INF-GEO 4310 Imaging

Lecture 08.09.2011

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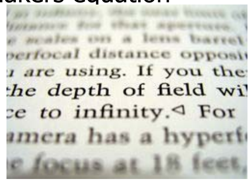
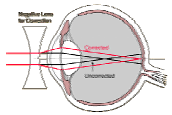
Geometrical Optics part II

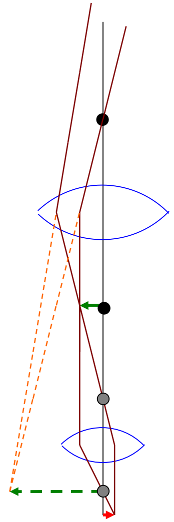
- ***Themes today:***
  - **Imaging by Refraction**
  - **Geometrical Optics: Diffraction**
  - **Geometrical Optics: Scattering**
- ***Literature:***
  - F. Albregtsen:
    "2. Reflection, refraction, diffraction, and scattering"
    (pages 37 - 82)

Geometrical Optics: Refraction

- Imaging by refraction
  - Refraction at a planar surface (Snell's Law)
  - Refraction at a single spherical surface
  - Thin lenses; "The lensmakers equation"
  - The camera;
  - "Depth-of-field"
  - The eye
  - The magnifier
  - The eyepiece
  - Microscopes
  - Telescopes
  - Multiple lens systems

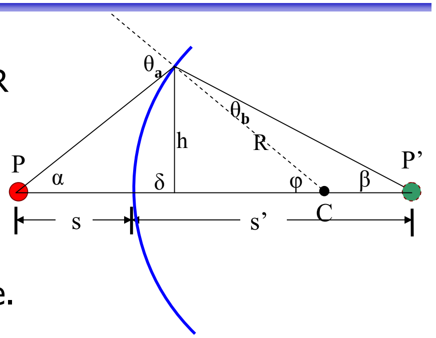






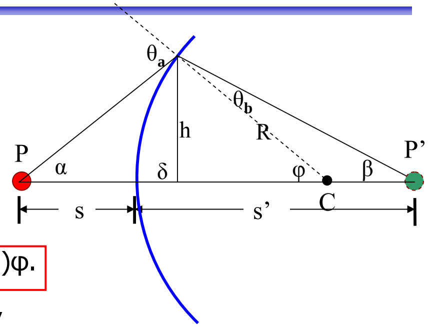
Refraction by spherical surface - I

- Convex surface with radius of curvature R facing incident light originating from P on the optical axis.
- A ray from P at an angle α to the axis is refracted at the surface.
- Angle of refraction is given by Snell's law, refracted ray crosses the optical axis at an angle β.
- All rays from P will intersect axis at the same point P', provided that the angle α is small.



Refraction by spherical surface - II

- Snell's law : $n_a \sin \theta_a = n_b \sin \theta_b$.
- Paraxial approximation
  => $n_a\theta_a = n_b\theta_b$.
- Combining this with $\theta_a = \alpha + \varphi$
  gives $\theta_b = (\alpha + \varphi)\, n_a/n_b$.
- Substituting this into $\varphi = \beta + \theta_b$
  we get $(n_a\alpha + n_b\beta) = (n_b - n_a)\varphi$.
- Tangents of α, β, and φ are simply
  $tg(\alpha) = h/(s+\delta)$, $tg(\beta) = h/(s'-\delta)$, $tg(\varphi) = h/(R-\delta)$.
- If the angle α is small, so are β and φ.
- Under the paraxial approximation,
  δ may be neglected compared to s, s', and R.
- => $\alpha = h/s$, $\beta = h/s'$, $\varphi = h/R$.



Refraction by spherical surface - III

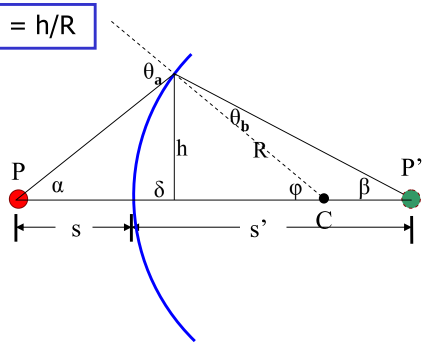
- Substituting $\alpha = h/s,\ \beta = h/s',\ \varphi = h/R$

  into

  $(n_a\alpha + n_b\beta) = (n_b - n_a)\varphi$

  we get general **object-image relation**, for single spherical surface:

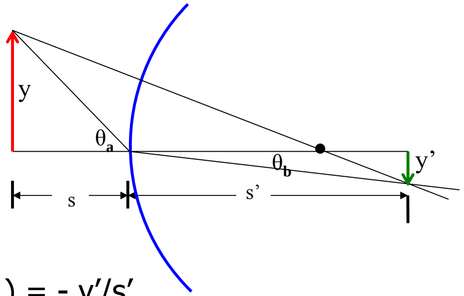
$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$



- Same expresssion, no matter if n_a is greater or less than n_b.
- Does not contain the angle α :
  => all light rays coming from P will intersect at P'.

Imaging by spherical surface

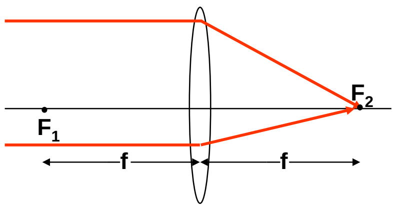
- Given
  (y, s, R)
- What is
  - the size y'
  - position s'
  - of the real image ?



- We see that
  $tg(\theta_a) = y/s$ and $tg(\theta_b) = -y'/s'$.
- Snell's law
  $n_a \sin(\theta_a) = n_b \sin(\theta_b)$
- Small angles => $n_a\, y/s = -\, n_b\, y'/s'$.
- Magnification: $m = y'/y = -(n_a s')/(n_b s)$.
  - (negative sign => inverted image)

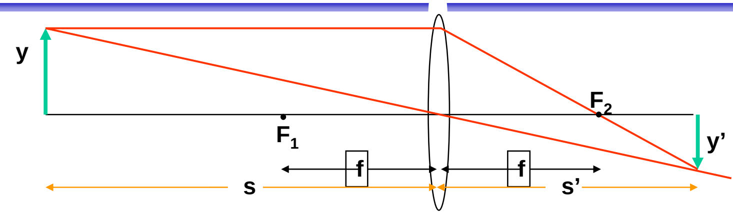
Thin lenses

- Thin lens: two refracting surfaces close enough to neglect distance between them.
- Centers of curvature of spherical surfaces lie on and define optical axis.
- The first and second focal points are on either side of the lens.



- Focal length: distance from focal point to middle of lens.
  - Focal length of convex lens is positive
  - Focal length of concave lens is negative.

Object-image relation

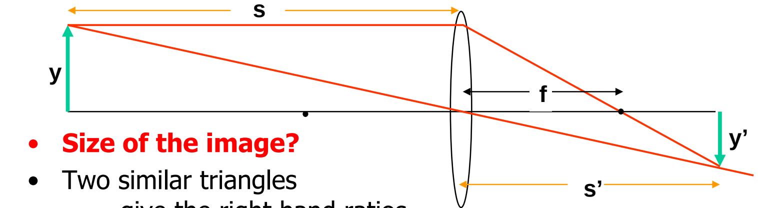


- We have two pairs of similar triangles, giving:

$$\frac{y'}{y} = \frac{s'}{s} \quad \text{and} \quad \frac{y'}{y} = \frac{s'-f}{f} \quad \text{implying} \quad \frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

- This is known as the "*object-image relation*".

Image size and magnification



- **Size of the image?**
- Two similar triangles give the right hand ratios.
- Then use "*object-image relation*".
- The **magnification** is

  m= y'/y

- If object is far away:

  s >> f => m << 1

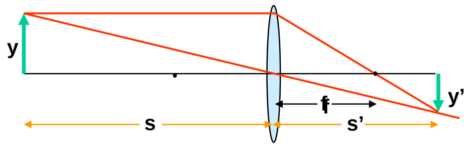
$$\frac{y'}{y} = \frac{s'}{s} \Rightarrow y' = \frac{y\,s'}{s}$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow s' = \frac{s\,f}{(s-f)}$$

$$\Rightarrow \quad y' = \frac{y\,f}{(s-f)}$$

Size of image of the Moon ?

- What is the size the Moon imaged by lens having f = 50 mm?
  - Moon has a diameter of 3476 km
  - Distance to Moon is 384 405 km.
- s = 384 405 km, f = 50 mm, y =3 476 km



- Then

y' = y f / (s-f) = 3476 km x 50 mm / (384405 km - 50 mm) = 0.45 mm.

- This modest image fills less than 0.02 % of the 24 x 36 mm film area!

"The lensmakers equation"

- Lens has two surfaces => **object-image relation** applied twice:

$$\frac{n_a}{s_1} + \frac{n_b}{s_1'} = \frac{n_b - n_a}{R_1} \quad , \quad \frac{n_b}{s_2} + \frac{n_a}{s_2'} = \frac{n_a - n_b}{R_2}$$

s_1 = distance to object, s_2' = distance to final image. $s_2 = -s_1'$.
Set $n_a = 1$, $n_b = n$. We get

$$\frac{1}{s_1} + \frac{n}{s_1'} = \frac{n-1}{R_1} \quad , \quad -\frac{n}{s'_1} + \frac{1}{s_2'} = \frac{1-n}{R_2}$$

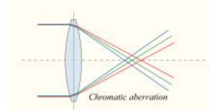
- Adding equations:

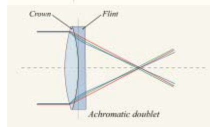
$$\frac{1}{f} = \frac{1}{s_1} + \frac{1}{s'_2} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

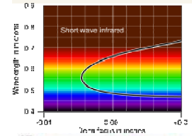
- Note that:
  - The two focal lengths are always equal, despite different curvatures.

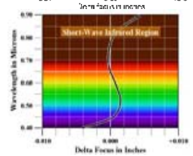
Chromatic aberration

- Index of refraction depends on wavelength => images at different λ focus at different distances.
- **Achromatic lens**
  - two materials (e.g. crown and flint) bonded
  - focus two wavelengths into same focal plane
  - Reduces chromatic aberration
- **Apochromatic lens**
  - more than two lenses of different materials
  - focus three λ (e.g. R,G,B) into same plane
  - order of magnitude better than achromat



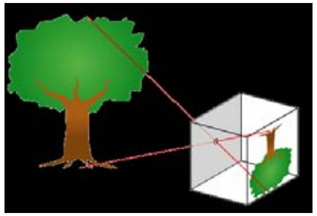






The camera *(lat.: small room)*

- A camera consists of:
  - light-tight box
  - lens (several elements)
  - adjustable aperture
  - controllable shutter
  - film or electronic detectors.



pinhole camera

- Fixed focal plane =>
  - lens closer to detector for distant object
  - lens farther away from detector for nearby object.

- This is very different from imaging in the eye.

f-number and exposure

- Energy per unit area in the focal plane is proportional to
  - aperture area and length of the exposure time interval.
- f-number of camera lens N = f/D.
- Intensity in focal plane is proportional to $(D/f)^2$.
- Changing D by √2 changes intensity by a factor of 2.
- f-numbers are often related by √2
  - 1/500 s at f/4
  - 1/250 s at f/5.6
  - 1/125 s at f/8

  } all correspond to the same exposure.
- Shorter exposure times
  - minimize motion-blurring
  - allows larger effective lens aperture, giving
    - better resolution of details in the image
    - reduced **depth of field** and **depth of focus**.

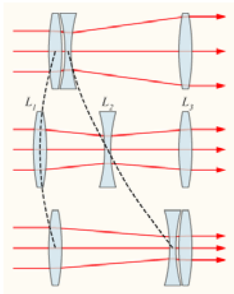
Field of view (FOW)

- Using 24 x 36 mm film,
  FOW measured along the diagonal will be
  - 75º for f = 28 mm (wide angle, landscapes)
  - 47º for f = 50 mm ("normal")
  - 25º for f = 105 mm (ideal for portraits)
  - 8º for f = 300 mm (full moon is 1/2º).
- Replace 24 x 36 mm film with digital detector,
  => smaller registered field of view.
  - *correspond to approximately 1.5 times longer focal lengths in cameras using 24 x 36 mm film.*

Same scene, different focal lengths

Zoom lenses

- **Digital zoom:** Cropping and enlarging an image
  - always lower quality than optical zoom, no resolution gained.
- **Optical zoom:** Several lens elements are used
  - focus and focal length can be varied, maintaining focal plane.
- Described by ratio of focal lengths:
  - a 20 to 200 mm zoom is a 10:1 or "10×" zoom.
- Two parts:
  - a fixed-focal-length lens (L3)
  - an ***afocal*** zoom lens system (L1 + L2)
    - does not focus the light
    - alters the size of a beam
    - alters overall magnification.



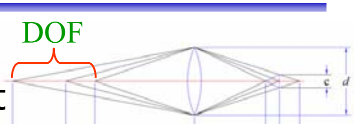
The Hitchcock zoom

- Zooming can manipulate perspective in time sequences.
- If the camera is pulled away from the object
  - while lens zooms in to maintain Field Of Wiew, or vice versa,
  - the size of the foreground objects will be constant,
  - but background details will change size relative to foreground.
- Continuous perspective distortion is counter-intuitive:
  - Perspective change without a size change is highly unsettling.
- Special effect used in Hitchcock's Vertigo
  - hence called "Vertigo-" or "Hitchcock-zoom".
  - Also used in "Jaws", "ET", ...


Camera is close, zoomed out


Camera is further away, zoomed in

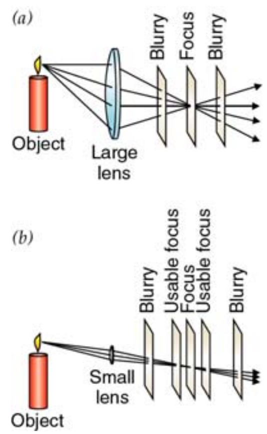
Depth of field



- DOF = distance in front of and beyond the focused object that appears to be in focus.
- A large DOF brings both foreground and background into focus.
- A small DOF will focus on part of an interesting object, and defocus a distracting background.

Lens aperture and Depth Of Field

- Large lens aperture D, low f/D - value:
  - Shorter exposure time
  - Focus more critical
  - Better angular resolution
- Small lens aperture D, high f/D-value:
  - Longer exposure time
  - Focus less critical
  - Poorer angular resolution

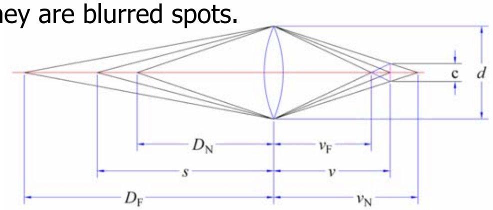


What determines DOF ?

- DOF is determined by three factors:
  – the focal length of the lens
  – the f-number of the lens aperture
  – the camera-to-object distance.
- Increasing the f-number (smaller aperture) increases the DOF.
  – reduces the amount of light transmitted
  – increases diffraction
  – reduces angular resolution
  => There is a practical limit to the reduction of aperture.

Near and far limits of DOF

- An object at distance *s* from the lens is in focus at image distance *v*.
- Objects at D_F and D_N are in focus at image distances v_F and v_N .
- At the image distance *v*, they are blurred spots.



- When blur spot diameter is equal to the acceptable circle of confusion *c (COC)*, the near and far limits of DOF are at D_N and D_F.
- From similar triangles we see that

$$\frac{v_N - v}{v_N} = \frac{c}{d} \qquad \frac{v - v_F}{v_F} = \frac{c}{d}$$

Limits of DOF from N=f/D

- "*f*-number" is given by focal length *f* and aperture diameter *d* :
  N = f/d.
- Substituting for d we get the focus limits on the image side of the lens

$$v_N = \frac{f\,v}{f - N\,c} \qquad v_F = \frac{f\,v}{f + N\,c}$$

- Thin lens equation:

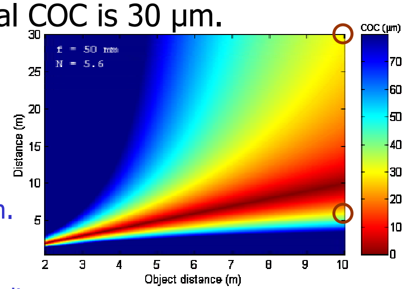
$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

- Substituting this give limits of DOF in terms of
  – focal length f
  – "f-number" N
  – object distance s
  – circle of confusion *c*

$$D_N = \frac{s\,f^2}{f^2 + N\,c\,(s - f)} \qquad D_F = \frac{s\,f^2}{f^2 - N\,c\,(s - f)}$$

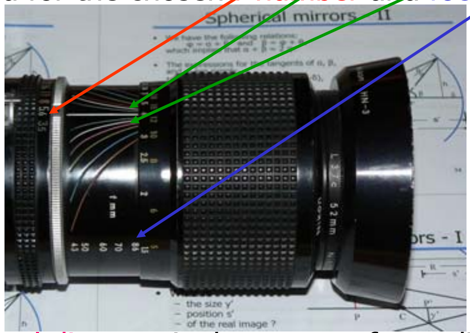
Practical limits of DOF

- DOF beyond the object is always greater than DOF in front of the object.
- For longer focal lengths the ratio tends towards unity.
- For the 35-mm format, a typical COC is 30 µm.
- A practical example:
  f = 50 mm
  N = 5.6
  s = 10 m
  c = 30 µm
  - DOF ranges from 6 m to 30 m.
  => DOF extends
  4 meters in front of and
  20 meters beyond the focus distance.
  - A smaller/larger COC gives a larger/smaller DOF.



DOF of a camera zoom lens

- On an old fashioned zoom lens, far and near limits of DOF indicated for the chosen f-number and focal length.



- Hyperfocal distance is the nearest focus distance at which the far limit of the DOF extends to infinity.

The hyperfocal distance

- **The hyperfocal distance is the nearest focus distance H at which the far limit of the DOF extends to infinity.**

$$D_F = \frac{s f^2}{f^2 - N c (s - f)}$$

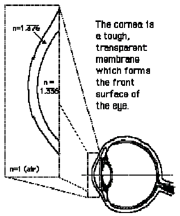
- Setting the far limit D_F to infinity and solving for s gives us H:

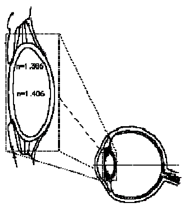
$$s = H = \frac{f^2}{N c} + f \approx \frac{f^2}{N c}$$

- Focusing the camera at the hyperfocal distance gives the largest possible DOF for a given *f*-number.
- You will see the hyperfocal distance again later!

The eye

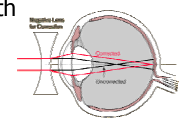
- Eye is nearly spherical and about 2.5 cm in diameter.
  - *cornea* protects the eye, performs much of the focusing
  - *iris* and *pupil* controls how much light will be let through
  - *lens* produces a sharp image
  - *retina* contains photo detectors
  - *optic nerve* transmits signals to brain.
- Lens
  - responsible for ca 20 % of the refraction.
  - focal length, f ≈ 1.5 cm.
- Focusing
  - Radial ligaments around the lens stretch it to a flattened disc
    - focus on far-away objects.
  - If ring-shaped muscle around the radial fibers is relaxed, lens becomes more spherical and its focal length is shortened
    - Focus on closer objects.
  - **Lens - retina distance is constant, unlike fixed-focal-length lenses.**
  - **Focusing power in "dioptres", d, where d = 1/f (f in meters)**
  - **Eye lens ≈ 67 d, cornea responsible for 45 d.**

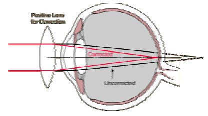


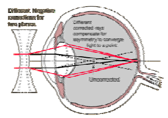


Accomodation and defects

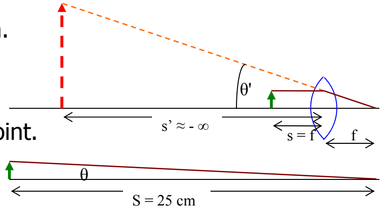
- *Accommodation* – an automatic ability to alter the focal length
  - is affected by ageing.
  - young individuals may alter focal power by up to 4 dioptres.
- *Presbyopia* - near point recedes as one grows older
- *Myopia* (near-sighted) - infinity focused in front of retina.
  - Corrected by diverging lens (f = 1/d < 0).
    - moves virtual image of distant object at or inside far point.
- *Hyperopia* (far-sighted) - infinity focused behind retina.
  - Corrected by converging lens (f = 1/d > 0).
    - forms virtual image of nearby object at or beyond near point.
- *Astigmatism* – different focus in horizontal and vertcal plane.
  - Remedied by lenses that are not rotationally symmetric.







The magnifier



- Max viewing angle is obtained at ≈ 25 cm.
- Converging lens forms virtual image
  - More distant => larger than object.
- Then object may be moved inside near point.
- Angular magnification M given by
  - ratio of the angle θ' (with the magnifier) to the angle θ (without the magnifier).
- Assume that the eye is relaxed => virtual image placed at infinity.
- Assuming that θ (given in radians) = sin(θ) =tg(θ):

Magnification:

$$\theta = \frac{y}{25\ cm} \qquad \theta' = \frac{y}{f}$$

$$M = \frac{\theta'}{\theta} = \frac{y/f}{y/25cm} = \frac{25cm}{f}$$

- Simple magnifiers give M ≈ 5.
- For more magnification, microscope is needed.

The eyepiece

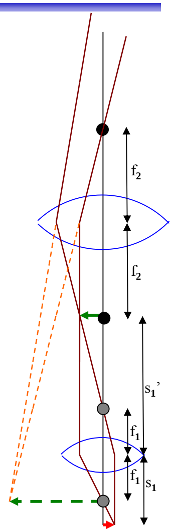
- Concave mirrors and lenses form real images.
- To inspect real image, we may use a second lens.
- This magnifier produce an enlarged virtual image.
  - Found in telescopes and microscopes.
- Telescope and microscope oculars are compound lenses
  - corrected for chromatic and geometric aberrations.

Optical microscope

- Object **just outside** first focal point of the objective => real, inverted and enlarged image.
- This image **just inside** first focal point of eyepiece => a final virtual image.
- Lateral magnification of objective: $m_1 = -s_1'/s_1$.
  - If object close to focal point: $m_1 = -s_1'/f_1$.
- Angular magnification of eyepiece: $M_2 = (25\ cm)/f_2$, (if real image is close to focal plane).
- Total angular magnification M:

$$M = m_1 M_2 = -\frac{(25cm)s_1'}{f_1 f_2}$$

  - Negative sign indicates that image is inverted.
  - Use objectives of different f_1 to vary magnification.

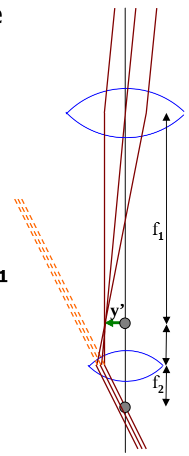


Refracting telescope

- Objective lens forms real, inverted and reduced image
  - distant object => real image at second focal point.
- Real image at the first focal point of eyepiece
  - => final virtual image at infinity
  - => objective to eyepiece distance = sum of focal lengths.
- Angle subtended at the eye by the final virtual image $\theta' = y'/f_2$ (As with the magnifier)
- Angle of object when viewed by unaided eye $\theta = -y'/f_1$
- Magnification:

$$M = \frac{\theta'}{\theta} = -\frac{y'/f_2}{y'/f_1} = -\frac{f_1}{f_2}$$

  - Negative sign indicates that image is inverted.
  - Use eyepieces of different f_2 to vary magnification.



Multiple lenses

- Focal length for two thin lenses in contact (achromat):

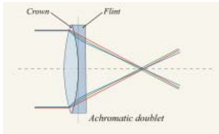
$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \quad \Rightarrow \quad f = \frac{f_1 f_2}{f_1 + f_2}$$

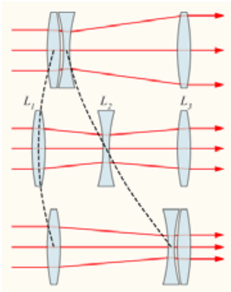
- *Back focal length* (BFL) of two lenses separated by *d:*

$$BFL = \frac{f_2(d - f_1)}{d - (f_1 + f_2)}$$

(d -> 0 => BFL -> f)

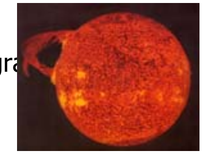
- $d = f_1 + f_2$ => BFL is infinite. (collimating afocal)
  - Alters width of beam
  - Found in zoom lenses & simple telescopes
- Magnification given by: $M = -\frac{f_1}{f_2}$
- $f_1 > 0$, $f_2 > 0$ => M < 0 & inverted image (two convex lenses).
- $f_1 > f_2 > 0$ => M > 0 & upright image (convex plus concave lens).

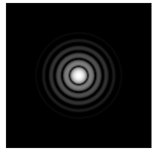




Geometrical Optics: Diffraction

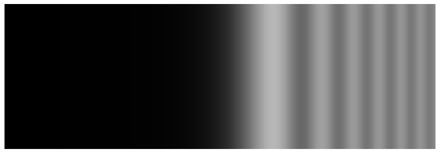
- Fraunhofer diffraction pattern
  - Single slit, twin slit, multiple slits
  - Diffraction grating, spectrograph, spectroheliograph, spectrograph
- Diffraction profile of circular aperture
  - Airy disc and Rayleigh criterion
  - Effect of central obstruction
- The smallest visible detail
  - In a high quality camera
  - In a compact digital camera
  - In a mobile-phone camera
- Depth of focus
- Convolving PSF and sampling aperture





Diffraction by an edge

- Light can bend around corners.
- When a point source casts shadow of a straight edge, the edge of the shadow is not a step edge.
  - some light in the area expected to be in shadow
  - alternating bright and dark fringes in illuminated area.
- Result of interference between many light waves
  - (Huygens' Principle).



Near- and far-field

- **Fresnel (near-field)** diffraction
  - Both light source and observation plane are close to the aperture.
  - Curvature of the wave fronts must be taken into account.
  - Fresnel diffraction effects later in the course.
- **Fraunhofer (far-field)** diffraction
  - Wave fronts at aperture and observation plane considered planar.
  - usually => light source and observation plane far from slit.
  - We may use collimating lenses
    - lens having light source in its primary focal point will collimate the beam before it reaches the aperture;
    - lens behind the aperture may collimate the beam traveling towards the observation plane.
- The near/far-field limit is at the hyperfocal distance !!!

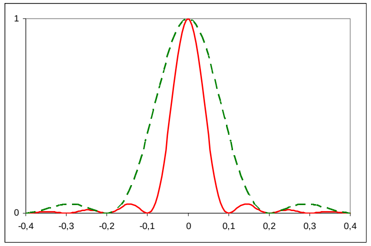
Single slit Fraunhofer diffraction

- Diffraction pattern of long, narrow slit:
  - a central bright band,
    - may be much wider than width of slit,
  - bordered by parallel dark and bright bands
    - with rapidly decreasing intensity.

$$I(\theta \mid a, \lambda) = \left(\frac{\sin[\pi a (\sin\theta)/\lambda]}{\pi a (\sin\theta)/\lambda} \right)^2 I_0$$

a = the width of the slit
λ = the wavelength
I_0 = intensity at $\theta = 0$.

- Visible light: $\lambda < a$, and $\sin(\theta) \approx \theta$.
  => first minimum at $\theta = \lambda/a$.
  => width of central bright band proportional to λ / a .

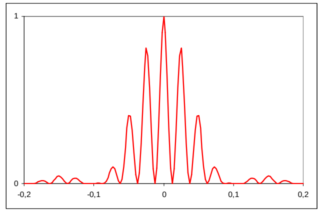


single slit
$-\pi/8 < \theta < \pi/8$
$a = 5\lambda$ (dashed line)
$a = 10\lambda$ (solid line)

Twin slit Fraunhofer diffraction

- Diffraction pattern from two slits
  a = slit width
  d = distance between slits
  λ = monochromatic wavelength
  I_0 = intensity at $\theta = 0$.

$$I(\theta \mid a, d, \lambda) = \cos^2(\pi d (\sin\theta)/\lambda) \left(\frac{\sin[\pi a (\sin\theta)/\lambda]}{\pi a (\sin\theta)/\lambda} \right)^2 I_0$$



- Single-slit pattern acts as envelope on cosine interference pattern of two slits.
  - Local maxima may be suppressed by envelope minima.

two slits
$-\pi/16 < \theta < \pi/16$
$a = 10\lambda$, $d = 4a$

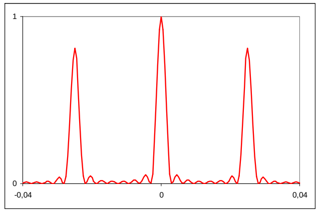
- Width of envelope proportional to λ / a .

Multiple slit diffraction

- Constructive interference for diffracted rays at an angle θ that arrive in observation plane with path length difference = integer number of λ: $d \cdot \sin(\theta) = m\lambda, \quad m = 0, \pm 1, \pm 2, \ldots$

- Diffraction pattern from N slits
  a = width of slit
  d = distance between slits
  λ = monochromatic wavelength
  I_0 = intensity at $\theta = 0$.

$$I(\theta \mid a, d, \lambda, N) = \left[\frac{\sin\left(\frac{N\pi d}{\lambda}\sin\theta\right)}{\sin\left(\frac{\pi d}{\lambda}\sin\theta\right)} \right]^2 \cdot \left(\frac{\sin\left[\frac{\pi a}{\lambda}\sin\theta\right]}{\frac{\pi a}{\lambda}\sin\theta} \right)^2 I_0$$



$N = 8$ slits
$-\pi/80 < \theta < \pi/80$
$a/\lambda = 10$
$d = 4a$.

- Principal maxima
  - same positions as in the two-slit case
  - width proportional to 1/N.
- N-1 minima between pair of principal maxima
  - secondary maxima get smaller as N increases.

Diffraction grating

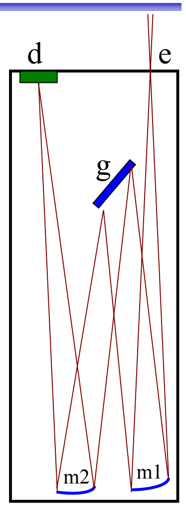
- An assembly of narrow slits - or grooves in planar (or curved) mirror.
- Gratings for λ = 400 to 700 nm usually have about 1000 lines/mm, corresponding to d on the order of 1/1000 mm = 1000 nm.
- When a beam is incident on a grating with an angle θ_i (measured from the normal of the grating), it is diffracted into several beams.
  - Specular reflection beam is called zero order ($m = 0$).
  - Other orders given by non-zero integers m in **grating equation**.

$$d \cdot [\sin\theta_m(\lambda) + \sin\theta_i] = m\lambda, \quad m = 0, \pm 1, \pm 2, \ldots$$

d = groove period
λ = wavelength of incident light
$\theta_m(\lambda)$ = value of the diffracted angle in order m.

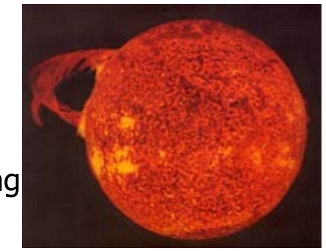
A slit spectrograph

- Light is focused onto entrance slit (e).
- Tilted concave collimating mirror (m1) reflects onto a plane reflecting diffraction grating (g).
- Dispersed light of some order *m* from grating is focused by a second concave mirror (m2) onto detector array (d).
- Special-purpose spectrographs are complex
  - to avoid
    - internal reflections
    - unwanted straylight.



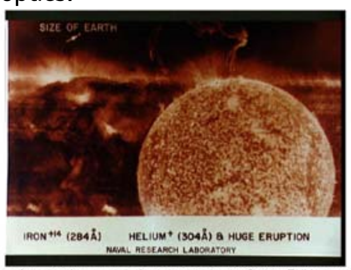
Spectroheliograph

- Produces monochromatic images of the Sun.
- An image of the Sun is focused on a plane.
- A narrow slit lets light into a spectrograph.
- Spectrograph produces a spectrum
  - of the portion of the solar disk imaged on the entrance slit at the same image scale as input image.
- Capture spectrum within an exit slit.
- Image scanned across entrance slit.
- Moving detector behind exit slit at same speed as the image is moving monochromatic image is recorded.



Slitless spectrograph

- Gives co-temporal spectra of all parts of extended objects.
- Concave grating produces image at all wavelengths.
  - Spectral resolution given by reflective grating.
  - Angular resolution given by telescope optics.
- X and λ are same direction.
- Images at different λ overlap.
- Spectral and spatial information mixed into complicated image.



Part of an "overlappograph" image taken by the S082A EUV spectroheliograph on the Skylab Apollo Telescope Mount, January 1, 1974 (*JSC Digital Image Collection*)

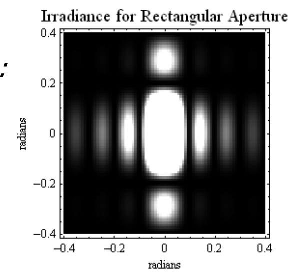
Diffraction by rectangular aperture

- An a x b aperture gives two orthogonal 1-D diffraction patterns:

$$I(\theta,\varphi\,|\,a,b,\lambda)=\left(\frac{\sin[\pi a(\sin\theta)/\lambda]}{\pi a(\sin\theta)/\lambda}\right)^2\left(\frac{\sin[\pi b(\sin\varphi)/\lambda]}{\pi b(\sin\varphi)/\lambda}\right)^2 I_0$$

- The widths of the central bright band are inversely proportional to the ratio of the size of the aperture (*a,b)* to the wavelength λ.

Diffraction pattern of rectangular aperture:
Horizontal aperture a = 10 λ
Vertical aperture b = 5 λ
For a ± 0.4 radians range of θ and φ.

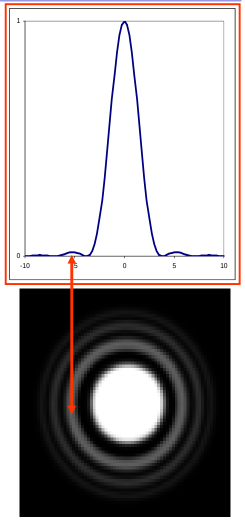


Diffraction profile of circular aperture

- Diffraction limits resolution and image quality.
- Diffraction by circular aperture will image point source as a small bright disc (Airy-disc)
  - surrounded by dark and bright rings,
  - intensity of the decrease rapidly
- Angular point spread function (PSF) of circular aperture given by:

$$F_0(\theta) = 4\left[\frac{J_1\left(\pi \frac{\theta}{\beta_0}\right)}{\pi \frac{\theta}{\beta_0}}\right]^2$$

θ given in radians,
$\beta_0 = \lambda/D$
J_1 = the first order Bessel function
$F_0(0) = 1$.



Airy disc and diffraction rings

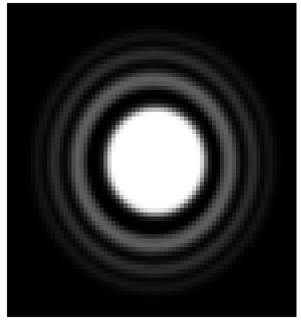
- Angular radii of first **dark** rings:

$$\sin\theta_1 = 1.22\frac{\lambda}{D} \quad \sin\theta_2 = 2.23\frac{\lambda}{D} \quad \sin\theta_3 = 3.24\frac{\lambda}{D}$$

- Angular radii of **bright** rings:

$$\sin\theta = 1.63\frac{\lambda}{D} \quad \sin\theta = 2.68\frac{\lambda}{D} \quad \sin\theta = 3.70\frac{\lambda}{D}$$

  - About 85% of energy within Airy disk
  - peak intensity of 1. ring ≈ 1.7%
  - peak intensity of 2. ring ≈ 0.4%

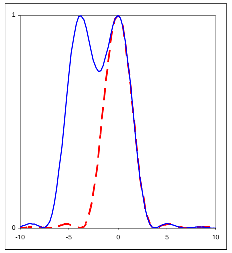


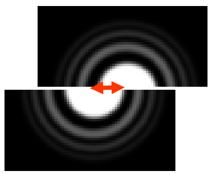
Rayleigh criterion

- Cross-section through diffraction-limited image
  - two equally bright point sources at infinity
  - angular separation between point sources:

$$\sin\theta = 1.22\frac{\lambda}{D}$$

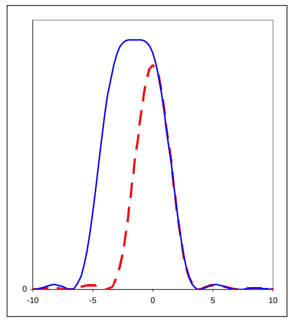
  - Corresponds to overlaying two patterns
  - maximum of first on first minimum of second.
  - 27 % "dip" between the peaks.
- This is the "Rayleigh criterion".
  - For small angles, sin θ = θ.





Sparrow criterion

- If the point sources are moved closer than the Rayleigh criterion, the "dip" will become shallower, until it becomes a flat plateau.
- This angular separation is the Sparrow criterion.
- The limit when two point sources "melt together".
- Sparrow: θ = 0.952 λ/D.



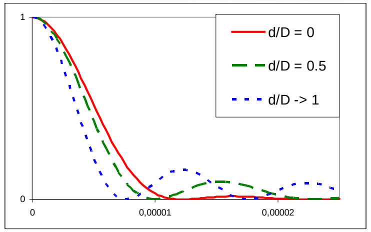
Circular aperture with central obstruction

- The diffraction profile of an annular aperture is given by the continuous circular symmetric function

$$F(\theta \mid \delta) = \frac{1}{\left(1-\delta^2\right)^2}\left[\left(\frac{2J_1(v)}{v}\right) - \delta^2\left(\frac{2J_1(\delta v)}{\delta v}\right)\right]^2$$

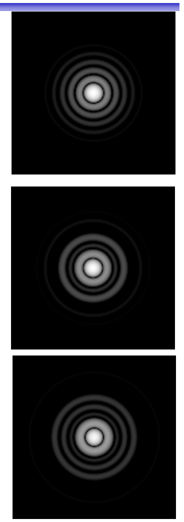
J_1 is the first order Bessel function
v = πθD/λ
θ is given in radians
δ = d/D
d = diameter of central obstruction
D = diameter of the aperture

- Central obstruction gives
  - ➢ a slightly narrower central disk
  - ➢ more energy in the bright rings
  - ➢ some energy is blocked out.



Airy patterns with central obstruction

- Airy pattern for obstructions of 0 %
  20 %
  33 %
- From 0 to 1/3 radial obstruction, maximum intensity of first diffraction ring increases from 1.7 % to 5.4 %.
- Obstruction affects both
  - the position of the minima
  - intensity of maxima in the ring pattern.



Smallest detail visible ... to the eye

- Pupil diameter ≈ 2 mm in bright light.
- Angular resolution of human eye ≈ 60 lines per degree.
  - 120 alternating black and white lines of equal thickness.
- A4 "landscape" paper at 30 cm distance covers 50º x 40º.
  - 3 000 black and 3 000 white vertical stripes should be resolved
  - 2 400 black and 2400 white horizontal stripes should be resolved.
- Rayleigh criterion for D=2.5 mm at λ=550 nm:

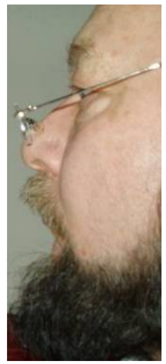
$$\sin\theta = \frac{1.22\,\lambda}{D} = \frac{1.22\,x\,550\,x10^{-9}}{2.5\,x10^{-3}} = 2.7\,x10^{-4}$$

- Converting from radians to degrees:

$$\theta = 2.7\,x10^{-4}\,\frac{180}{\pi} \approx \frac{1}{60}^{\circ}$$

Accomodation distance ... Near point

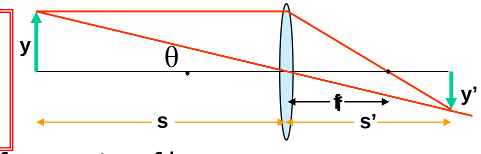
= The closest distance we may focus sharply with the unaided eye.
- 7 cm for a 10 year old,
- 10 cm for a 20 year old,
- 14 cm for a 30 year old,
- 22 cm for a 40 year old,
- 40 cm for a 50 year old,
- 100 cm for a 60 year old.



- 6 000 dots / 11 inches, ≈ 550 dpi.
- 47 yrs => s ≈ 30 cm.
  - printer better than 600 dpi is a waste.
- 20 yrs => s = 10 cm.
  - can inspect the printout at 10 cm distance
    - Will need 1 200 dpi (common for printing high quality images).

Smallest detail visible by a camera

- Focal length f = 35 mm.
- f/D = 3 => D = f/3.
- Object is s = 3.5 meters away.
- Wavelength is λ = 500 nm.

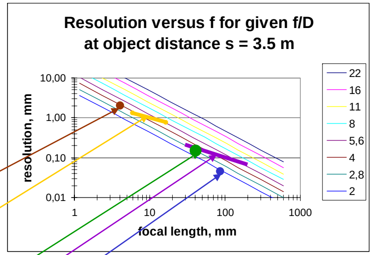


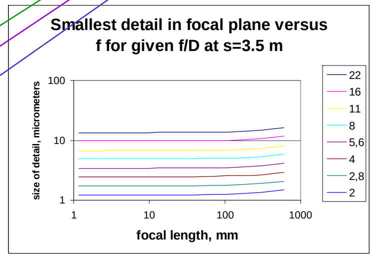
- Just resolvable angle, as seen from center of lens, given by Rayleigh criterion:
  $\sin(\theta) = 1.22\ \lambda/D = 1.22 \bullet 500 \bullet 10^{-9} \bullet 3 / 35 \bullet 10^{-3} = \underline{5.23 \bullet 10^{-3}}$.
- **Q:** What is distance y between two just resolved points on object ?
- **A:** Given by: tg (θ) = (y/s).
  For small angles $\sin(\theta) = tg(\theta) = \theta$, when θ given in radians.
  $y = 3.5 \bullet 5.23 \bullet 10^{-3}$ m $= 1.83 \bullet 10^{-4}$ m ≈ **<u>0.2 mm.</u>**
- **Q:** What is the corresponding distance y' in the focal plane?
- **A:** $y' = y \bullet f / (s-f)$
  $y' = 0.2 \bullet 35 / (3500–35)$ mm ≈ 0.002 mm = **<u>2 μm.</u>**

Resolution and detail – medium distance

- Object and image resolution depends on aperture and distance.
- At s = 3.5 meters:

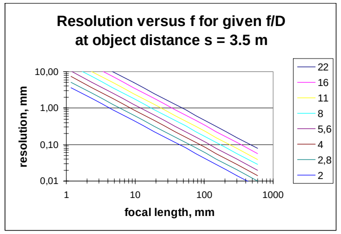
Mobile phone camera: Δy=2.1 mm
Compact digital zoom: Δy=0.6-1.1 mm
Compact 35 mm: Δy=0.2 mm
Digital SLR zoom: Δy=0.06-0.3 mm
"old fashioned" 85 mm: Δy = 0.05 mm





Object resolution versus distance

- For a given lens and f/D, size of smallest resolvable object detail increases linearly with distance.
- Keeping f/D constant, size of smallest resolvable object detail decreases linearly with focal length.
- Keeping focal length constant, size of smallest resolvable object detail increases linearly with f/D.



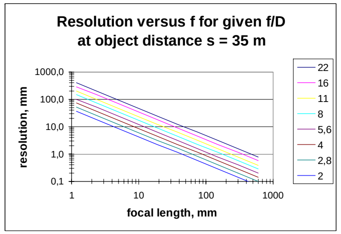
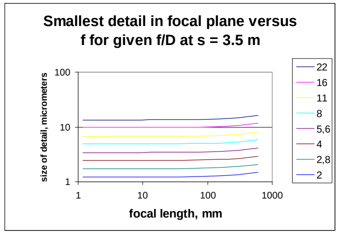
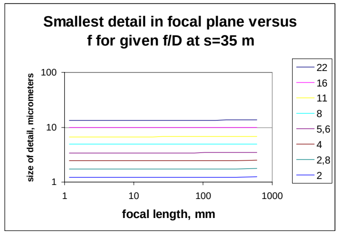


Image resolution versus distance

- For given lens and f/D, size of smallest resolvable detail in image is independent of object distance, except when f is comparable to s.
- Keeping f/D constant, size of smallest resolvable object is independent of focal length, except when f is comparable to s.
- Keeping focal length constant, size of smallest resolvable object detail increases linearly with f/D.





Diffraction limited depth of focus

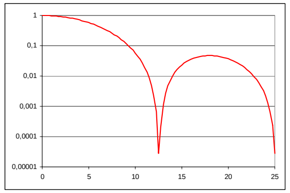
- Intensity of diffraction pattern near focus:

$$I(u,v)=\left(\frac{2}{u}\right)^2\left[1+V_0^2(u,v)+V_1^2(u,v)-2V_0(u,v)\cos\left\{\frac{1}{2}\left(u+\frac{v^2}{u}\right)\right\}-2V_1(u,v)\sin\left\{\frac{1}{2}\left(u+\frac{v^2}{u}\right)\right\}\right]I_0$$

u is along optical axis, v orthogonal to optical axis.

- Along optical axis:

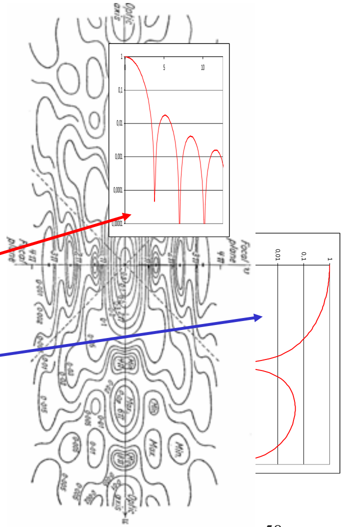
$$I(u,0)=\left(\frac{2}{u}\right)^2\left[2-2\cos\left\{\frac{u}{2}\right\}\right]I_0=\left(\frac{\sin(u/4)}{u/4}\right)^2 I_0$$



- Symmetric about the focal plane.
  - First minimum at u/4 = ± π, => **z = ± 2λ(f/D)²** from focal plane.
  - f/D = 5.6, λ=500 nm => z = ± 0.032 mm
  - f/D = 22, λ=500 nm => z = ± 0.5 mm
  - f/D = 5.6, λ=1000 nm => z = ± 0.25 mm.
- **Depth of focus is independent of physical dimensions of the optics.**

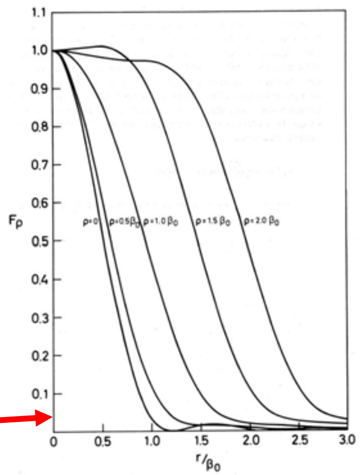
Intensity along optical axis

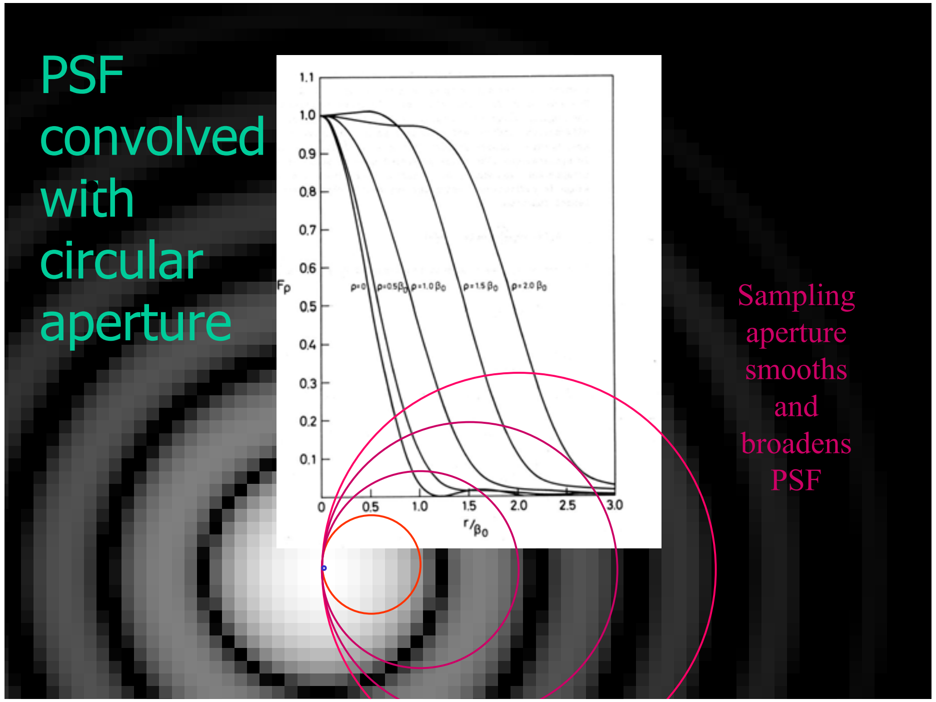
- A contour plot of the intensity I(u,v) in a meridional plane near the focus of a converging spherical wave diffracted by a spherical aperture.
- Vertical u-axis is the optical axis, and the horizontal v-axis is in the focal plane.
- The maxima and minima along the v-axis correspond to bright and dark rings of focal plane diffraction pattern.
- Maxima and minima along the u-axis illustrate "depth-of-focus".
  - *Contour plot from M. Born and E. Wolf: "Principles of Optics", Pergamon Press, 4th. Ed., 1970.*



Convolving PSF and sampling aperture

- PSF determines resolution.
- Ideal:
  - Point sampling in focal plane
  - Sampling theorem => density
- Reality:
  - Extended sampling aperture
  - Fixed, non-overlapping
  - Movable, overlapping
    - Rectangles
    - Circles
  - Sampling aperture * PSF





Geometrical Optics: Scattering

- What is scattering?
- Some effects of scattering
  - Atmospheric blurring and straylight in images
  - Turbidity in liquids
  - Subsurface scattering in non-metallic materials and in tissues
- Doppler-shifted straylight

What is scattering?

- Scattering causes radiation to deviate from a straight trajectory.
  - microscopic irregularities in surfaces
  - non-uniformities in transparent media
- *Elastic scattering* : no (or a very small) loss or gain of energy
- *Inelastic scattering* : some change in energy
- *Absorption* : substantial or complete loss of energy
- *Single scattering* : one localized scattering center.
  - treated as a random phenomenon, described by probability distribution.
- *Multiple scattering* : radiation is scattered several times.
  - randomness of interaction averaged out by large number of events
  => deterministic angular distribution of intensity - PSF.
- Observed blurred image:
  - *convolution* of true image with PSF (diffraction + scattering).

Inverse scattering problem

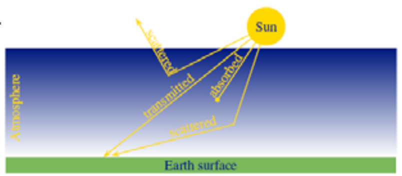
- A difficult challenge!
- Observe blurred object + scattering around it.
- Determine
  - scattering parameters (PSF)
  - distribution of radiation before scattering (true object).
- In general, the inverse is not unique.
- PSF can be determined by observing image of some well-known object through the same scattering medium.
- PSF then used to deconvolve image of unknown object.

Wavelength dependence of scattering

- **Rayleigh scattering**
  - in transparent solids, liquids and gases.
  - wavelength dependence ~ λ^{-4}
- **Blue sky**: Blue light is scattered much more than red light.
  - We observe blue light coming from all directions of the sky.
  - At higher altitudes, less scattering particles => sky is much darker.
- **Red sunset**: Sunlight must pass through greater air mass.
  - More scattering of blue light, little scattering of red light => red-hued sky.
- **Mie scattering**
  - scattering by spheres larger than Rayleigh range.
  - wavelength dependence ~ $1/\lambda$.
  - shape of scattering center significant
    - theory only applies well to spheres, spheroids and ellipsoids.

Atmospheric scattering

- Object is illuminated by sunligh



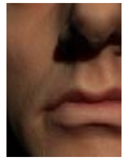
- Incident radiation detector:
  - Specular reflection
  - Diffuse reflection
  - radiation scattered in the air:
    - scattered before reaching object
    - specular and diffuse reflection, scattered onto detector.
- Important to shield detector to minimize straylight.
- Even with shielding, scattering will be present.
- Corrections important
  - In high precision measurements (e.g., astrophysics, …)
  - Remote sensing (radiation passing twice through atmosphere)

High density of scatterers

- Vapors:
  - an object that is seen through mist or fog will look blurred.
  - at some distance the object will disappear into the background fog.
- Water:
  - Particles / organisms act as scatterers
  - cause haziness that indicate water quality
  - ***turbidity*** can be measured using *Secchi disk*
    - lowered into water until it can no longer be seen.

- Translucent solids:
  - light penetrates non-metallic surface and scatters inside material
  - either absorbed or leaving the material at a different location.
  - This phenomenon is called ***subsurface scattering (SSS)***.
  - The effect is a "softer" image than a metallic surface would give.

- Tissues:
  - human skin, salmon fillets, etc show ***subsurface scattering***
    - may depend on wavelength, condition of tissue, etc.
  - Thus, measuring SSS may be useful for
    - quality inspection of e.g., fish and meat
    - medical diagnostic work.



Straylight integral

- Straylight causes a blurring of the image (here the Sun).
- Given
  - circular symmetric PSF, Ψ(r),
  - true intensity $\Phi_c(p')$
- Observed intensity I(p) given by integral equation:

$$I(p)=\int_{\oplus}\Phi_C(p')\Psi(r)d\omega$$

  - p and p' are directions in the sky
  - r is the angle between them.
  - Integration is performed over the solid angle of the Sun.

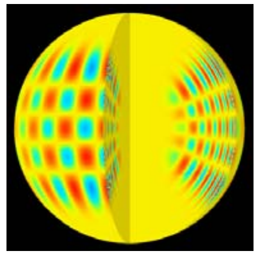
Doppler shifted straylight

- Different parts of the Sun have different line-of-sight velocities.

=> Observed intensity contain straylight with different Doppler velocities.

$$I(d,\lambda)=2\int_{\rho_0}^{\rho_1}\int_0^{\alpha_0}\Phi_C(a)(1-I_C(a))\exp\left[-(\lambda-\Delta\lambda)^2/w^2(d)\right]\Psi(\rho)\,d\rho\,d\alpha$$

  - Φ_c = true continuum intensity distribution across the solar disc
  - w = Doppler width of Gaussian absorption line profile
  - I_c = central intensity of absorption line
  - d = distance from the centre of the solar disc
  - λ = wavlength within a spectral line
  - Ψ(r) = circular symmetric PSF, :



- Straylight introduce errors = 0.1 – 1.0 m/s,
- Amplitudes of global solar oscillations ≈ 0.1 m/s.
  => Error ≈ velocity oscillation signal.