) {
    __m512d s1 = _mm512_setzero_pd();
    for(int i=0; i<n/8; i++) {
        __m512d av = _mm512_loadu_pd(&a[8*i]);
        s1 = _mm512_add_pd(cv,s1);               // sum = av + sum
    }
    __m256d l1 = _mm512_castpd512_pd256(s1);   // l1 =  1  2  3  4  5  6  7  8
    __m256d h1 = _mm512_extractf64x4_pd(s1,1); // h1 =  5  6  7  8
    __m256d s2 = _mm256_add_pd(h1,l1);         // s2 =  6  8 10 12
    __m256d s3 = _mm256_hadd_pd(s2, s2);       // s3 = 14 14 22 22
    __m128d l3 = _mm256_castpd256_pd128(s3);   // l3 = 14 14
    __m128d h3 = _mm256_extractf128_pd(s3,1);  // h3 = 22 22
    __m128d s4  = _mm_add_sd(l3,h3);           // s4 = 36 36
    return _mm_cvtsd_f64(s4);                  // return 36
}
//hadd_pd(a,b) =(a0+a1, b0+b1, a2+a3, b2,b3)

We can reduce a vector of width w in ln2(w) sums.
For 8 doubles it takes 3 sums.
For 16 floats it takes 4 sums.
Transposing a 4x4 float matrix

// row0  1  2  3  4
// row1  5  6  7  8
// row2  9 10 11 12
// row3 13 14 15 16

tmp0 = _mm_unpacklo_ps(row0, row1);  // 1  5  2  6
tmp2 = _mm_unpacklo_ps(row2, row3);  // 9 13 10 14
tmp1 = _mm_unpackhi_ps(row0, row1);  // 3  7  4  8
tmp3 = _mm_unpackhi_ps(row2, row3);  // 11 15 12 16

row0 = _mm_movelh_ps(tmp0, tmp2);    // 1  5  9 13
row1 = _mm_movehl_ps(tmp2, tmp0);    // 2  6 10 14
row2 = _mm_movelh_ps(tmp1, tmp3);    // 3  7 11 15
row3 = _mm_movehl_ps(tmp3, tmp1);    // 4  8 12 16

Scalar operations to transpose a matrix go as 2*n*(n-1) but only n*ln2(n) with SIMD

<table>
<thead>
<tr>
<th>nxn matrix</th>
<th>4x4</th>
<th>8x8</th>
<th>16x16</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIMD ops</td>
<td>8</td>
<td>24</td>
<td>64</td>
</tr>
<tr>
<td>SIMD ops + R/W</td>
<td>16</td>
<td>40</td>
<td>96</td>
</tr>
<tr>
<td>Scalar ops + r/w</td>
<td>24</td>
<td>120</td>
<td>480</td>
</tr>
</tbody>
</table>
Gather and Scatter

Intel has created micro-code which essentially implements the following:

```c
int n = 8;  // or 4 with AVX512 (AVX2)

void gather(double *a, double *b, int *idx) {
    for(int i=0; i<n; i++) b[i] = a[idx[i]];
}

void scatter(double *a, double *b, int *idx) {
    for(int i=0; i<n; i++) b[idx[i]] = a[i];
}
```

Where `n` is eight (four) with AVX512 (AVX2)
Gather and scatter example

double  a[16] = {0, -1, 2, -3, 4, -5, 6, -7, 8, -9, 10, -11, 12, -13, 14, -15};
int idx[8] = {1, 3, 5, 7, 9, 11, 13, 15};
double b[8];
gather(a, b, idx); // b = {-1, -3, -5, -7, -9, -11, -13, -15}
memset(a, 0, 16*sizeof(double));
scatter(a, b, idx);
//a = {0, -1, 0, -3, 0, -5, 0, -7, 0, -9, 0, -11, 0, -13, 0, -15};

__m256i vidx = _mm256_loadu_si256((__m256i*)idx);
__m512d bv   = _mm512_i32gather_pd (vidx, a, 8);
   _mm512_storeu_pd(b, bv);
memset(a, 0, 16*sizeof(double));
_mm512_i32scatter_pd(a, idx, bv, 8);
Vector extensions for C

// works with GCC, ICC, and Clang, but to MSVC
#include <stdio.h>

typedef float float4 __attribute__((vector_size(sizeof(float)*4)));

void foo(float *a, float *b, float *c, int n) {
    int i;
    for(i=0; i<(n&-4); i+=4) { //(n&-4) rounds n down to a multiple of 4
        float4 av = *(float4*)&a[i]; // Read four floats from a
        float4 bv = *(float4*)&b[i]; // Read four floats from b
        float4 cv = av + bv;       // add four floats from a and b
        *(float4*)&c[i] = cv;       // write four floats
    }
    for(; i<n; i++) c[i] = a[i] + b[i];
}

int main(void) {
    float a[8], b[8], c[8];
    for(int i=0; i<8; i++) a[i] = i, b[i] = i;
    foo(a, b, c, 8);
    for(int i=0; i<8; i++) printf("%.0f ", c[i]);
    puts("\n");
}
Dot Product and an AoSoA

typedef float float4 __attribute__((vector_size(sizeof(float)*4)));

struct vec3 {
    float x; float y; float z;
};

struct vec3SSE {
    float4 x; float4 y; float4 z;
};

float dot(struct vec3 a, struct vec3 b) {
    return a.x*b.x + a.y*b.y + a.z*b.z;
}

float4 dotSSE(struct vec3SSE a, struct vec3SSE b) {
    return a.x*b.x + a.y*b.y + a.z*b.z;
}

//convert an array of structs to a struct of arrays
struct vec3SSE aos2soa(struct vec3 *a) {
    struct vec3SSE a2;
    for(int i=0; i<4; i++) a2.x[i] = a[i].x, a2.y[i] = a[i].y, a2.z[i] = a[i].z;
    return a2;
}

Goal: get the dot product of array of 3D vectors (struct vec3)
Dot Product and an AoSoA

vec3 a[8];
vec3SSE b[2];

x y z
b[0] = aos2soa(&a[0])
a0: 0 1 2
a1: 1 2 3
a2: 2 3 4
a3: 3 4 5
b[1] = aos2soa(&a[4])
a4: 4 5 6
a5: 5 6 7
a6: 6 7 8
a7: 7 8 9

float4 x: 0 1 2 3
float4 y: 1 2 3 4
float4 z: 2 3 4 5
float4 x: 4 5 6 7
float4 y: 5 6 7 8
float4 z: 6 7 8 9

for(int i=0; i<8; i++) printf("%.0f ", dot(a[i], a[i]));
//5 14 29 50 77 110 149 194

for(int i=0; i<2; i++) {
  float4 t = dotSSE(b[i], b[i]);
  for(int j=0; j<4; j++) printf("%.0f ", t[j]);
}
//5 14 29 50 77 110 149 194

See “Extending a C-like Language for Portable SIMD Programming”
http://compilers.cs.uni-saarland.de/papers/leissa_vecimp_tr.pdf