IN5550: Neural Methods in Natural Language Processing
Sub-lecture 3.1
Multi-layered neural networks

Andrey Kutuzov, David Samuel

University of Oslo



Contents

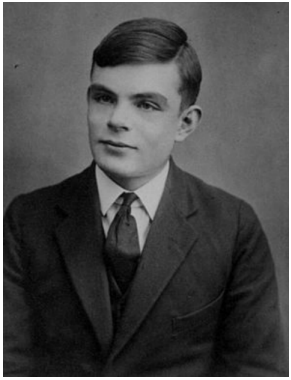
1. Is it like brain?
2. Let's talk math
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Is it like brain?

'Machines of this character can behave in a very complicated manner when the number of units is large.'

Alan Turing, 'Intelligent Machinery'

[Turing, 1948]



Is it like brain?

Brain metaphor

- Why ‘artificial neural networks’?



Is it like brain?

Brain metaphor



- Why ‘artificial neural networks’?
- ...because it seems neurons in human brain act similarly:

Is it like brain?

Brain metaphor



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Is it like brain?

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  - ... or completely relaxed (‘silent’);

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  - depending on input signals, a neuron can be more or less activated,
  - ... or completely relaxed ('silent');
  - the whole system is distributed across many neurons.

Is it like brain?

...But brain metaphor is not optimal



- First artificial neural networks were based loosely on knowledge of human brain at the time
- ...but we have long abandoned any real connection to neuroscience.

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- It's less cool but more convenient to think about ANNs in terms of linear algebra concepts:

Is it like brain?

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- First artificial neural networks were based loosely on knowledge of human brain at the time
- ...but we have long abandoned any real connection to neuroscience.
- It's less cool but more convenient to think about ANNs in terms of linear algebra concepts:
  - vectors and matrices
  - sequential algebraic operations on them.

Is it like brain?

The most basic neuron...

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Let's talk math

Recall the equation for a linear classifier:

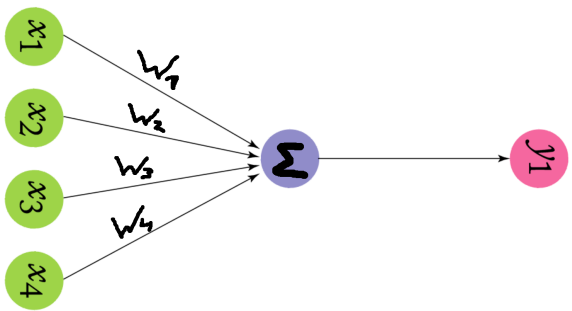
$$\hat{\boldsymbol{y}} = \boldsymbol{x}\boldsymbol{W}(+\boldsymbol{b}) \tag{1}$$

Let's talk math

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Suppose we have 4 features, binary classification task and no bias term $\boldsymbol{b}$:

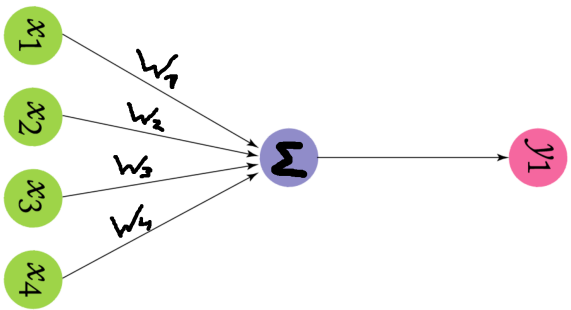


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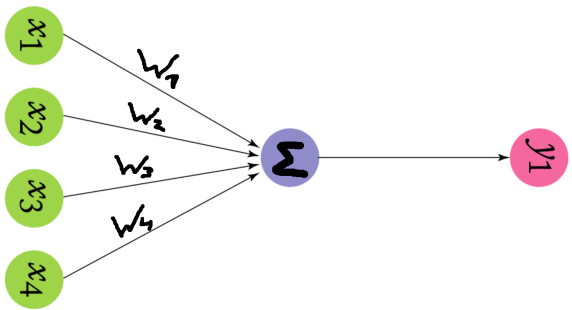
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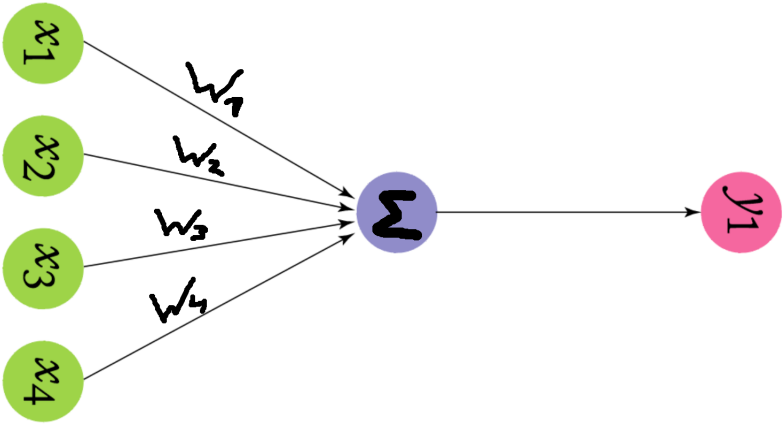
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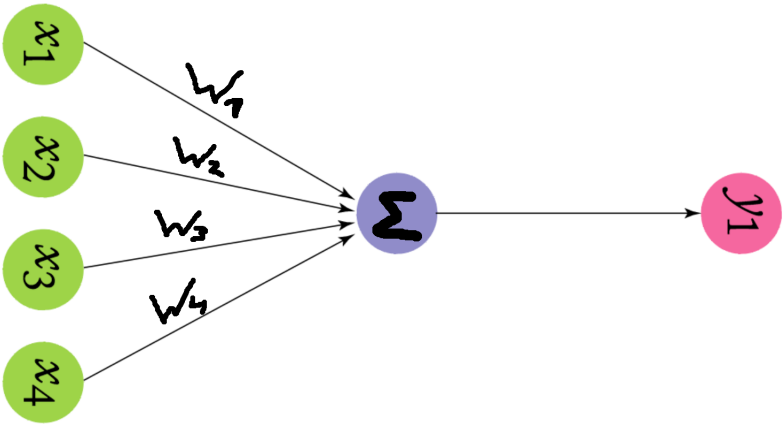


- input feature vector $\boldsymbol{x}$ is multiplied by the weights $\boldsymbol{W}$;
- here, $\boldsymbol{W}$ is a vector, so the result is a scalar value $\hat{y}_1$.

Let's talk math

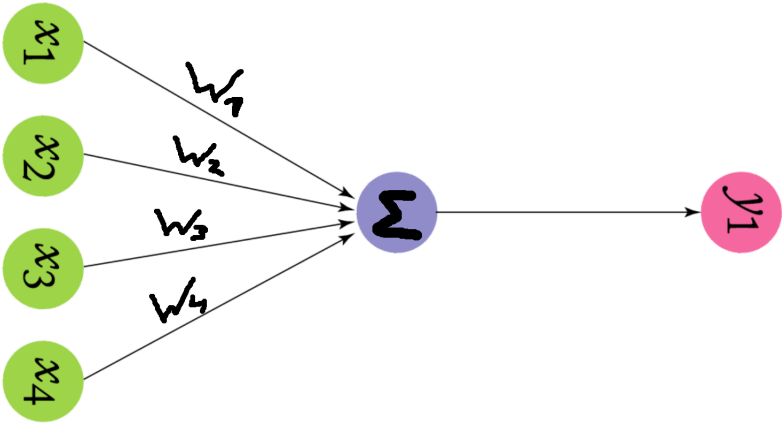


Let's talk math



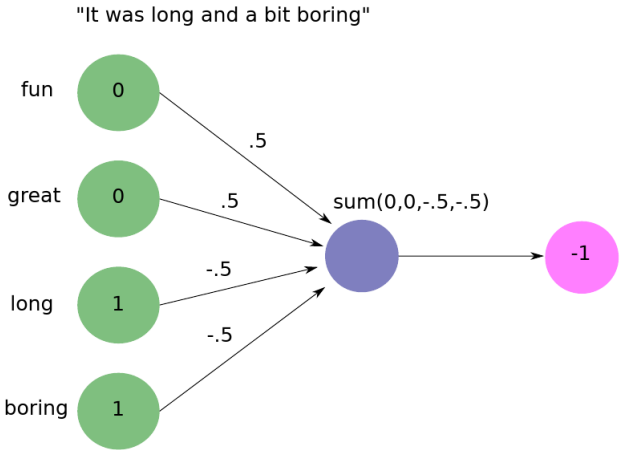
- In fact this is already a simple neural network...
- with one neuron Σ as a computational unit.

Let's talk math



- In fact this is already a simple neural network...
- with one neuron Σ as a computational unit.
- Σ takes 4 input values and returns their weighted sum as output value.

Let's talk math



Sentiment analysis with only one neuron and binary bag-of-words.

Let's talk math

Small improvements

- This is a linear architecture
  $\hat{\boldsymbol{y}} = \boldsymbol{x}\boldsymbol{W}$
- We can make this classifier predict vectors instead of scalars...

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Small improvements

- This is a linear architecture
  $\hat{\boldsymbol{y}} = \boldsymbol{x}\boldsymbol{W}$
- We can make this classifier predict vectors instead of scalars...
  - ...by making $\boldsymbol{W}$ a matrix;
  - ...and thus using a row of several neurons, instead of only one.

Let's talk math

Stack of linear classifiers

Additionally we can stack classifiers...

Let's talk math

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- by feeding the prediction of one classifier to another one:

Let's talk math

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- $\hat{\boldsymbol{y}} = (\boldsymbol{x}\boldsymbol{W}^{\mathbf{0}})\boldsymbol{W}^{\mathbf{1}}$

Let's talk math

Stack of linear classifiers

Additionally we can stack classifiers...

- by feeding the prediction of one classifier to another one:
- $\hat{y} = (xW^0)W^1$
- $\hat{y} = ((xW^0)W^1)W^2$

Let's talk math

Stack of linear classifiers

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Let's talk math

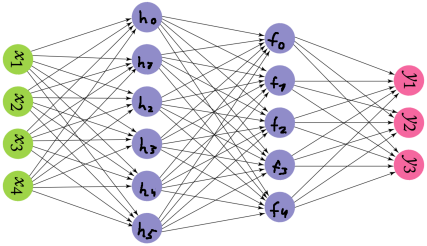
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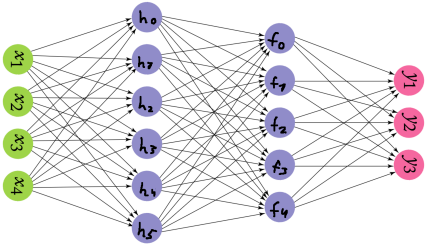
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- ...etc. Can be arbitrarily deep
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- it is multiplied by the next weight matrix (shapes must match).

Can even use both multiple neurons and multiple layers.

Let's talk math



Let's talk math



Looks much like a 'deep neural network'

Stacked linear classifier with multiple computational units at each layer:

$$h = \boldsymbol{x}\boldsymbol{W^0} \tag{2}$$

$$f = \boldsymbol{h}\boldsymbol{W^1} \tag{3}$$

$$\hat{\boldsymbol{y}} = \boldsymbol{f}\boldsymbol{W^2} \tag{4}$$

Let's talk math

- But any stack of linear classifiers is still a linear classifier :-(

Let's talk math

- But any stack of linear classifiers is still a linear classifier :-(
- Still can't handle XOR and other non-linear problems.

Let's talk math

Non-linear transformations of the input data make the desired difference.

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Non-linearities

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$$relu(\boldsymbol{x}) = \begin{cases} \boldsymbol{x}, & \text{if } \boldsymbol{x} \geq 0 \\ 0, & \text{otherwise} \end{cases} \rightarrow [0, \infty]$$

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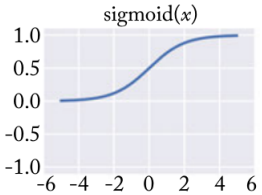
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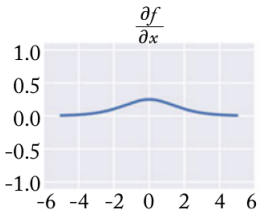
- Hyperbolic tangent:

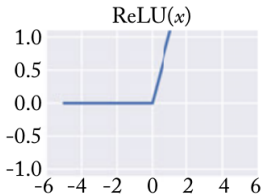
$$\tanh(\boldsymbol{x}) = \frac{e^{2\boldsymbol{x}} - 1}{e^{2\boldsymbol{x}} + 1} \rightarrow [-1, 1]$$

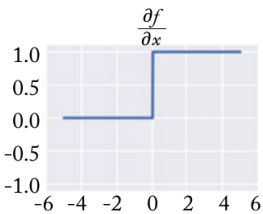
Non-linearities

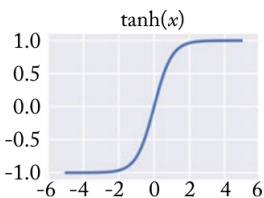
Popular activation functions...

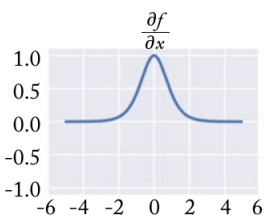






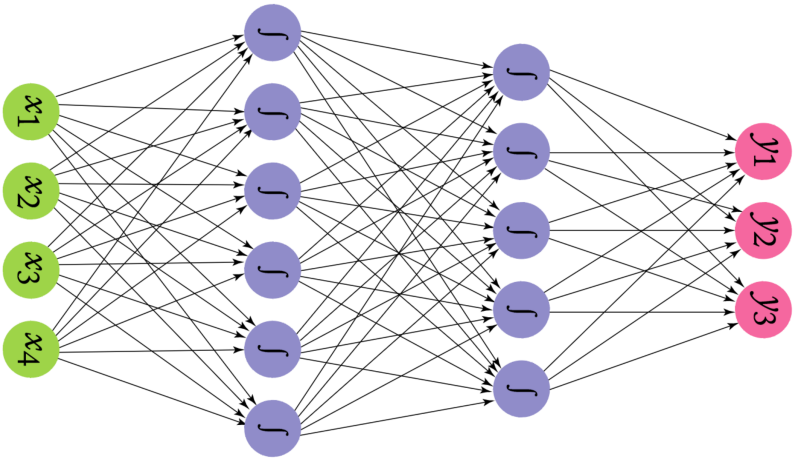






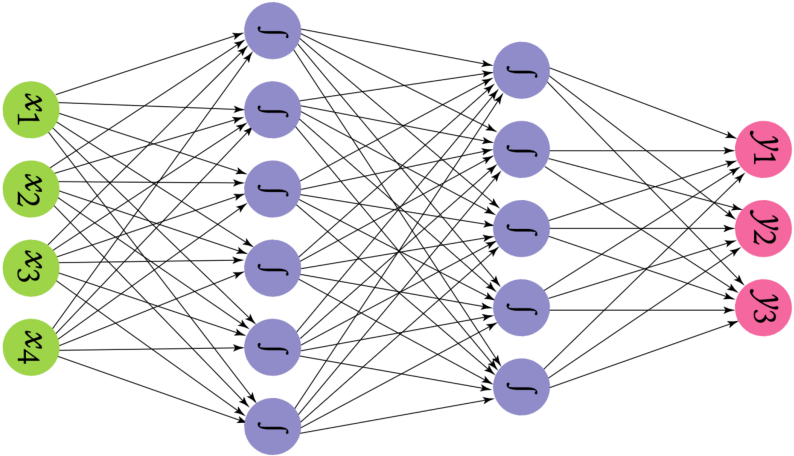
...and their derivatives.

Non-linearities



The only difference between this deep neural network and a stack of linear classifiers: non-linearities ($\int$) between linear transformations.

Non-linearities



The only difference between this deep neural network and a stack of linear classifiers: non-linearities ($\int$) between linear transformations.

In the next sub-lecture 3.2: why 'feed-forward' and 'fully connected'?

References I


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Deep sparse rectifier neural networks.
In Proceedings of the Fourteenth International Conference on Artificial Intelligence and Statistics, pages 315–323.


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