IN5550: Neural Methods in Natural Language Processing
Sub-lecture 2.2
Linear classifiers

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Linear classifiers

Simple linear function

$$f(\boldsymbol{x}; \boldsymbol{W}, \boldsymbol{b}) = \boldsymbol{x} \cdot \boldsymbol{W} + \boldsymbol{b} \tag{1}$$

$$\theta = \boldsymbol{W}, \boldsymbol{b} \tag{2}$$

- Function input:
  - feature vector $\boldsymbol{x} \in \mathbb{R}^{d_{in}}$;
  - each training instance is represented with d_{in} features;
  - for example, some properties of the documents.

Linear classifiers

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- Function input:
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  - each training instance is represented with d_{in} features;
  - for example, some properties of the documents.
- Function parameters θ:
  - matrix $\boldsymbol{W} \in \mathbb{R}^{d_{in} \times d_{out}}$
    - d_{out} is the dimensionality of the desired prediction (number of classes)
  - bias vector $\boldsymbol{b} \in \mathbb{R}^{d_{out}}$
    - bias 'shifts' the function output to some direction.

Linear classifiers

Training of a linear classifier

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Linear classifiers

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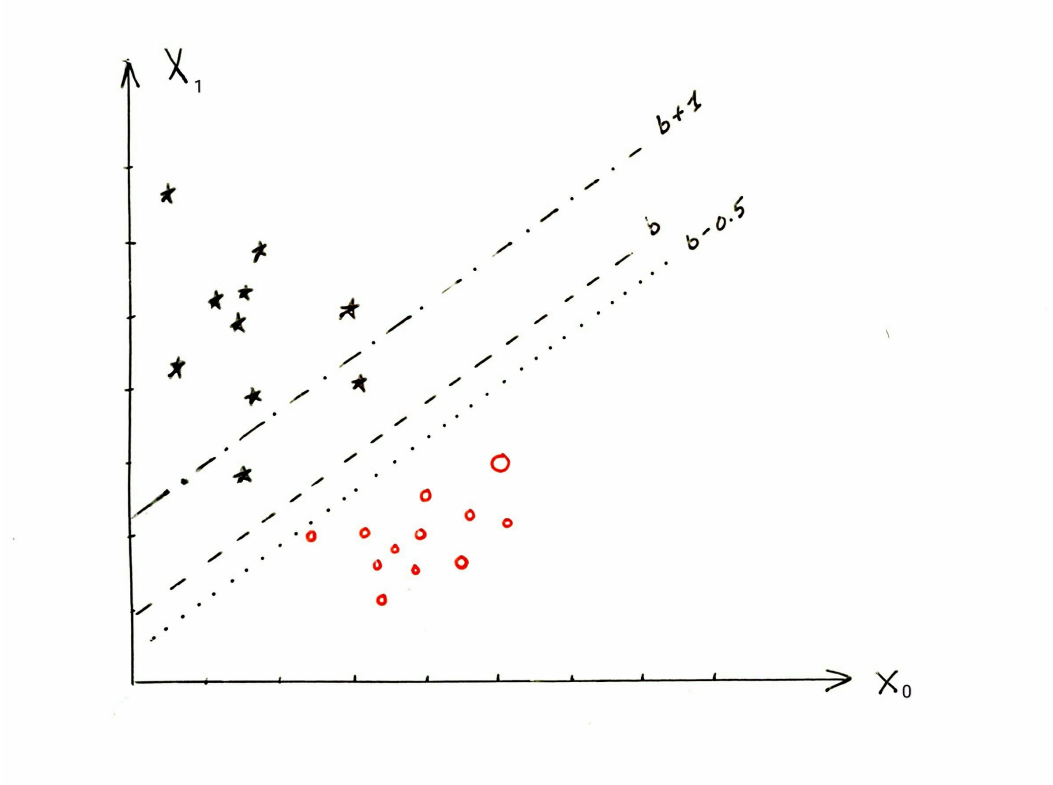
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- Training is finding the optimal θ.
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- Ideally, $\hat{\boldsymbol{y}} = \boldsymbol{y}$

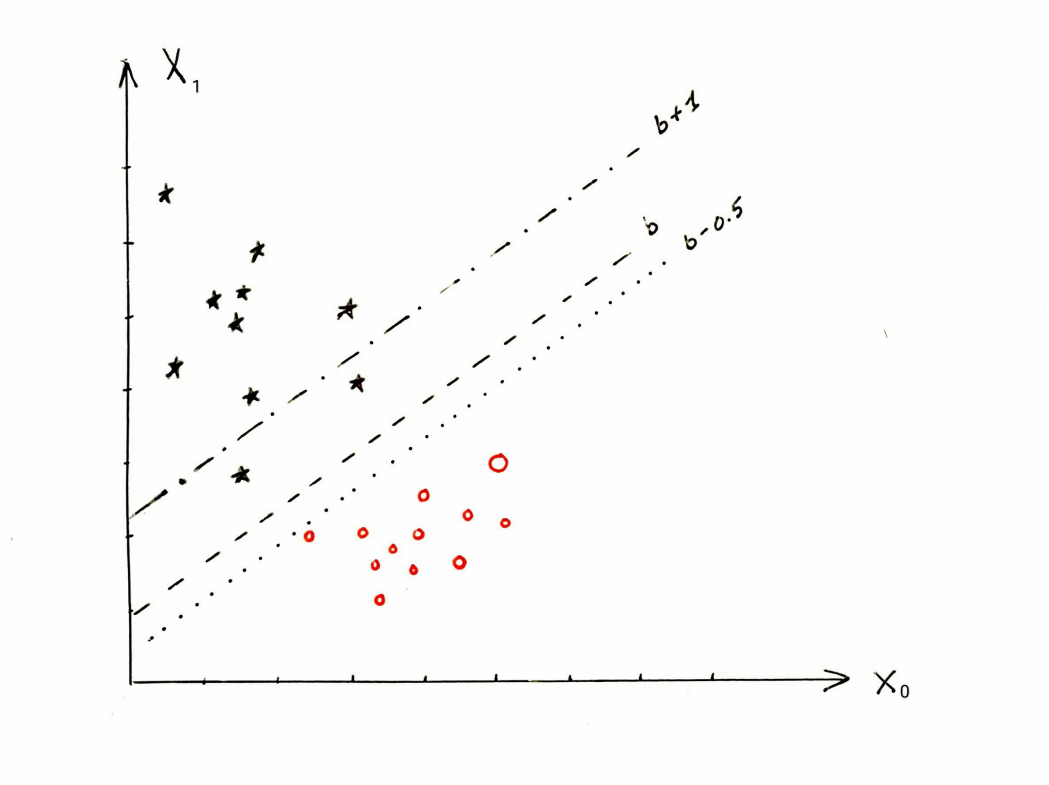
Linear classifiers

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Linear classifiers

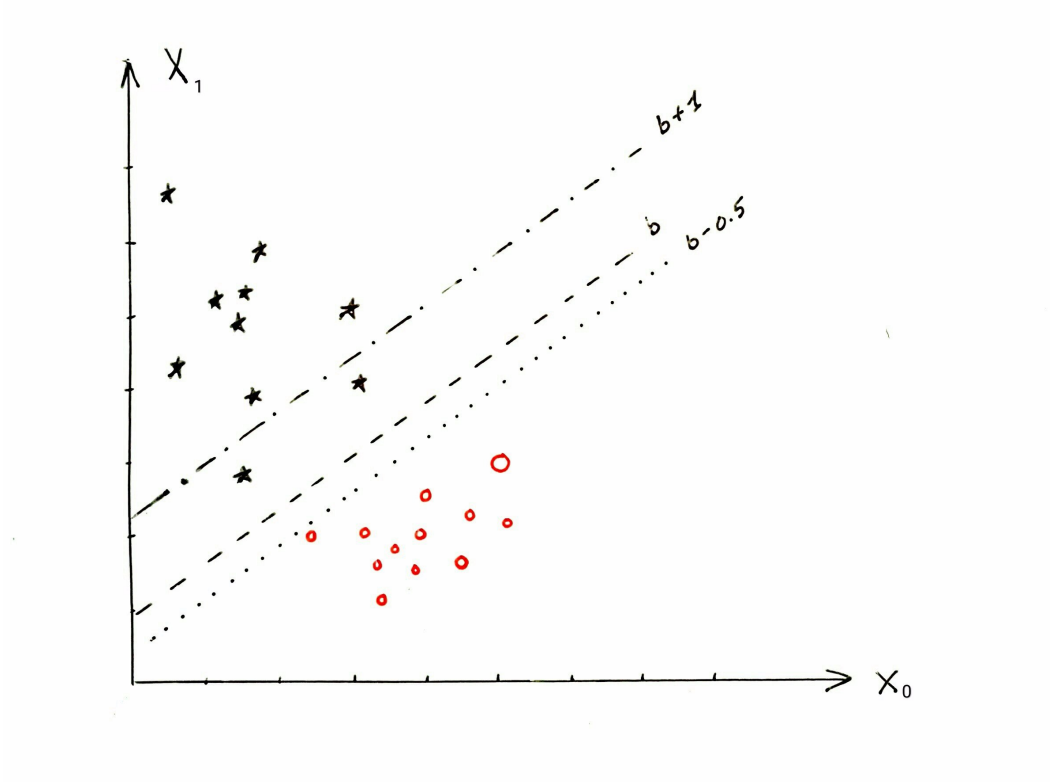
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Linear classifiers

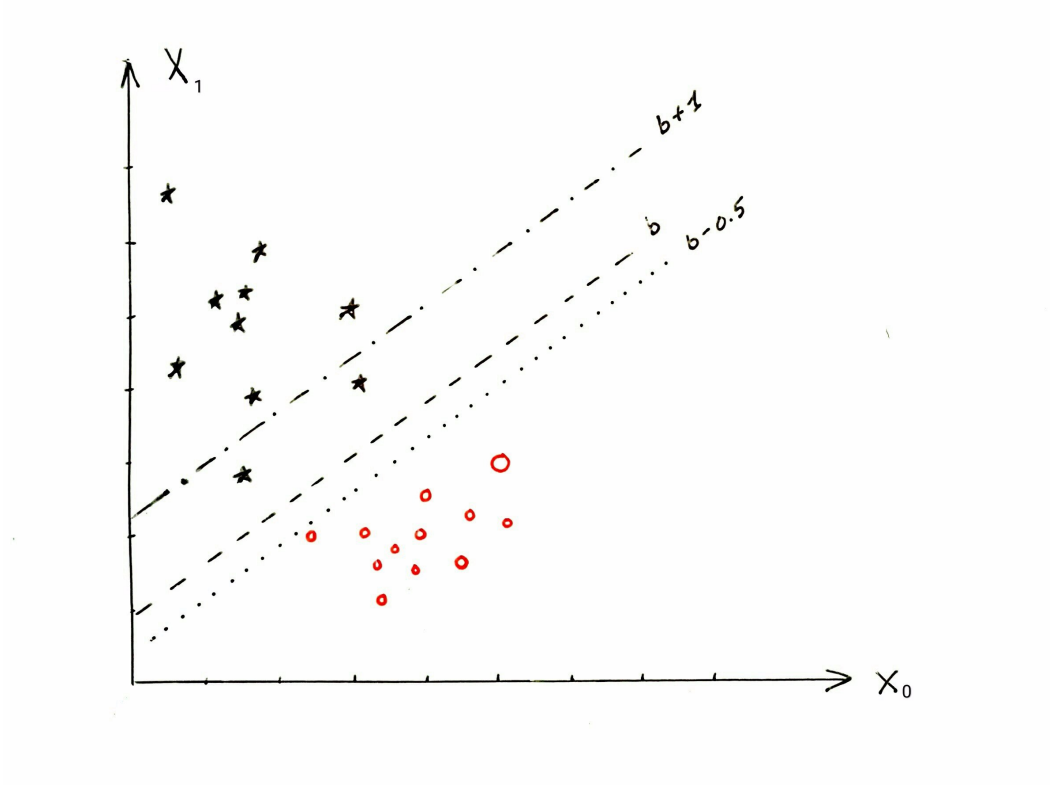
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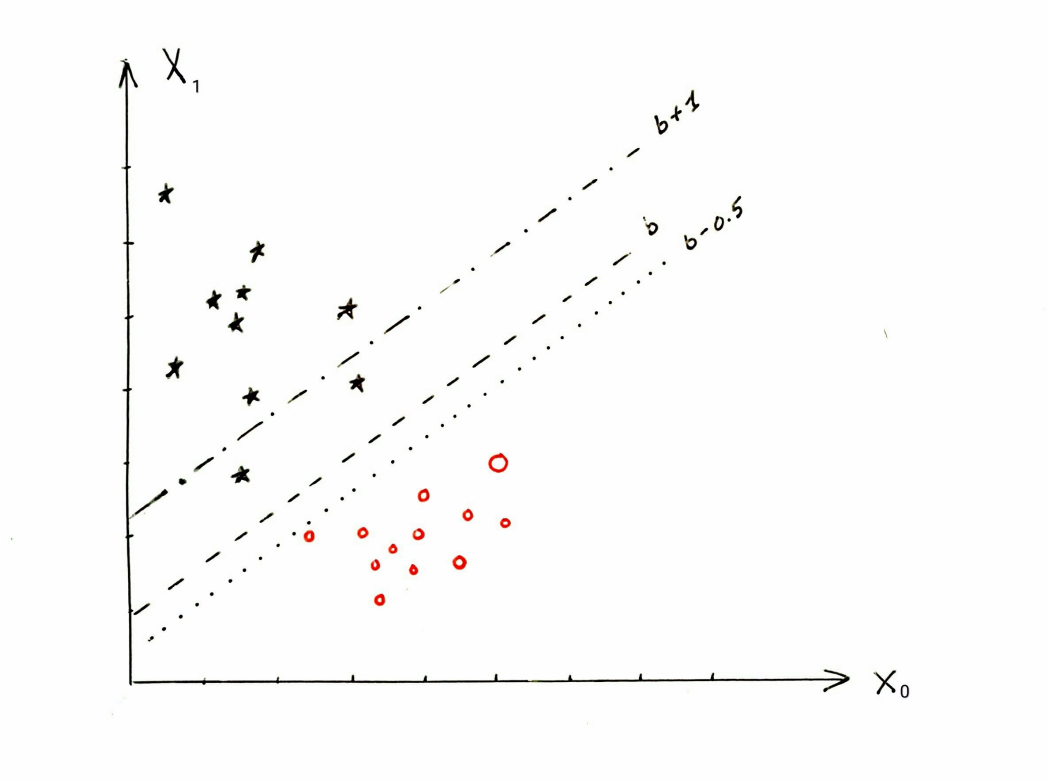
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- NB: the dataset on the plot is linearly separable.
- **Question: lines with 3 values of b are shown. Which is the best?**

Linear classifiers

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- This schema is called 'bag of words' (BoW).
  - for example, if we have 1000 words in the vocabulary:
  - $\boldsymbol{x}_i \in \mathbb{R}^{1000}$

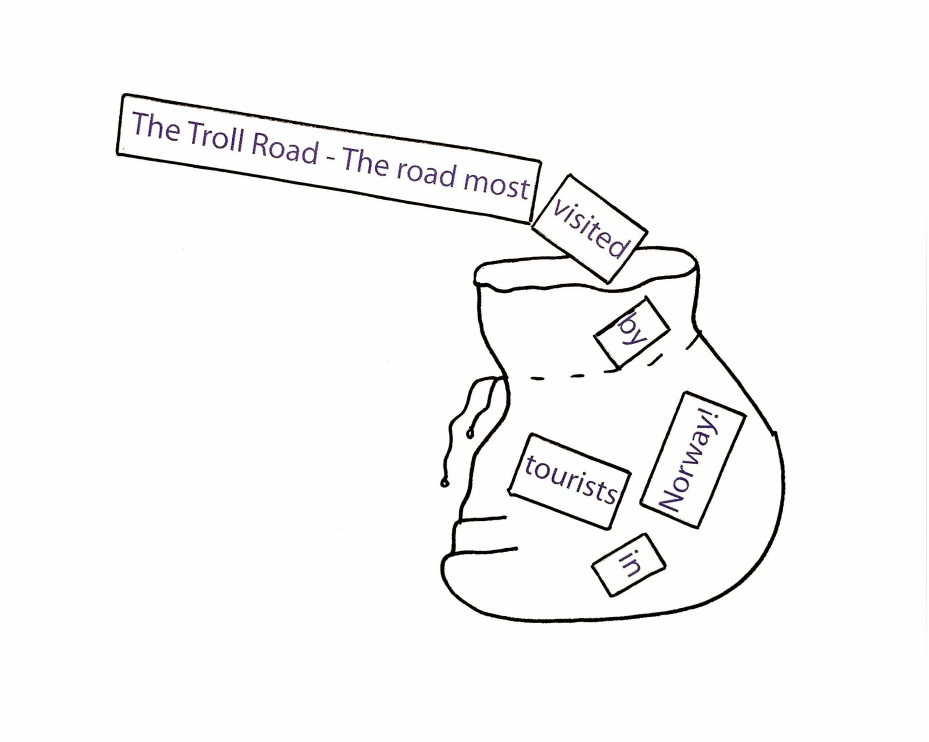
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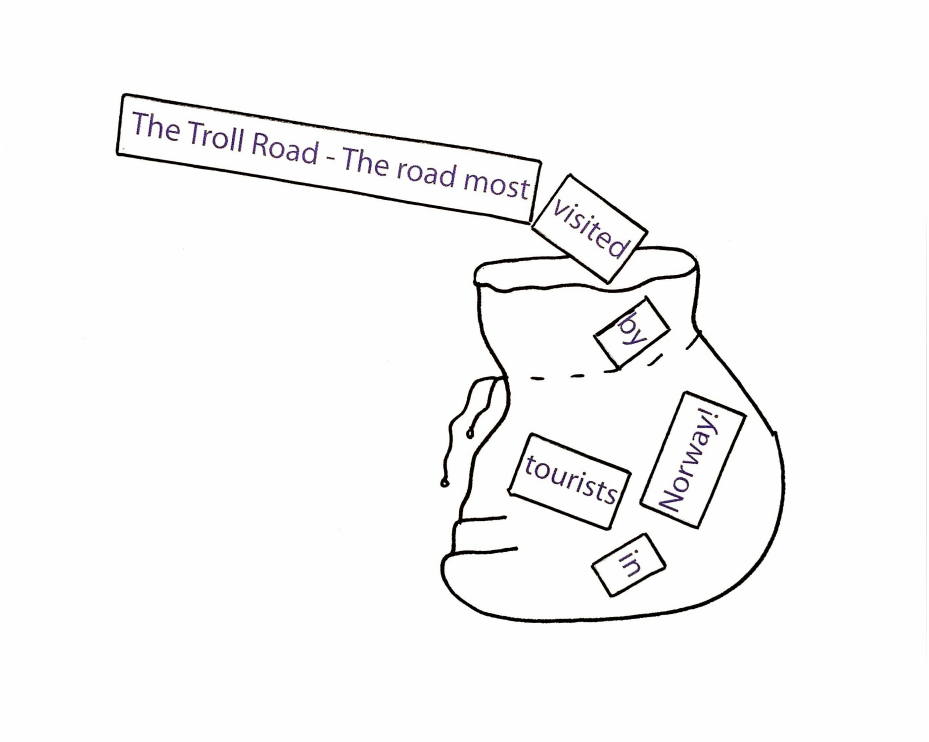
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Linear classifiers



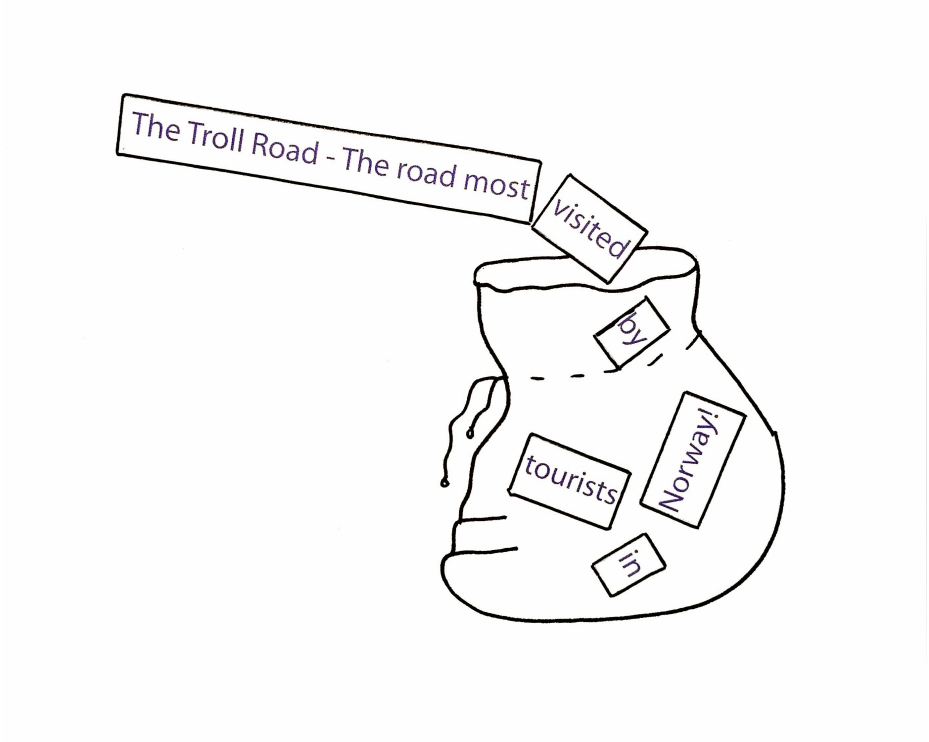
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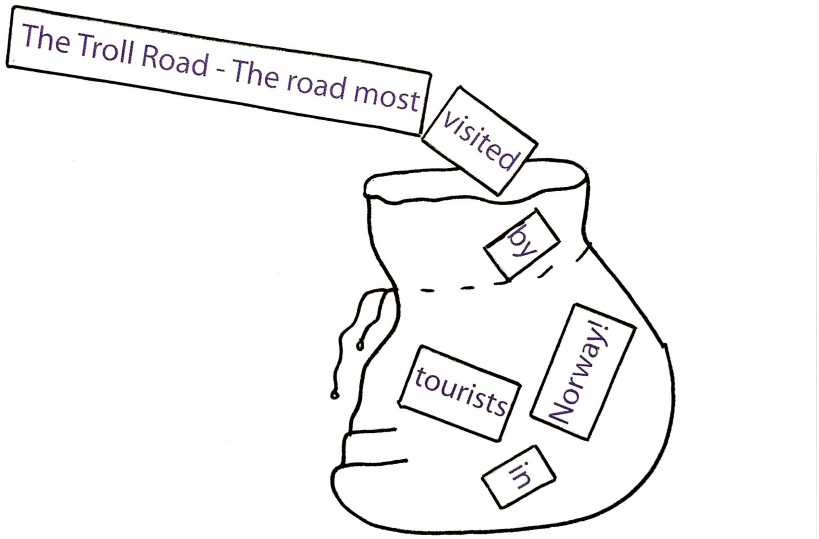
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  - D extracted from the text above contains 10 words (lowercased): $\{$*'-'*, *'by'*, *'in'*, *'most'*, *'norway'*, *'road'*, *'the'*, *'tourists'*, *'troll'*, *'visited'*$\}$.

Linear classifiers



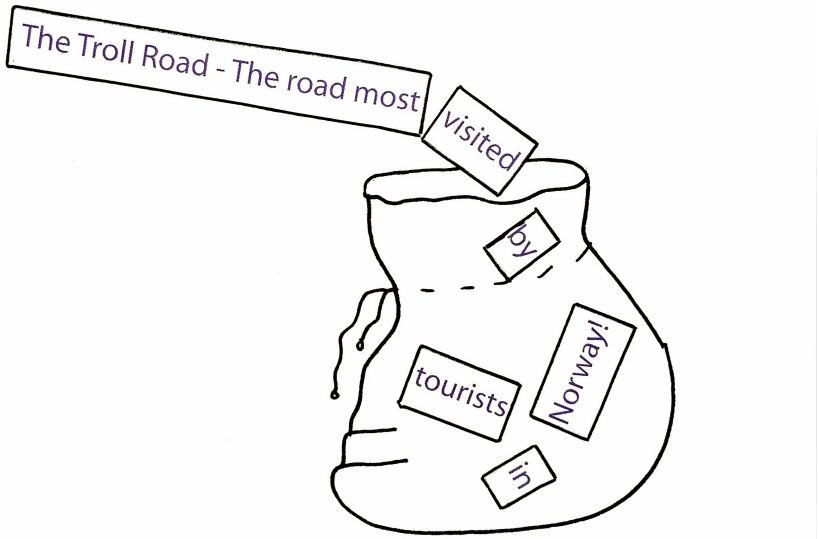
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  - etc...
  - $\boldsymbol{i} = [1, 1, 1, 1, 1, 2, 2, 1, 1, 1]$ ('*the*' and '*road*' mentioned 2 times)

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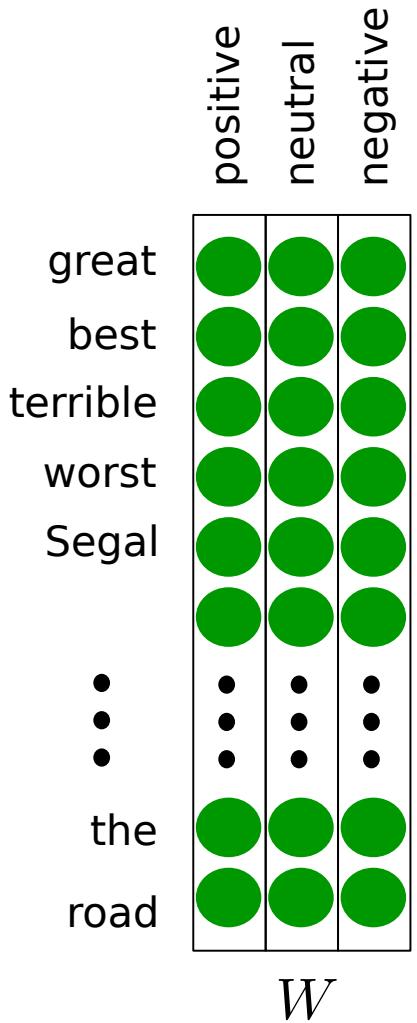
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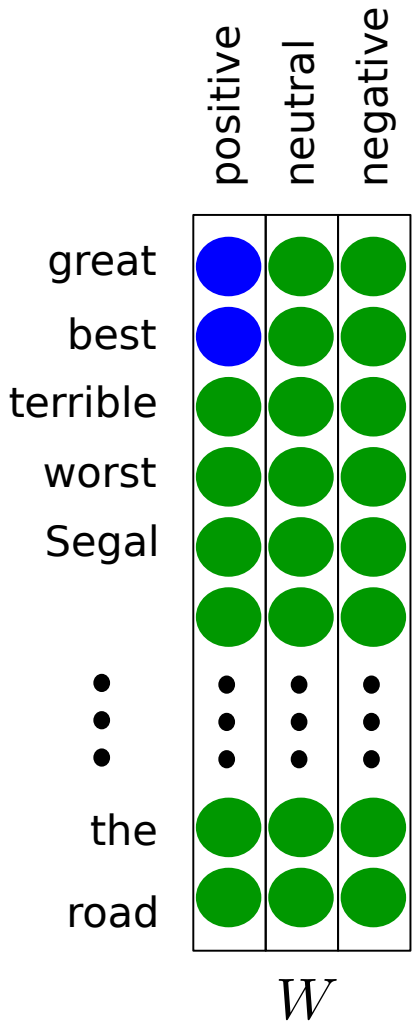
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- Feature engineering is deciding what features of the instances we will use during the training.

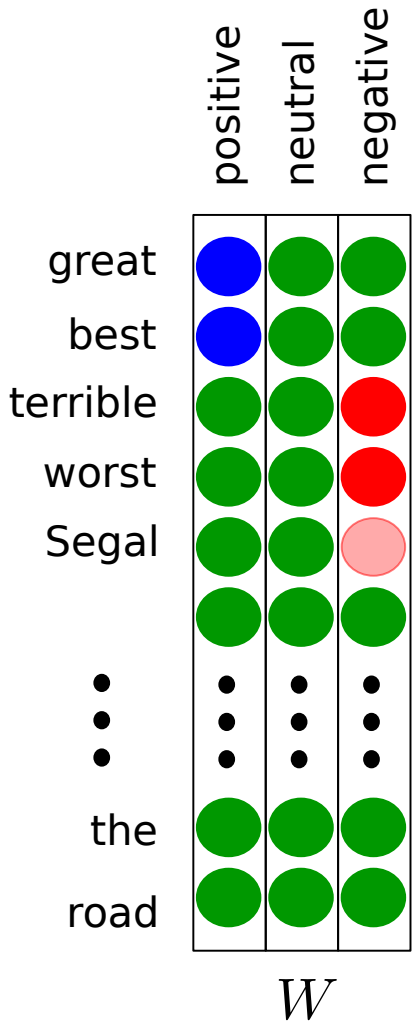
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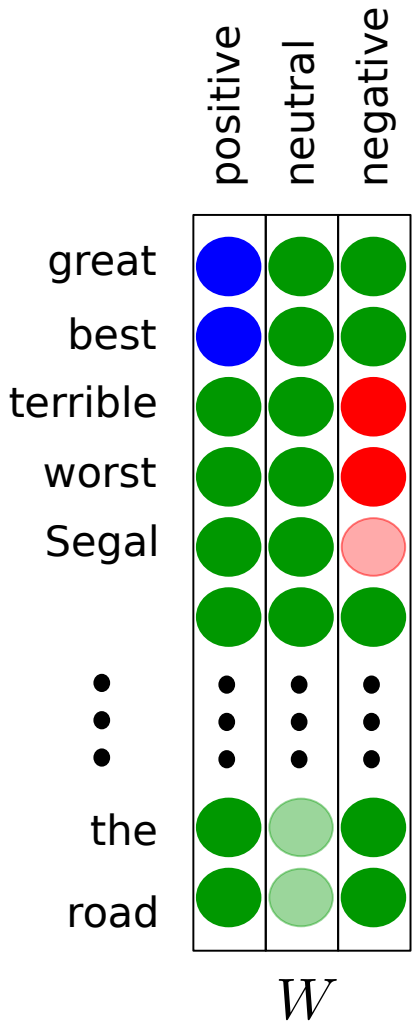
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Linear classifiers

$$f(\boldsymbol{x}; \boldsymbol{W}, \boldsymbol{b}) = \boldsymbol{x} \cdot \boldsymbol{W} + \boldsymbol{b}$$

Output of binary classification

Binary decision ($d_{out} = 1$):

Linear classifiers

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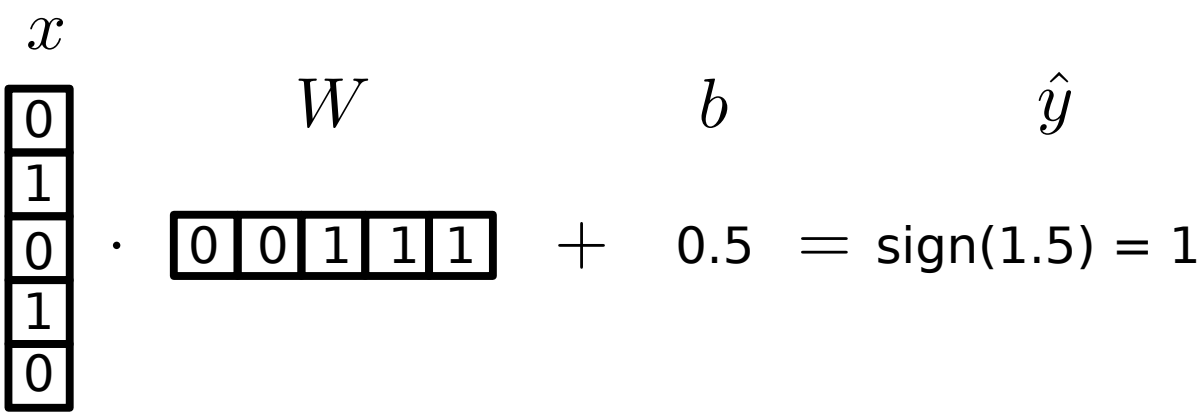
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- NB: the model can output any number, but we convert all negatives to -1 and all positives to 1 (*sign* function).

$$\theta = (\boldsymbol{W} \in \mathbb{R}^{d_{in}}, \boldsymbol{b} \in \mathbb{R}^1)$$

Linear classifiers

$$f(x; W, b) = x \cdot W + b$$



Linear classifiers

$$f(\boldsymbol{x}; \boldsymbol{W}, \boldsymbol{b}) = \boldsymbol{x} \cdot \boldsymbol{W} + \boldsymbol{b}$$

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Linear classifiers

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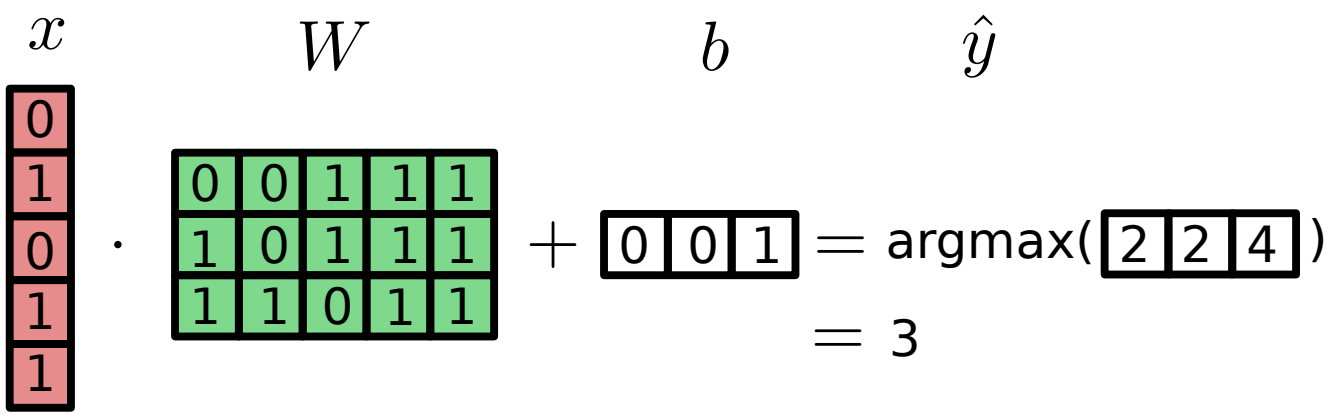
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- The prediction $\hat{\boldsymbol{y}}$ is also a one-hot vector of k components.
- The component corresponding to the correct language has the value of 1, others are zeros, for example:
  $\hat{\boldsymbol{y}} = [0, 0, 1, 0]$ (for $k = 4$)

$$\theta = (\boldsymbol{W} \in \mathbb{R}^{d_{in} \times d_{out}}, \boldsymbol{b} \in \mathbb{R}^{d_{out}})$$

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Linear classifiers

Log-linear classification

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Linear classifiers

Log-linear classification

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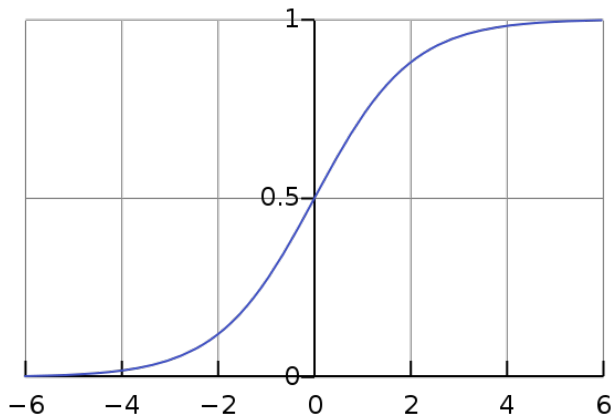
Log-linear classification

If we care about how confident is the classifier about each decision:

- Map the predictions to the range of $[0, 1]$...
- ...by a squashing function, for example, sigmoid:

$$\hat{y} = \sigma(f(\boldsymbol{x})) = \frac{1}{1 + e^{-(f(\boldsymbol{x}))}} \tag{3}$$

- The result is the probability of the prediction!



$\sigma(\boldsymbol{x})$

Linear classifiers

- For multi-class cases, log-linear models produce probabilities for all classes, for example:
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- But often it is more convenient to transform scores into a probability distribution, using the softmax function:

$$\hat{\boldsymbol{y}} = softmax(\boldsymbol{x}\boldsymbol{W} + \boldsymbol{b}) \tag{5}$$

$$\hat{\boldsymbol{y}}_{[i]} = \frac{e^{(\boldsymbol{x}\boldsymbol{W}+\boldsymbol{b})_{[i]}}}{\sum_j e^{(\boldsymbol{x}\boldsymbol{W}+\boldsymbol{b})_{[j]}}} \tag{6}$$

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- $\hat{\boldsymbol{y}} = \mathit{softmax}([0.4, 0.1, 0.9, 0.5]) = [0.22, 0.16, 0.37, 0.25]$
  - (all scores sum to 1)

Linear classifiers

$$f(x; W, b) = x \cdot W + b$$

