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 University of OsloIN4180 - Analog Microelectronics Design

## Basic Operational Amplifier Design and Compensation - Part 2 Compensation and stability

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- PMOS diff input stage
- Numbers realistic transistor widths
- Length 1-2 times minimum
- Output buffer may not be needed for capacitive loads

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- Gain for diff pair - 1 . stage

$$
A_{v 1}=g_{m 1}\left(r_{d s 2} \| r_{d s 4}\right)
$$

- Typical gain 50-100

$$
\lambda
$$

- Gain of common source - 2. stage

$$
=\frac{k_{d s}}{2 L \sqrt{V_{D S}-V_{e f f}+\Phi_{0}}}
$$

$$
A_{v 2}=-g_{m 7}\left(r_{d s 6} \| r_{d s 7}\right)
$$

- Typical gain 50-100

$$
k_{d s}=\sqrt{\frac{2 K_{s} \varepsilon_{0}}{q N_{A}}}
$$

- Gain of source follower - output buffer

$$
A_{v 3}=\frac{g_{m 8}}{G_{L}+g_{m 8}+g_{s 8}+g_{d s 8}+g_{d s 9}}
$$

$$
\begin{aligned}
& r_{d s} \cong \frac{1}{\lambda I_{D_{\gamma}}} \\
& g_{s 8}=\frac{1}{2 \sqrt{V_{S B}+\left|2 \varphi_{F}\right|}}
\end{aligned}
$$

- Gain $\approx 1$
- Not needed for capacitive loads

$$
g_{m 1}=\sqrt{2 \mu_{n} C_{o x} \frac{W}{L} I_{D}}=\sqrt{2 \mu_{n} C_{o x} \frac{W}{L} \frac{I_{b i a s}}{2}}
$$

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Frequency response - First order model

$$
\text { at midband freq } C_{e q} \text { dominates }
$$

$$
A_{1}=g_{m 1} \frac{1}{s C_{e q}}=g_{m 1} \frac{1}{s C_{C} A_{2}}
$$

$$
A_{v}=\frac{v_{\text {out }}}{v_{\text {in }}}=A_{1} A_{2} A_{3} \approx g_{m 1} \frac{1}{s C_{C} A_{2}} \cdot A_{2} \cdot 1=\frac{g_{m 1}}{\underline{s C_{C}}}
$$

$$
\omega_{t a}=\frac{g_{m 1}}{C_{C}}=\frac{I_{D 5}}{V_{e f f 1} C_{C}}
$$

Unit-gain frequency proportional to $g_{m}$ assuming $A_{3}=1$

$$
\begin{aligned}
& A_{1}=g_{m 1} Z_{\text {out } 1} \\
& =g_{m 1}\left(r_{d s 2}\left\|r_{d s 4}\right\| \frac{1}{s C_{e q}}\right)
\end{aligned}
$$

$\mathrm{Q}_{3}$

and

## Feedback stability



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Feedback stability

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## Frequency response - First order model



Midband frequencies

- Below unit-gain frequency
- Above frequencies without compensation effects
- Ignore all C except $C_{c}$
- Ignore $R_{c}$ which only has effect at $\omega_{t a}$



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Frequency response Second order model

$R_{1}=r_{d s 4} \| r_{d s 2}$ and $C_{1}=C_{d b 2}+C_{d b 4}+C_{g s 7}$

$$
R_{2}=r_{d s 6} \| r_{d s 7} \text { and } C_{2}=C_{d b 7}+C_{d b 6}+C_{L 2}
$$

$\qquad$

Differential-inpu
first stage

Common-source second stage

- Assume $R_{C}=0$ give transfer function

$$
\begin{aligned}
& \frac{v_{\text {out }}}{v_{\text {in }}}=\frac{g_{m 1} g_{m 7} R_{1} R_{2}\left(1-\frac{s C_{C}}{g_{m 7}}\right)}{1+s a+s^{2} b} \\
& a=\left(C_{1}+C_{C}\right) R_{2}+\left(C_{1}+C_{C}\right) R_{1}+g_{m 7} R_{1} R_{2} C_{C} \\
& \quad b=R_{1} R_{2}\left(C_{1} C_{2}+C_{1} C_{C}+C_{2} C_{C}\right)
\end{aligned}
$$

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- Assume widely separated poles

$$
D(s)=\left(1+\frac{s}{\omega_{p 1}}\right)\left(1+\frac{s}{\omega_{p 2}}\right) \approx 1+\frac{s}{\omega_{p 1}}+\frac{s^{2}}{\omega_{p 1} \omega_{p 2}}
$$

- Dominant pole

$$
\begin{aligned}
& \omega_{p 1} \\
& =\frac{1}{R_{1}\left[C_{1}+C_{C}\left(1+g_{m 7} R_{2}\right)\right]+R_{2}\left(C_{1}+C_{C}\right)} \\
& \quad \approx \frac{1}{R_{1} C_{C}\left(1+g_{m 7} R_{2}\right)} \\
& \quad \approx \frac{1}{g_{m 7} R_{1} R_{2} C_{C}}
\end{aligned}
$$

- Non-dominant pole

$$
\begin{array}{ll}
\omega_{p 2} & \begin{array}{l}
\text { •Increasing } g_{m 7} \\
\\
=\frac{\rightarrow \text { increased pole distance }}{C_{1} C_{2}+C_{1} C_{C}+C_{2} C_{C}} \\
\\
\approx \frac{g_{m 7}}{C_{1}+C_{2}}
\end{array} \\
\quad \text { •Pole splitting compensation }
\end{array}
$$

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$$
\frac{v_{\text {out }}}{v_{\text {in }}}=\frac{g_{m 1} g_{m 7} R_{1} R_{2}\left(1-\frac{s C_{C}}{g_{m 7}}\right)}{1+s a+s^{2} b} \quad \Rightarrow \omega_{z}=-\frac{g_{m 7}}{C_{C}}
$$

I use $\beta=1$ (max feedback) in this analysis

- Right half-plane $\rightarrow$ negative phase shift with decreased PM
- Stability issues
- Hard to get rid of, but pole distance is increased with $\mathrm{g}_{\mathrm{m} 7}$
- Have to make $R_{C}>0$
- Zero with some resistive element

$$
\omega_{z}=-\frac{1}{C_{C}\left(1 / g_{m 7}-R_{C}\right)}
$$

- May eliminate that zero by setting

$$
R_{C}=\frac{1}{g_{m 7}}
$$

- Alternatively try to cancel $\omega_{\mathrm{p} 2}$ with $\omega_{\mathrm{z}}$

$$
\frac{g_{m 7}}{C_{1}+C_{2}}=-\frac{1}{C_{C}\left(1 / g_{m 7}-R_{C}\right)} \Rightarrow R_{C}=\frac{1}{g_{m 7}}\left(1+\frac{C_{1}+C_{2}}{C_{C}}\right)
$$

- "Overcompensation" might even be wise:

$$
\omega_{Z}=1.7 \omega_{t}
$$

$$
R_{C} \gg 1 / g_{m 7} \Rightarrow \omega_{Z} \approx \frac{1}{R_{C} C_{C}} \quad \omega_{t} \approx g_{m 7} / C_{C} \text { gives } R_{C}=\frac{1}{1.7 g_{m 7}}
$$

## Two-pole amplifier

- Dominant poles of two-stage amps

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General form:

$$
A_{v}(s)={\frac{N^{\prime}}{D}=\frac{A_{0}\left(1+\frac{s}{\omega_{z}}\right)}{\left(1+\frac{s}{\omega_{P 1}}\right)\left(1+\frac{S}{\omega_{P 2}}\right)}}_{\text {ZEROS }}^{(10 S}
$$

From two-stage second order analysis:

$$
\begin{aligned}
& A(s)=\frac{V_{\Delta a r}}{V_{1 r}}=\frac{-g_{m} R_{2}\left(1+s \frac{C_{G 1)}}{g_{m 1}}\right)}{1+s a+s^{2} b} \\
& a=R_{s}\left[C_{G s 1}+C_{G 011}\left(1+g_{n 1} R_{2}\right)\right]+R_{2}\left(C_{G D 1}+C_{2}\right) \\
& B=R_{s} R_{2}\left(C_{G 01} C_{a 11}+C_{a 11} C_{2}+C_{G 01} C_{2}\right)
\end{aligned}
$$

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## Opamp compensation

- Dominant-pole compensation
- Forcing a feedback system to have 1. order response up to loop unitgain frequency $\omega_{t}$
- First order system unconditional stable with > 90 phase margin
- Lead compensation
- Adding zero, $\omega_{z}$, just above $\omega_{\mathrm{t}}$
- May improve PM with $20^{\circ}$

Dominant pole comp using miller Cc


Dominant-pole +


## Compensation procedure



## Dominant pole

- From first order model $C_{C}$ and $\omega_{t}$ is given as:

$$
L(s) \approx A(s) \frac{Z_{1}}{Z_{1}+Z_{2}}
$$

$$
\omega_{t}=L_{0} \omega_{p 1}=\beta \frac{g_{m 1}}{C_{C}}
$$

$$
\beta=\frac{Z_{1}}{Z_{1}+Z_{2}}
$$

Two stage opamp small signal model

$$
L(s) \approx A(s) \frac{Z_{1}}{Z_{1}+Z_{2}}
$$

setting unit-gain frequency close to second pole

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## Compensation procedure

Lead compensation - controlling Zero

$$
\omega_{z} \approx \frac{-1}{C_{C}\left(\frac{1}{g_{m 7}}-R_{C}\right)}
$$

Several possibilities for $R_{C}$ :

Two stage opamp small signal model


$$
R_{C}=\frac{1}{g_{m 7}}->\omega_{z}=\infty
$$

$$
R_{C}>\frac{1}{g_{m 7}} \quad \text { RHPZ }->\text { LHPZ and cancel } \omega_{p 2}
$$

$R_{C} \gg \frac{1}{g_{m 7}}$
Moving LHPZ to a frequency slightly higher than $\omega_{t}$ (wo $R_{C}$ )
Recommended to get more PM (20-30 degrees)

$$
\omega_{p 2}=\frac{g_{m 7} C_{C}}{C_{1} C_{2}+C_{1} C_{C}+C_{2} C_{C}}=\frac{-1}{C_{C}\left(\frac{1}{g_{m 7}}-R_{C}\right)} \Rightarrow R_{C}=\frac{1}{g_{m 7}}\left(1+\frac{C_{1}+C_{2}}{C_{C}}\right)
$$

## $R_{C}$ as transistor

- Compensation resistor
- Replaced by transistor in triode region


$$
R_{C}=r_{d s}=\frac{1}{\mu_{n} C_{o x} \frac{W}{L} V_{e f f}}
$$

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## Opamp compensation design stategy

1. Start with $C_{C}^{\prime}=\left(\beta \frac{g_{m 1}}{g_{m 7}}\right) c_{L}$
setting unit-gain frequency close to second pole
2. By simulation (SPICE, CADENCE) find frequency with $-125^{\circ}$ phase shift ( called gain A')

- This is our unit gain frequency $\omega_{t}$ target

3. Choose new $\mathrm{C}_{\mathrm{C}}$ such that $\omega_{t}$ is unit-gain freq of $\mathrm{L}(\mathrm{s})$

- $\quad \mathrm{C}_{\mathrm{C}}=\mathrm{C}_{\mathrm{C}}{ }^{\prime} \mathrm{A}^{\prime}$ giving $55^{\circ}$ phase margin
- A couple of simulation iterations may be necessary

4. Choose $\mathrm{R}_{\mathrm{C}}: \quad R_{C}=\frac{1}{1.7 \omega_{t} C_{C}} \quad$ Almost optimum lead compensation for any opamp

- Giving phase margin of $85^{\circ} \quad\left(+30^{\circ}\right)$ leaving $5^{\circ}$ for variations

1. Sometimes phase margins are not adequate, then increase $C_{C}$
2. Replace $\mathrm{R}_{\mathrm{C}}$ with a transistor $\quad R_{C}=\frac{1}{\mu_{n} C_{o x}\left(\frac{W}{L}\right)_{16} V_{\text {eff } 16}}$


## Opamp compensation Cadence example

- Find best compensation network $\mathrm{C}_{\mathrm{c}}$ and $\mathrm{R}_{\mathrm{c}}$ for:



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Find bias voltage:


Vbias1=2.3V give $84 \mu \mathrm{~A}$ tail current
Found by simple simulation run displaying tail current

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- Start with $\mathrm{Cc}=0.5 \mathrm{pF}$ and $\mathrm{Rc}=0$

Expressions
T

$0^{\circ}$ phase in CADENCE display is $-180^{\circ}$ actual phase shift

Find (180-125) $=55^{\circ}$ phase shift at $\omega t=50.1 \mathrm{MHz}$ with gain $A^{\prime}=3.7$

$$
C_{C}=C_{C}^{\prime} A^{\prime}=0.5 p F \cdot 3.7 \approx 1.9 p F
$$

- New simulation with $\mathrm{Cc}=1.9 \mathrm{pF}$ give
$-\omega_{\mathrm{t}}=44.7 \mathrm{MHz}$ with $\mathrm{A}^{\prime}=1.32$

$$
C_{C}=C_{C}^{\prime} A^{\prime}=1.3 p F \cdot 1.32 \approx 2.5 p F
$$

- New simulation with $\mathrm{Cc}=2.5 \mathrm{pF}$ give
$-\omega_{\mathrm{t}}=41 \mathrm{MHz}$ with $\mathrm{A}^{\prime}=1.2$

$$
C_{C}=C_{C}^{\prime} A^{\prime}=2.5 p F \cdot 1.2 \approx 3.1 p F
$$

- New simulation with $\mathrm{Cc}=3.1 \mathrm{pF}$ give

Marker at 55 deg phase margin

- Finding Rc

$$
R_{C}=\frac{1}{1.2 \omega_{t} C_{C}}=\frac{1}{1.2 \cdot 37.7 \cdot 10^{6} \cdot 3.1 \cdot 10^{-12}} \approx 7132 \Omega
$$

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- Adding compensation resistor Rc



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## Phase margins?



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- What to do?
- Book: increase Cc
- Try to decrease Rc

Expressions
[1]


