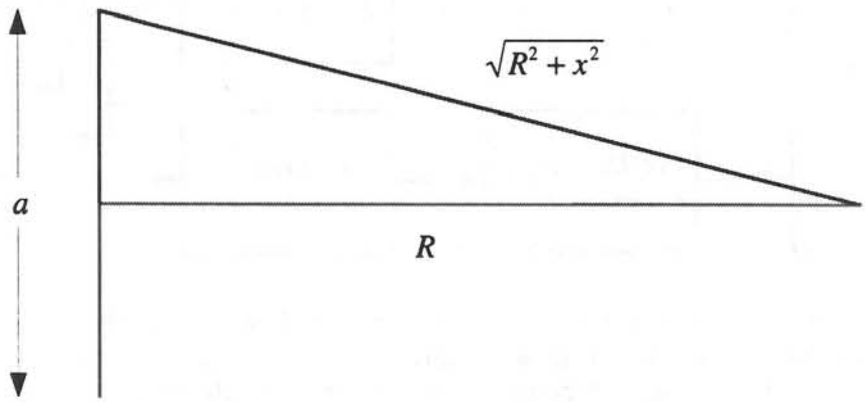
Apertures and Geometrical Optics

The Near Field/Far Field Crossover

Consider a uniform linear aperture of extent a excited with a monochromatic signal having wavelength λ. Consider a point target at a distance R from the aperture. The radiation from each incremental part of the aperture arrives at the target according to the propagation path length associated with the incremental aperture position and the target location. We can analyze this with the following figure.



(2.1)

The differential path length Δ associated with a point x on the aperture and a range R can be evaluated using simple geometry.

$$
\begin{aligned}
\Delta &= \sqrt{R^2 + x^2} - R \\
&= R\sqrt{1 + \left(\frac{x}{R}\right)^2} - R \\
&\cong \frac{x^2}{2R}
\end{aligned}
\tag{2.2}
$$

This differential error across the aperture is thus essentially quadratic, and can be reduced arbitrarily be increasing R. That is, in the far field the radiation from each point on the aperture arrives (essentially) coherently, adding constructively. As we move the point target closer to the aperture, the delay error increases inversely with R until, at some crossover range R_C between the near field and far field, it becomes non-negligible. We define this range $R = R_C$ (rather arbitrarily) as that for which the maximum error is

$$\Delta = \lambda/8 \tag{2.3}$$

As the maximum error will always be associated with the ends of the aperture, we substitute $x = a/2$ and use $\Delta = \lambda/8$ in eq. (2.2) to obtain

$$\boxed{\frac{R_C}{a} = \frac{a}{\lambda}} \tag{2.4}$$

That is, *the crossover range, measured in apertures, is equal to the aperture, measured in wavelengths.*

The far field is often called the *Fraunhofer region*, where we can ignore the differential phase terms associated with different propagation lengths. The near field is often called the *Fresnel region*, characterized by the (approximately) quadratic phase attributable to different propagation lengths from different aperture points.

Let us calculate the near field/far field crossover of a practical ultrasonic aperture operating at a center frequency of 3.5 MHz. Let

$$a = 28mm$$
$$\lambda = .44mm$$

This aperture might have 128 elements spaced at $\lambda/2$. The transition between the Fraunhofer region and the Fresnel region occurs at

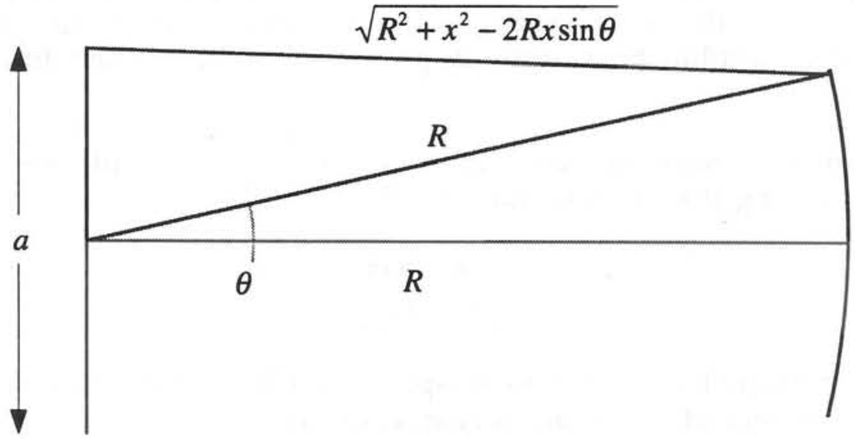
$$R_C = 1782mm$$

or almost two meters! *All modern diagnostic ultrasonic imaging occurs in the extreme near field.* This distinguishes the discipline from many other imaging technologies and presents a notable engineering challenge.

We note that there are many alternative definitions of R_C possible, and they can be found in the literature. Outside of a specific engineering context, they are all rather arbitrary. Our choice is consistent with a delay error of one eighth of a wavelength, which may be devastating in some applications and benign in others. The designer of near field imaging systems has to be constantly aware of systematic phase errors (and their sources, and their effects) across the aperture.

Azimuthal Resolution and the Rayleigh Criterion of a Linear Aperture

Consider again a uniform linear aperture of extent *a* excited with a monochromatic signal having wavelength λ. Consider a point target in the far field, moved around a circular arc of constant range. Because we are in the far field, there is negligible phase error when the angle of the target is zero (i.e., the radiation from each aperture point arrives in phase). But as we traverse the circular arc, the phase error increases until some parts of the aperture start contributing destructively.



(2.5)

The differential path length Δ between an aperture point x and the center, as our point target moves along the circular arc, is

$$\begin{aligned}\Delta &= \sqrt{R^2+x^2-2Rx\sin\theta}-R \\ &= R\sqrt{1+\left(\frac{x}{R}\right)^2-2\left(\frac{x}{R}\right)\sin\theta}-R\end{aligned} \tag{2.6}$$

Expanding this in terms of x/R and θ

$$\begin{aligned}\Delta &\cong -x\theta+\frac{x^2}{2R} \\ &\cong -x\theta\end{aligned} \tag{2.7}$$

As in the last section, we ignore the quadratic term on the basis of far field operation.

Destructive interference occurs when the magnitude of any differential path length error exceeds $\lambda/4$, so we want to calculate the angular extent for which

$$-\lambda/4 \le \Delta \le \lambda/4 \tag{2.8}$$

As the maximum error will always be associated with the ends of the aperture, we substitute $x = \pm a/2$ and use $\Delta = \pm\lambda/4$ in eq. (2.7) to get the angles associated with this maximum error.

$$\theta\big|_{\Delta=\pm\lambda/4} = \pm\frac{\lambda}{2a} \tag{2.9}$$

We define this angular extent as the angular resolution θ_R.

$$\boxed{\theta_R = \frac{\lambda}{a}} \tag{2.10}$$

That is, *the angular resolution, measured in radians, is the inverse of the aperture, measured in wavelengths.*

It is also convenient to define the ratio of the range and the aperture, which re-occurs in different contexts, as the f-number.

$$\boxed{f_\# = \frac{R}{a}} \tag{2.11}$$

The distance around the circular arc associated with θ_R is simply $R\theta_R$, yielding the azimuthal resolution u_R.

$$\boxed{u_R = f_\# \lambda} \tag{2.12}$$

That is, *the azimuthal resolution, measured in wavelengths, is the f-number.*

Let us return to eq. (2.7) to consider the diffraction pattern as we traverse an arc in the far field. Let the arc length $u = R\theta$. A differential path length of $\Delta = -x\theta$ will induce a differential phase of $-2\pi\Delta/\lambda = 2\pi xu/R\lambda$ (the sign arising because a positive differential path length results in a later arrival time). If each incremental part of the aperture is radiating a complex exponential of the form $\exp(j2\pi f_0 t)$, then the ratio of the signal received at our point target to the total excitation signal is