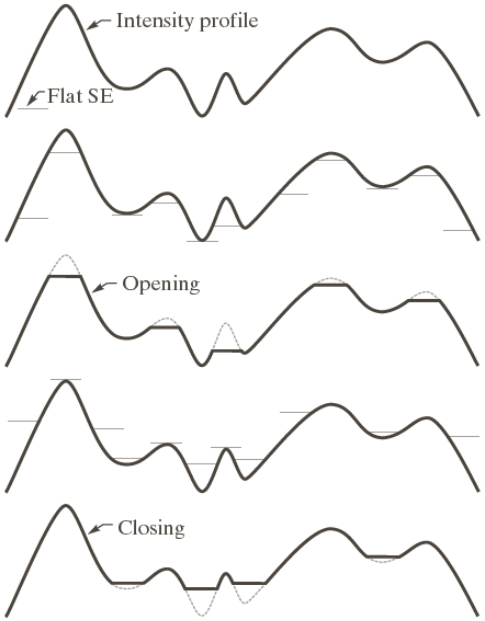
IN5520 – Digital Image Analysis

MORPHOLOGICAL IMAGE PROCESSING



Andreas Kleppe 10.11.2022
Modification of slides by Fritz Albregtsen

Curriculum

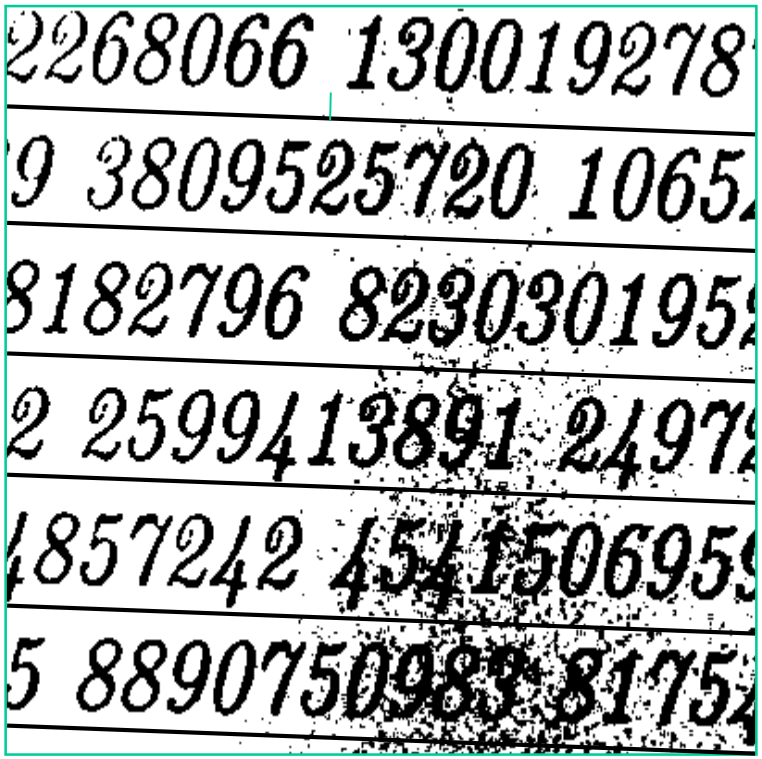
- G&W, "DIP", Ch. 9 except sections on Skeletons (9.5.7 in 3Ed), Pruning (9.5.8 in 3Ed), Reconstruction (9.5.9 in 3Ed, 9.6 in 4Ed), and Gray scale reconstruction (9.6.4 in 3Ed, pp 688 in 4Ed).
- Full lecture available online (video + slides).
- Today's lecture will focus on the main points.

Plan for today

- Before the break, lecture and Q&A about:
  - Binary morphology
- After the break, lecture and Q&A about:
  - Grey level morphology

Motivation

- Morphology is the study of shape.
- Binary morphological operations are useful after segmentation in order to improve segmentation of the objects.



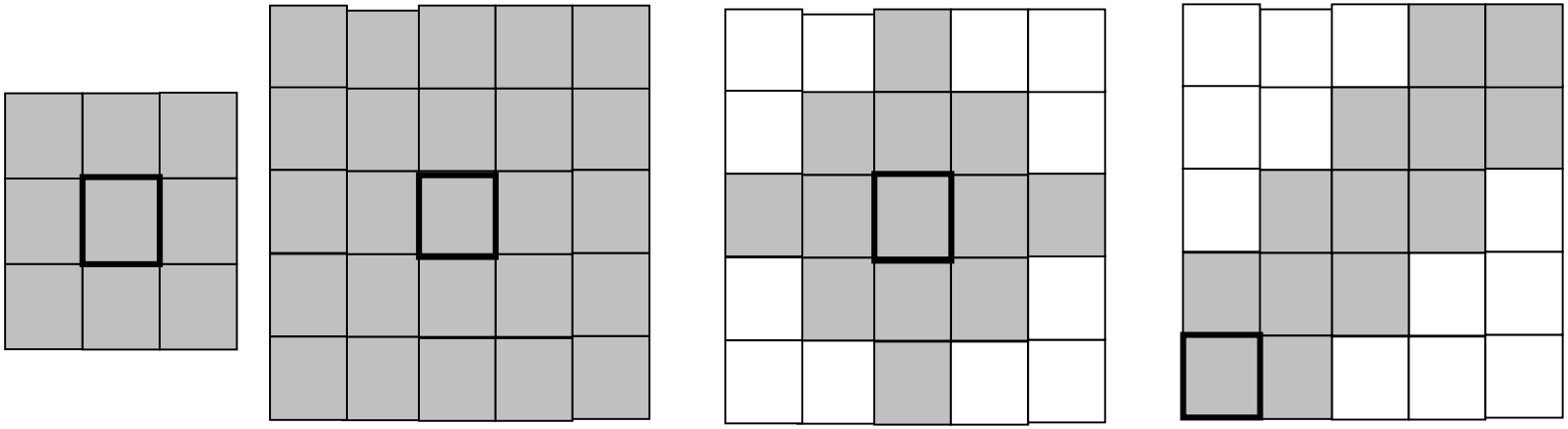
Some symbols have been fragmented.

Some symbols are connected with background noise.

Symbols can be connected with neighbouring symbols.

Need to remove lines or frames.

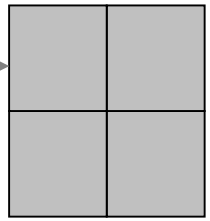
Structuring elements

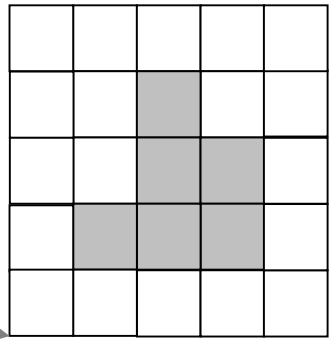


- A small matrix of pixels.
  - Often flat (e.g. 1s defining the elements and 0s defining not an element).
- Can have different sizes and shapes.
- Has an origin/centre.
  - The origin is a pixel position.
  - The origin *<u>can</u>* be outside the element.
  - The origin is often marked on the structuring element using □
  - Otherwise, we assume the centre pixel is the origin.

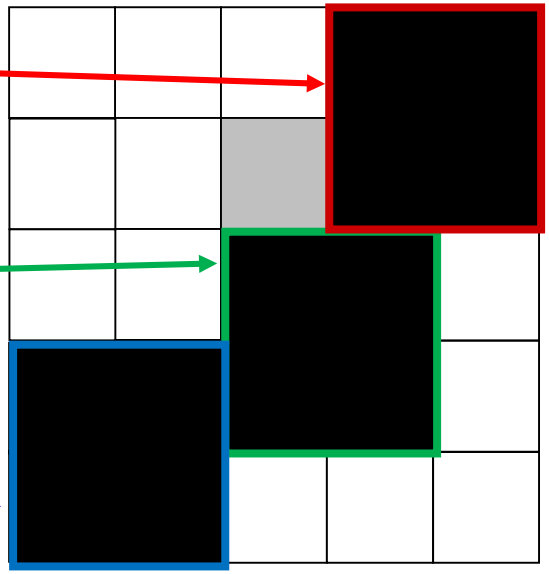
Hit vs. Fit

- Let the structuring element overlay a binary image containing an object at different pixel positions.





- The following cases arise:
  - Positions where the element does not overlap with the object.
  - Positions where the whole element fits inside the object
    - the element fits the object
  - Positions where the element partly overlaps the object
    - the element hits the object



Erosion of a binary image is fitting

- To compute the erosion of pixel (x,y) in image f with the structuring element S: place the structuring elements such that its origin is at (x,y). Compute

$$g(x,y) = \begin{cases} 1 & \text{if S fits f} \\ 0 & \text{otherwise} \end{cases}$$

- Erosion of the image f with structuring element S is denoted

  ε (f|S) = f ɵ S

- Erosion of a set f with the stucturing element S is defined as the position of all pixels x in f such that S is included in f when origin of S is at x.

$$f\ \theta\ \mathrm{S} = \{x | S_x \subseteq f\}$$



eroded by





gives



gives

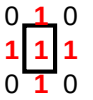


Edge detection using erosion

- Erosion removes pixels on the border of an object.
- We can find the border by subtracting an eroded image from the original image: $g = f - (f \ominus s)$
- The structuring element decides if the detected edge pixels will be 4-neighbours or 8-neighbours



Eroded by





gives





difference => edge pixels

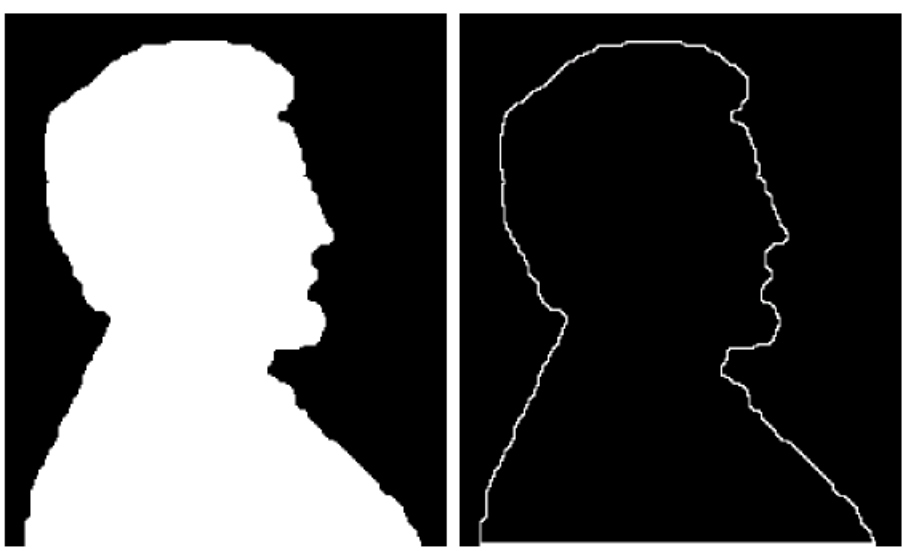




8-edgels – 4-edgels



Edge detection





$$f - (f \ominus S)$$

Example use: find border pixels in a region

Dilation of a binary image is hitting

- Place the 180 degrees rotated S such that origin lies in pixel (x,y) and use the rule:

$$g(x,y) = \begin{cases} 1 & \text{if } \hat{S} \text{ hits f} \\ 0 & \text{otherwise} \end{cases}$$

- The image f dilated by the structuring element S is denoted:

$$f \oplus S$$

- Dilation of a set f with a structuring element S is defined as the position of all pixels x such that the 180 degrees rotated S overlaps with at least one pixel in f when the origin is placed at x.

$$f \oplus S = \left\{ x \,|\, \hat{S}_x \cap f \neq \varnothing \right\}$$

0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	1	1	0	0
0	0	1	0	0	0	1	1	0	0	0
0	0	0	1	0	1	1	0	0	0	0
0	0	0	0	1	1	0	1	0	0	0
0	0	0	1	1	0	0	0	1	0	0
0	0	1	1	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0

Dilated by

1	1	1
1	1	1
1	1	1

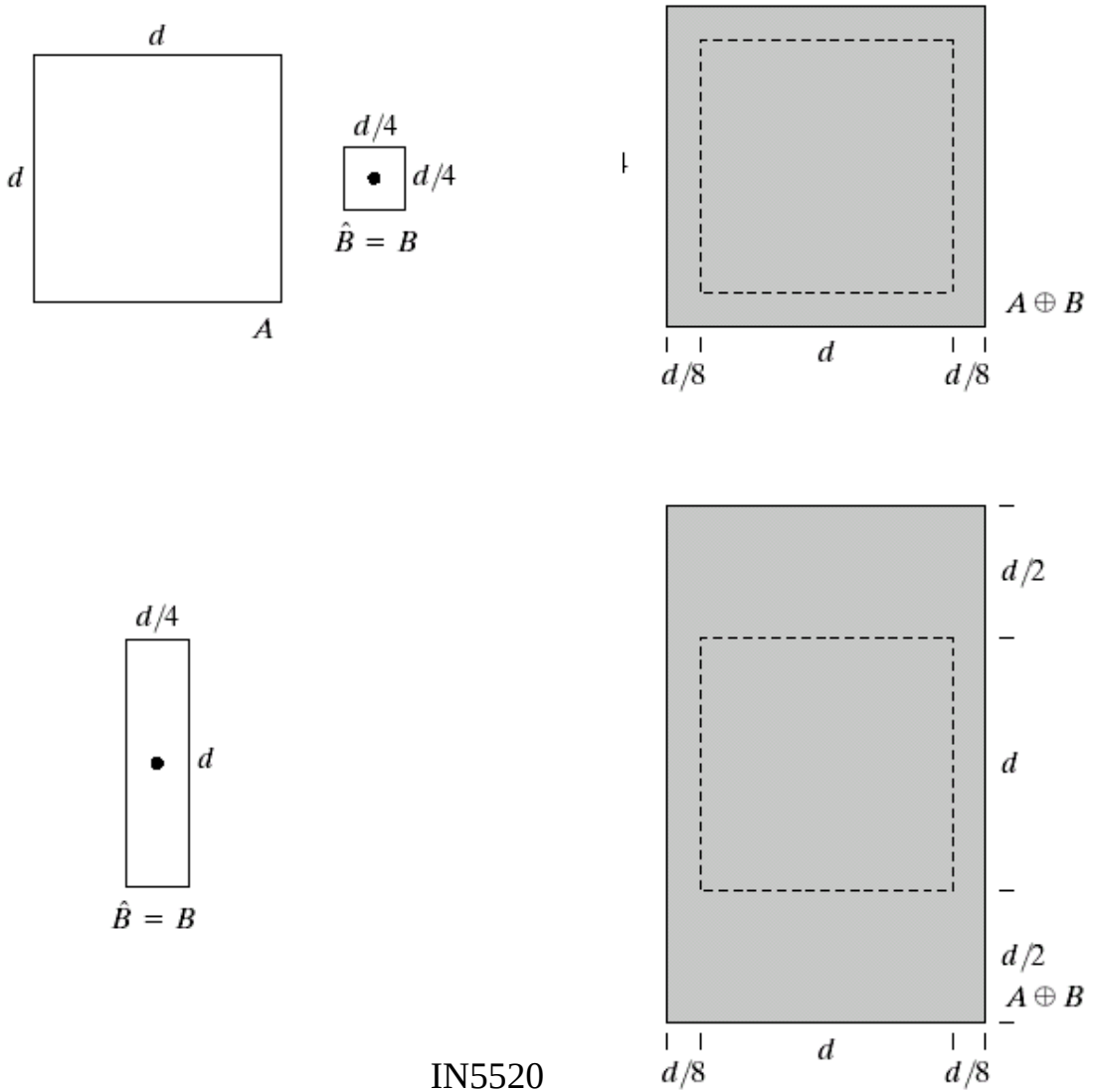
0	1	0
1	1	1
0	1	0

gives

1	1	1	0	0	0	1	1	1	1	0
1	1	1	1	0	1	1	1	1	1	0
1	1	1	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	1	1
0	1	1	1	1	1	0	1	1	1	1
0	1	1	1	1	0	0	0	1	1	1

0	1	0	0	0	0	0	1	1	0	0
1	1	1	0	0	0	1	1	1	1	0
0	1	1	1	0	1	1	1	1	0	0
0	0	1	1	1	1	1	1	0	0	0
0	0	0	1	1	1	1	1	1	0	0
0	0	1	1	1	1	0	1	1	1	0
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0	0	1	1	0	0	0	0	0	1	0

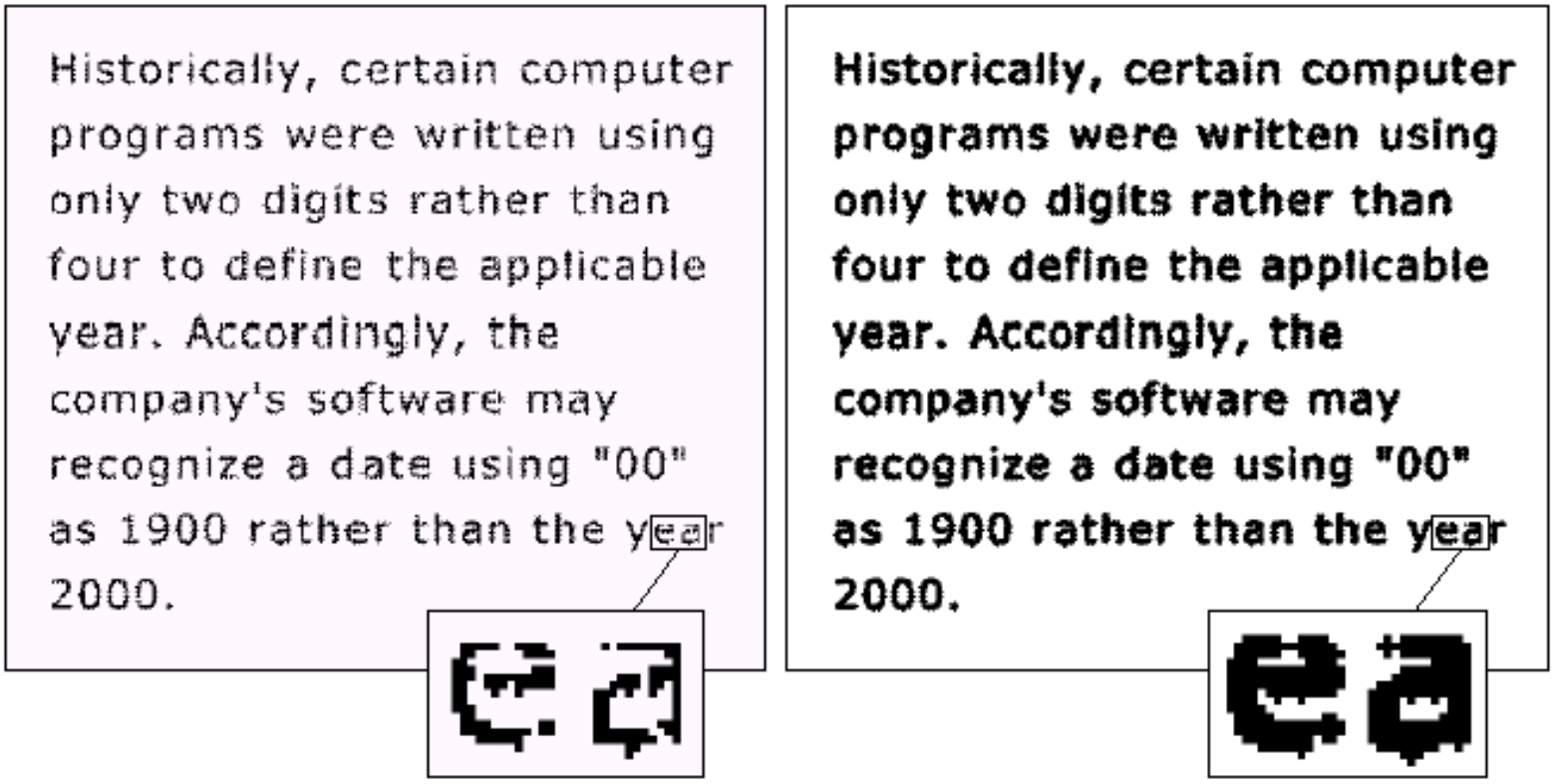
Dilation



Effects of dilation

- Expand the object borders
  - Both inside and outside borders of the object
- Dilation fills holes in the object
- Dilation smooths out the object contour
- Depends on the structuring element
- Bigger structuring element gives greater effect

Example of use of dilation – fill gaps



0	1	0
1	1	1
0	1	0

Combined operation I: Opening

- Erosion of an image removes all structures that the structuring element cannot fit inside, and shrinks all other structures.
- Dilating the result of the erosion with the same structuring element, the structures that survived the erosion (were shrunken, not deleted) will be restored.
- This is called morphological opening:

$$f \circ S = \left(f \ominus S \right) \oplus S$$

- The name tells that the operation can create an opening between two structures that are connected only by a thin bridge, without just shrinking the structures (as erosion alone would do).

Opening

- The operation can create an opening between two structures that are connected only in a thin bridge, without just shrinking the structures (as erosion alone would do).

$$f \circ S = \left(f \ \theta \ \mathrm{S} \right) \oplus S$$

original

0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	1	1	1	0
0	1	1	0	0	0	1	1	1	1	0
0	1	1	1	0	1	1	1	1	1	0
0	1	1	1	1	1	0	1	1	1	0
0	1	1	1	1	0	0	0	1	1	0
0	1	1	1	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0

1	1	1
1	1	1
1	1	1

erosion

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	1	0	0
0	0	1	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

dilated erosion

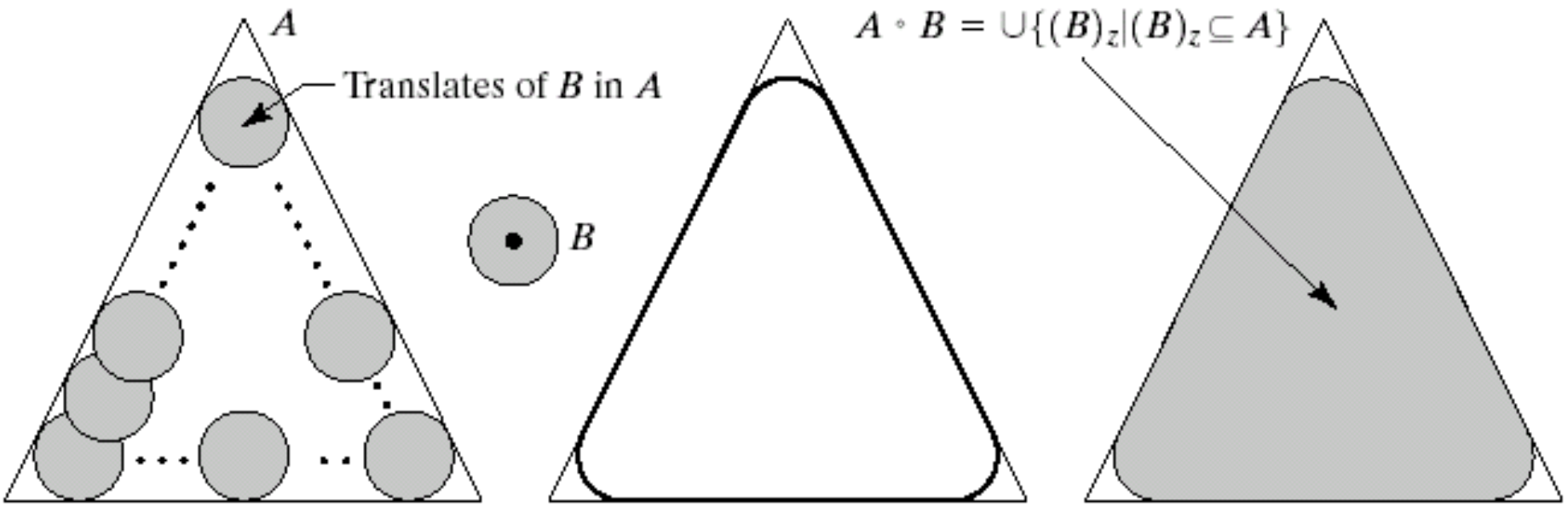
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	1	0
0	0	0	0	0	0	0	1	1	1	0
0	1	1	1	0	0	0	1	1	1	0
0	1	1	1	0	0	0	1	1	1	0
0	1	1	1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

difference

0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
0	0	0	0	0	1	1	0	0	0	0
0	0	0	0	1	1	0	0	0	0	0
0	0	0	0	1	0	0	0	1	1	0
0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0

Opening

- Imagine that you have a marker shaped like the structuring element.
- Place it orthogonally inside the object, drawing the interior of the objects wherever possible.
- What you have drawn is the result of the opening with that structuring element.



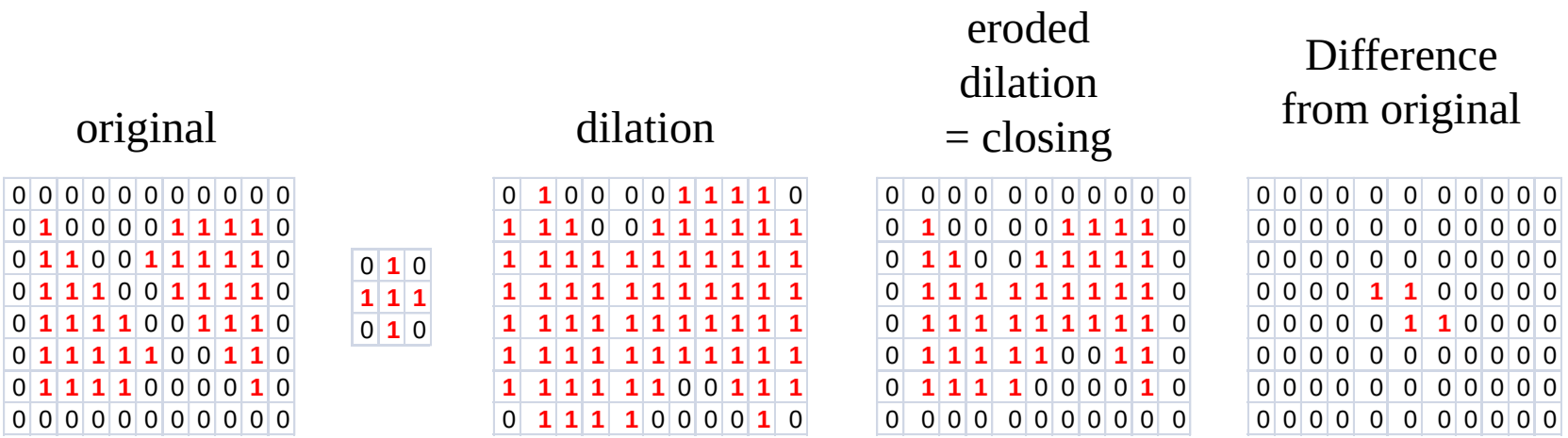
Combined operation II: Closing

- A dilation of an object grows the object and can fill gaps.
- If we erode the result with the same structuring element, the objects will keep their structure and form, but small holes filled by dilation will not appear.
- Objects merged by the dilation will not be separated again.
- Closing is defined as $$f \bullet S = \left(f \oplus S \right) \theta S$$
- This operation can close gaps between two structures without just growing their sizes like dilation alone would.

Closing

- This operation can close gaps between two structures without just growing their sizes like dilation alone would.

$$f \bullet S = \left(f \oplus S \right) \theta S$$



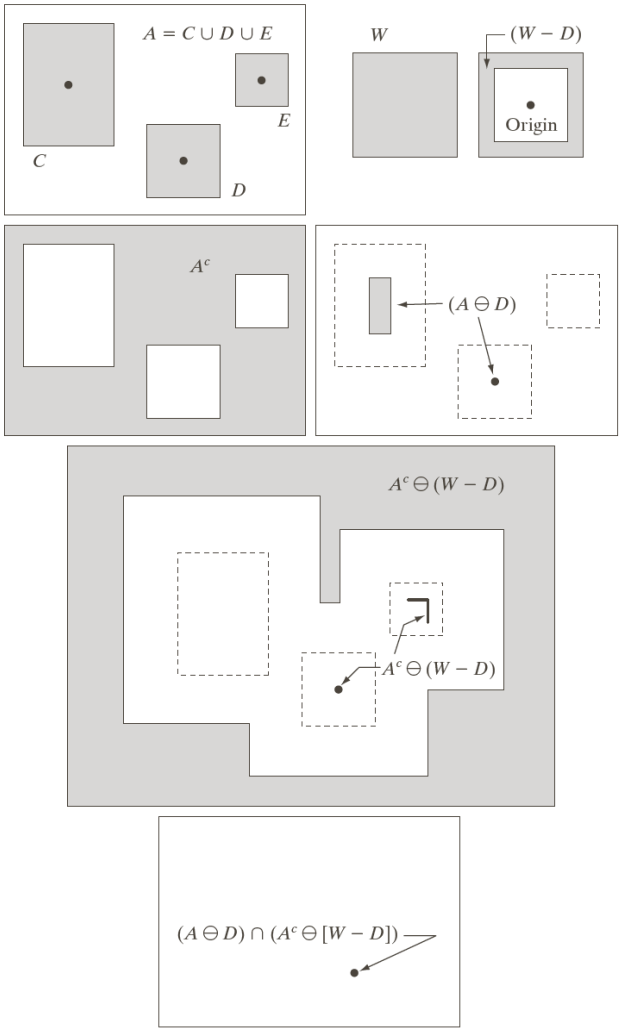
- Very different result from a simple dilation (e.g., slide 10)

"Hit or miss"- transformation

- Transformation used to detect a given pattern in the image – "template matching".
- Objective: Find location of shape D in set A.
- D can fit inside many objects, so we need to look at the local background W-D.
- First, compute the erosion of A by D, AθD (all pixels where D can fit inside A)
- To fit also the background: Compute A^C, the complement of A. The set of locations where D exactly fits is the intersection of AθD and the erosion of A^C by W-D, A^C θ(W-D).
- Hit-or-miss (HMT) is expressed as A ⊛ D:

$$(A\theta D)\cap\left[A_C\theta(W-D)\right]$$

Main use: Detection of a given pattern or removal of single pixels



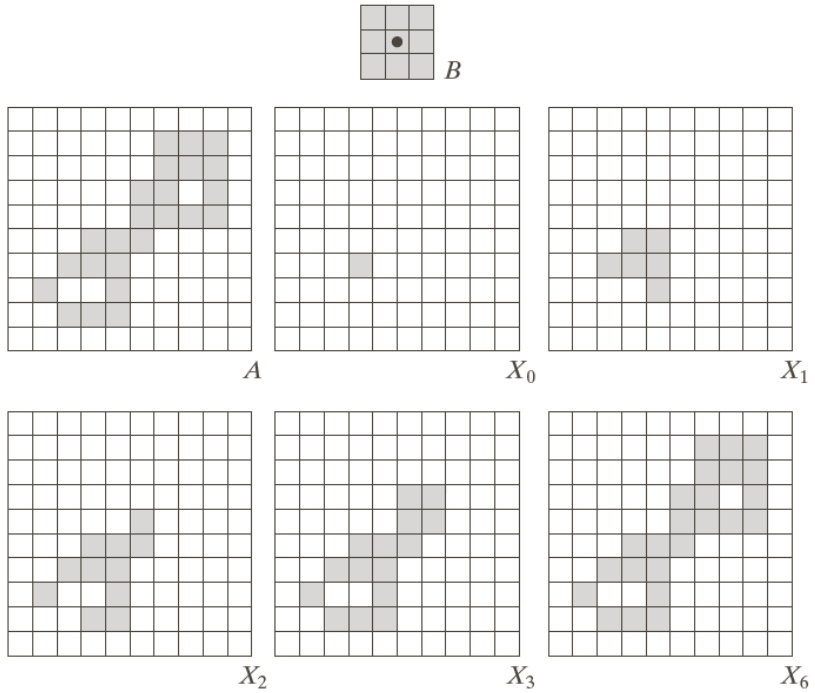
Extracting connected components

- Detects a connected object Y, in image A, given a point p in Y
  - Start with X_0, a point in Y
  - Dilate X_0 with either a square or plus
  - Let X_1 be only those pixels in the dilation that are part of the original region.
  - Continue dilating X_1 to give X_k until

$$X_k = X_{k-1}$$

$$X_k = X_{k-1} \oplus B \cap A$$

$$X_0 = p,\ k=1,2,3,$$



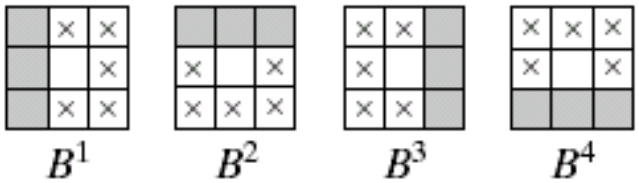
Computing convex hull using morphology

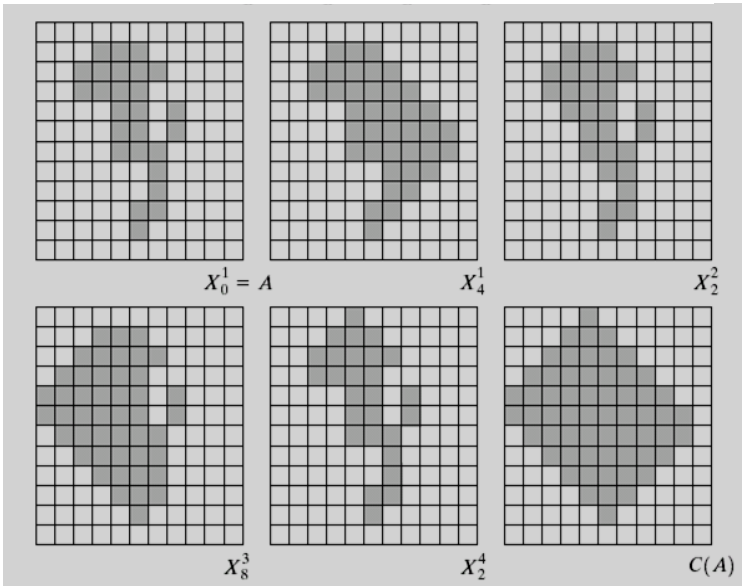
- The convex hull C of a set of points A may be estimated using the Hit-or-Miss transformation
- Consider the four structuring elements B^1-B^4.
- Apply hit-or-miss with A using B^1 iteratively until no more changes occur. Let D^1 be the result.
- Then do the same with B^2,..,B^4 to compute D^2…D^4 in the same manner.

$$X_k^i = \left(X_{k-1}^i \circledast B^i\right) \cup A; i = 1, 2, 3, 4;$$
$$k = 1, 2, 3, \ldots; X_0^i = A; \text{and}$$
$$D^i = X_{\text{conv}}^i.$$

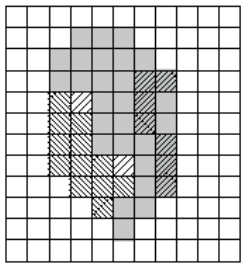
- Then estimate the convex hull as the union of all the D^is, i.e. C (A) = $\bigcup D_i$
- Does not always give the smallest convex set containing the original points (which it should to be the convex hull of the points).
- Can be improved by taking the intersection to the maximum dimension in x and y direction

C_res = C (A) ⋂ ROI(A)



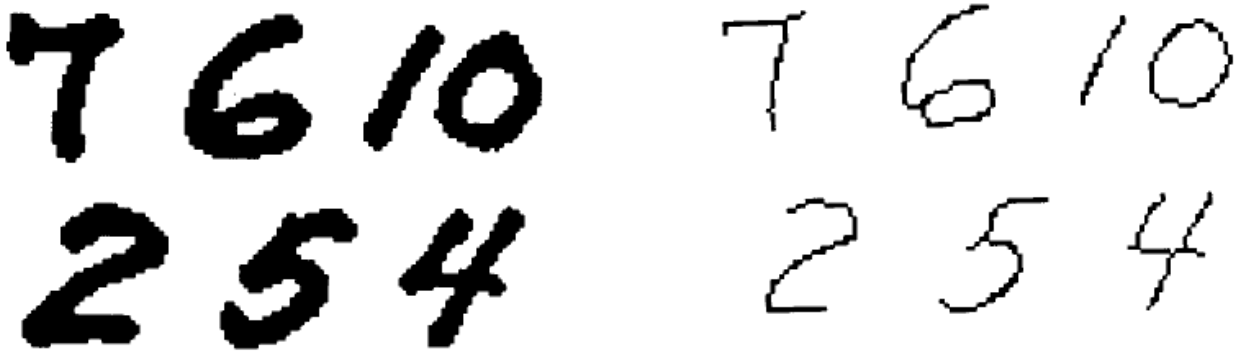


C_res



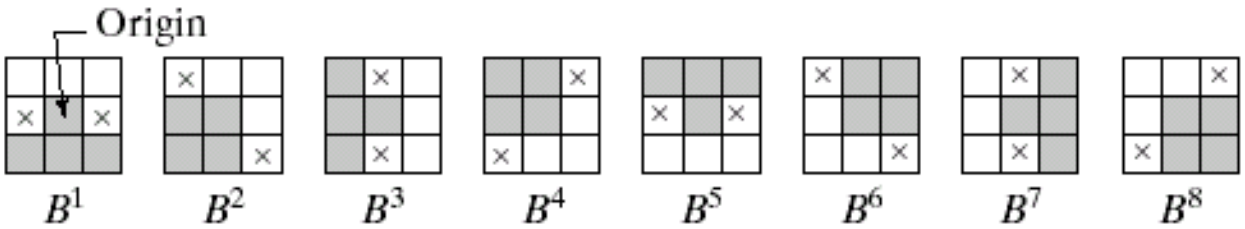
x in B means don't care

Region thinning and skeletons

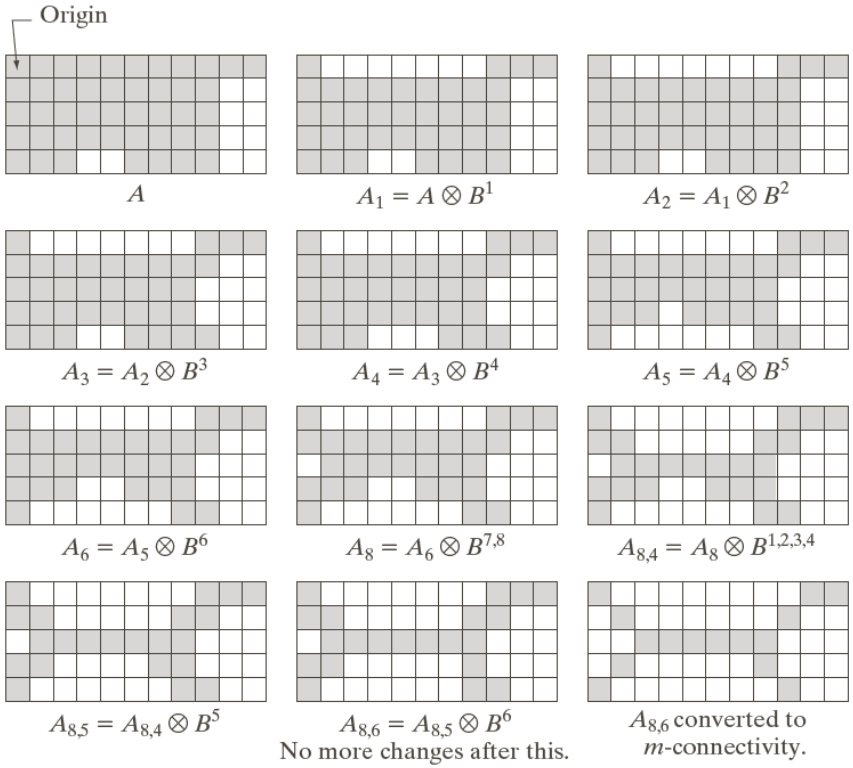


- Let the region object be described using an intrinsic coordinate system, where every pixel is described by its distance from the nearest boundary pixel.
- The skeleton is defined as the set of pixels whose distance from the nearest boundary is locally maximum.
- Many different methods for computing the skeleton exist.
- Shape features can later be extracted from the skeleton.
- Skeletonization may imply loss of information.
- *Thinning* is a possible procedure to compute the skeleton.

Thinning



- Thinning of set A with structuring element B (a set of n structuring elements, here n=8).
- First, thin A by one pass of B^1, then thin the result by one pass of B^2, and so on, until A is thinned with one pass of B^n.
- Then repeat the entire process until no further changes occur.



- $A \otimes B = A - (A \circledast B)$

- $A \otimes \{B\} =$
  $((...((A \otimes B^1) \otimes B^2)...) \otimes B^n)$

Thickening

- Thickening is the dual operator of thinning.

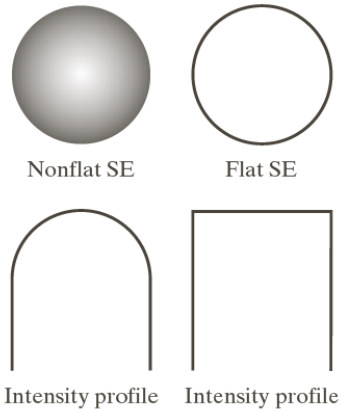
- It can be computed as a separate operation, but thickening the object is normally computed by thinning the background and then complementing the result:

  from $C=A^C$, thin C, then form C^C.

  (Example: Fig 9.22 in G&W)

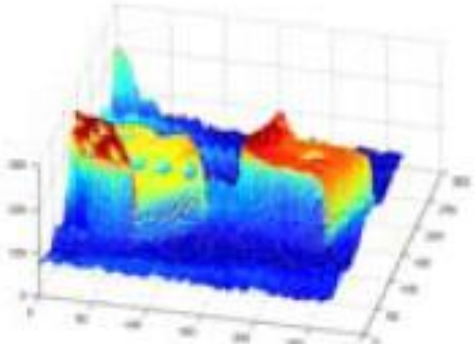
Grey level morphology

- As for the binary case, we will assume a flat structuring element
- Also, the centre pixel is the origin when not otherwise specified.



- Erosion and dilation then correspond to local minimum and maximum over the area defined by the structuring element.





Grey level erosion / dilation



- **Erosion**:
  - Place the structuring element with origin at pixel (x,y)
  - Chose the local **minimum** grey level in the region defined by the structuring element B
  - Assign this value to the output pixel (x,y)
  - Results in darker images and removal of light details:

$$[f \ominus B](x,y) = \min_{(s,t)\in B}\{f(x+s, y+t)\}$$

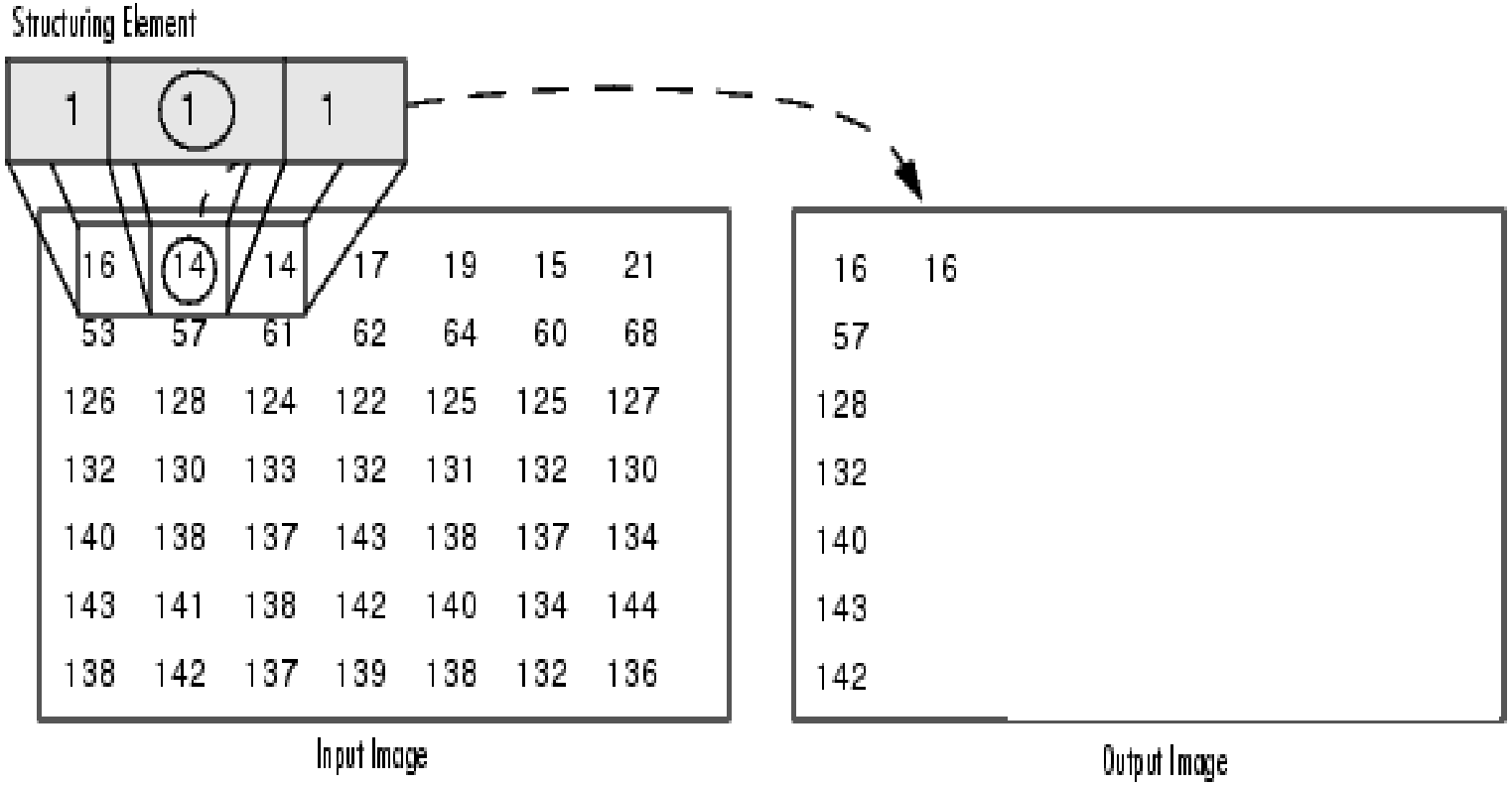


- **Dilation**:
  - Place the structuring element with origin at pixel (x,y)
  - Chose the local **maximum** grey level in the region defined by the reflected structuring element B
  - Assign this value to the output pixel (x,y)
  - Results in brighter images and removal of dark details:

$$[f \oplus B](x,y) = \max_{(s,t)\in B}\{f(x-s, y-t)\}$$



Grey level dilation example



Morphological Dilation of a Grayscale Image

Grey level opening and closing

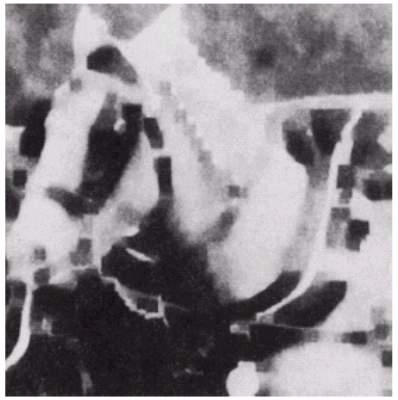
- Corresponding definition as for binary opening and closing





$$f \circ S = (f \ominus S) \oplus S$$
$$= \max(\min(f))$$

- Opening smooths out bright details



$$f \bullet S = (f \oplus S) \ominus S$$
$$= \min(\max(f))$$

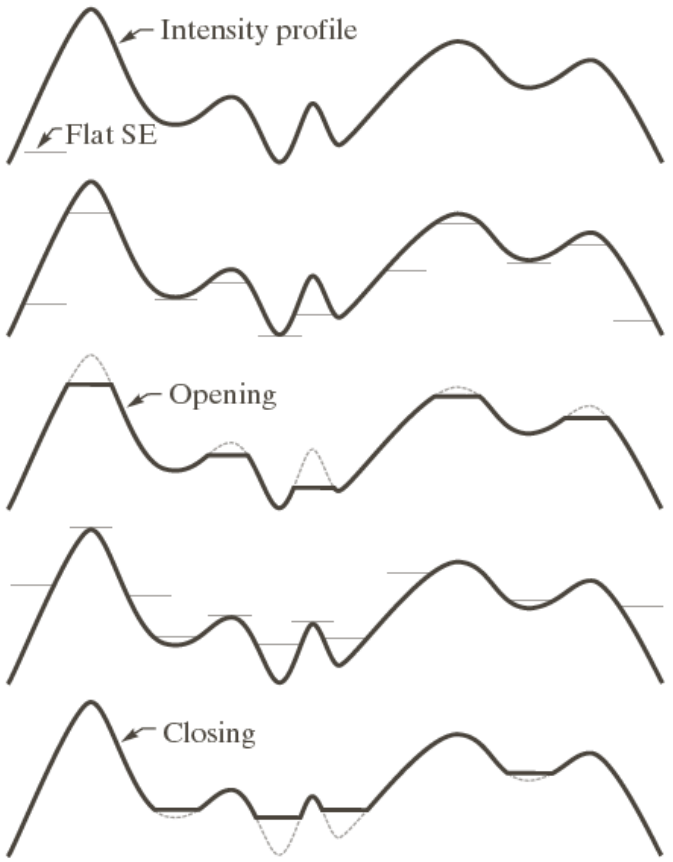
- Closing smooths out dark details

Grey level opening and closing

- Intensity values can be drawn as height curves over the (x,y)-plane.

- Opening of f by B:
  Push the structuring element up from below towards the curve f.
  The value assigned is the highest level B can reach.
  smooths bright values down.

- Closing:
  Push the structuring element down from above towards the curve f.
  The value assigned is the lowest level B can reach.
  smooths dark values upwards.



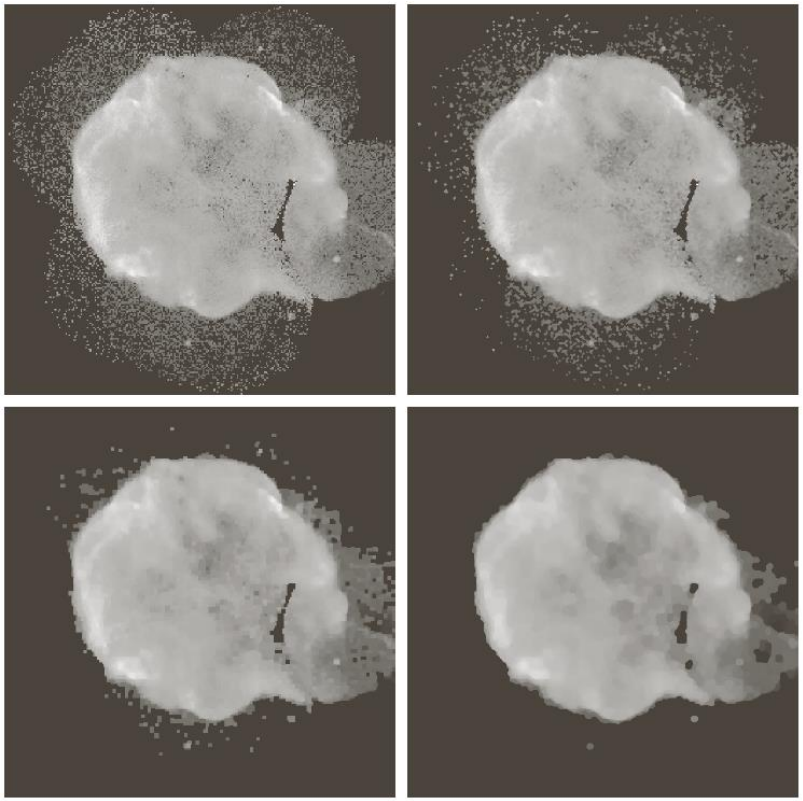
Morphological filtering

- Grey level opening and closing with flat structuring elements can be used to filter out noise.
- This is particularly useful for e.g. dark or bright noise.
- Remark: bright or dark is relative to surroundings, e.g. local extremas are filtered.
- To remove bright noise and then dark noise, do first opening, then closing:

  min(max(max(min(f(x,y)))))
- The size of the structuring element should reflect the size of the noise objects that we wish to remove.

Morphological filtering example

Original



Opening and closing by disk of size 1

Opening and closing by disk of size 5

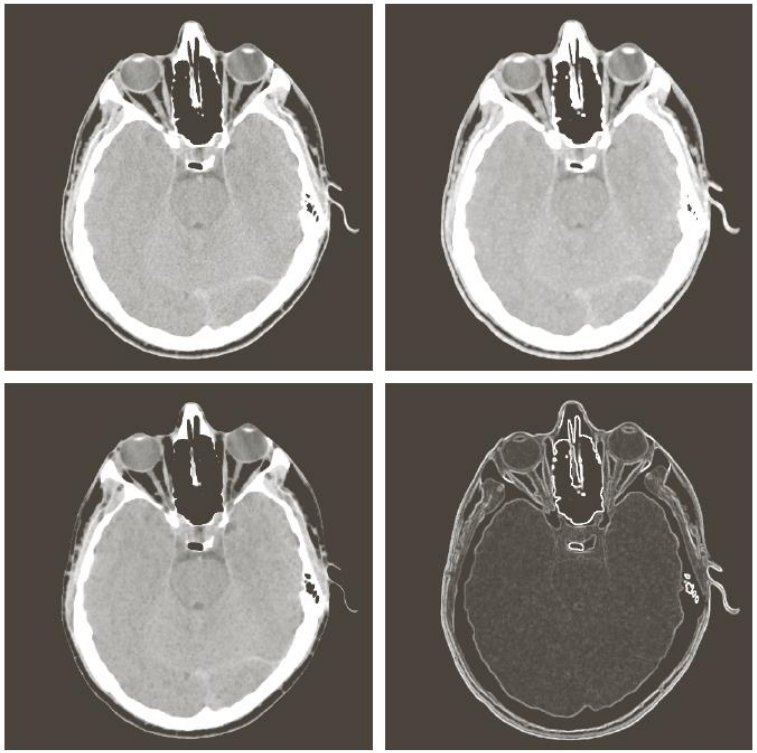
Opening and closing by disk of size 3

Morphological gradient

- Grey level dilation will give an image with equal or higher values.
- Grey level erosion will give an image with equal or lower values.
- This can be used for edge detection.
- Morphological gradient = $(f \oplus S) - (f \ominus S)$

  => gradient = dilation minus erosion

Morphological gradient



Erosion

Dilation

Gradient
= dilation-erosion

Top-hat transformation

- Purpose: detect (or remove) structures of a certain size.
- Top-hat: detects light objects on a darker background
  - also called white top-hat.
- Top-hat (image minus its opening):

$$f - (f \circ b)$$

- Bottom-hat: detects dark objects on a brighter background
  - also called black top-hat.
- Bottom-hat (closing minus image):

$$(f \bullet b) - f$$

- Very useful when correcting for uneven illumination or objects on a varying background.

Top-hat transformation example

Original, un-even background





Global thresholding using Otsu's method.

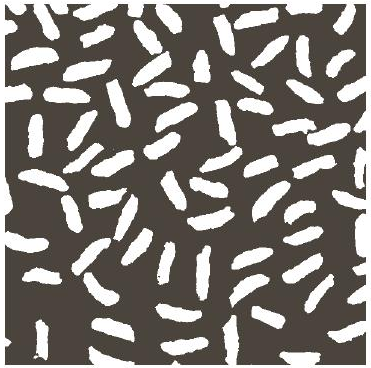
Objects in the lower right corner disappear. Misclassification of some background in upper left corner.

Opening with a 41x41 structuring element removes objects and gives an estimate of the background





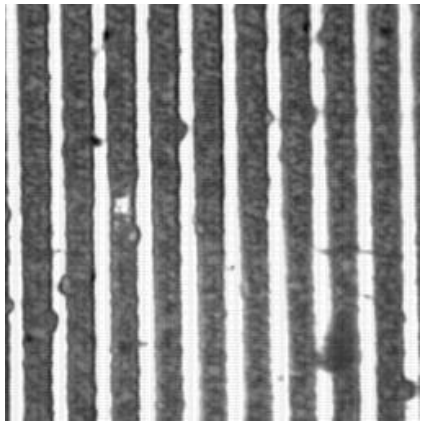
Top-hat transform (original – opening)



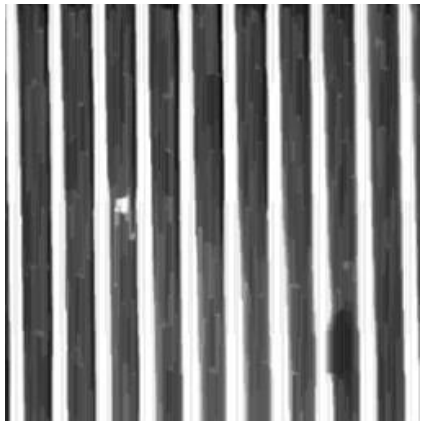
Top-hat thresholded with global Otsu threshold

An alternative to opening would be a moving 41x41 median filtering

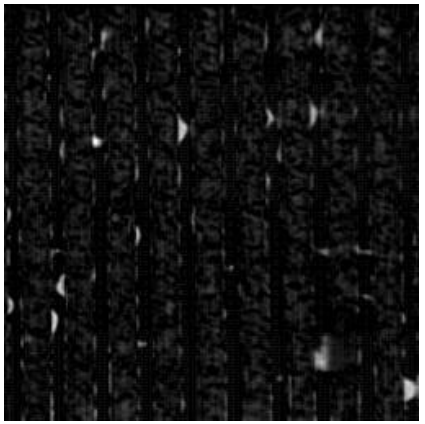
Fault detection using 'Bottom-hat'



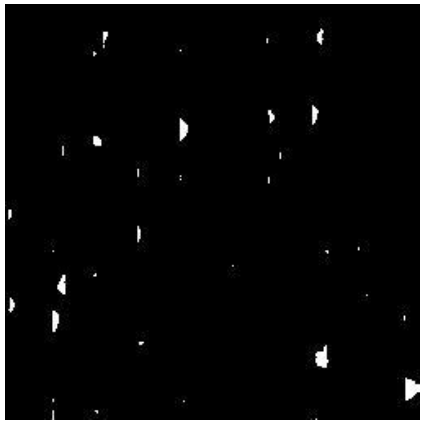
Original



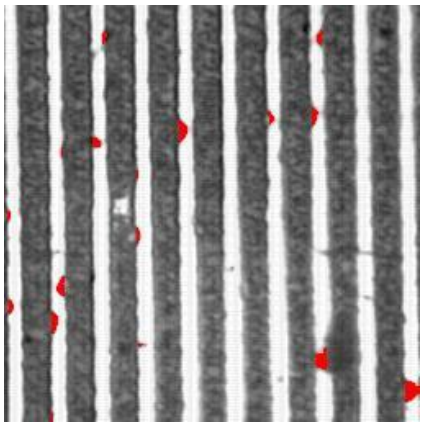
Closing' with vertical structuring element



Bottom-Hat = Closing – original



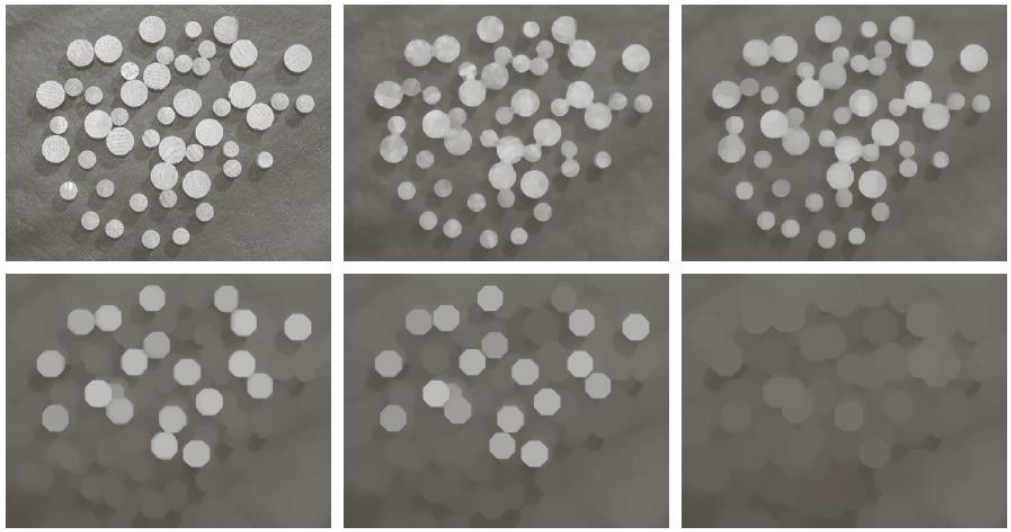
Threholded previous image



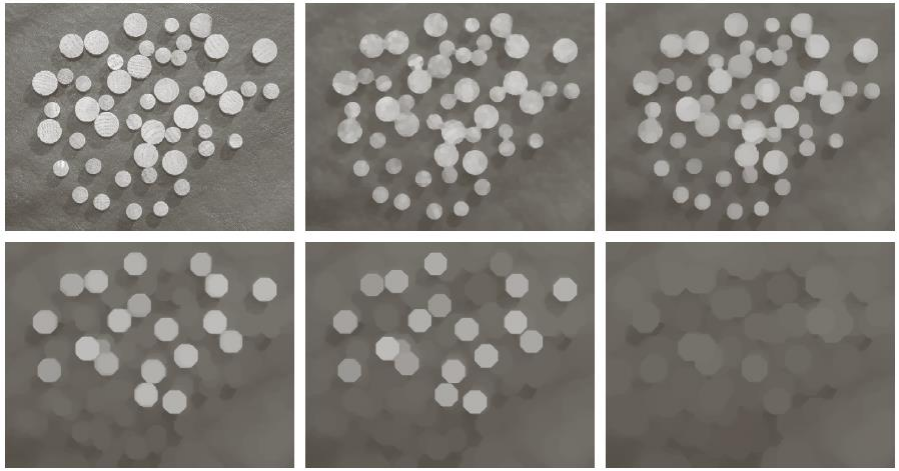
Error detection

Example application: Granulometry

- Granulometry: determine the size distribution of particles in an image.
- Assumption: objects with regular shape on a background.
- Principle: perform a series of openings
  with increasing radius r of structuring element
- Compute the sum of all pixel values after the opening.
- Compute the difference in this sum between radius r and r-1,
  and plot this as a function of radius.



Example application: Granulometry



a b c
d e f

FIGURE 9.41 (a) 531 × 675 image of wood dowels. (b) Smoothed image. (c)–(f) Openings of (b) with disks of radii equal to 10, 20, 25, and 30 pixels, respectively. (Original image courtesy of Dr. Steve Eddins, The MathWorks, Inc.)

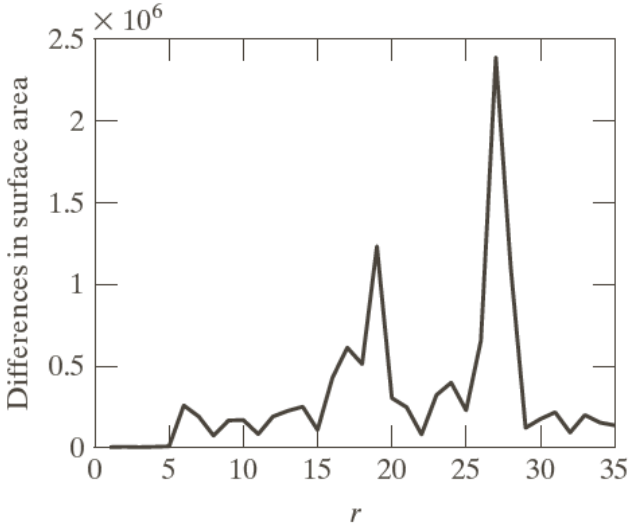


FIGURE 9.42 Differences in surface area as a function of SE disk radius, r. The two peaks are indicative of two dominant particle sizes in the image.

Learning goals

- Understand in detail binary morphological operations and selected applications:
  - Be able to perform basic operators (erosion, dilation, opening, closing) by hand and through its geometrical view.
  - Understand how morphological operations can be used to find e.g. edges, connected components, and convex hull.

- Understand in detail grey level morphological operations:
  - Be able to perform basic operators (erosion, dilation, opening, closing) by hand and through its geometrical view.
  - Understand top-hat and bottom-hat transformations and what they are used for.