Tutorial solutions:

Basic questions

1. The motion of the radar or the object is used to create/synthesize a larger aperture and thus to improve the cross-range resolution. Information in cross-range is obtained by the evolution of the phase (Doppler shift) during the time of analysis.
2. Range resolution depends of the signal bandwidth: $\Delta r = \frac{c}{2B}$.
3. a: $\Delta cr \approx r\lambda / d$. The larger is the aperture, the smaller is the resolution.
   b: $\Delta cr \approx d/2$
4. a: see above. Resolution deteriorates with distance.
   b: resolution independent of range.
5. When an object is approaching or moving away from the radar it changes the phase of the transmitted signal which results in a frequency shift at the receiver.
6. Range resolution: $\Delta r = \frac{c}{2B} = 1.5\text{m}$. Cross-range resolution: $\Delta cr \approx r\lambda / d = 500\text{ m}$ with $\lambda = 5\text{cm}$ (real aperture), $\Delta cr \approx d/2 = 50\text{cm}$ (synthetic aperture)
7. Foreshortening: range differences between two points located at different altitudes (i.e. foreslopes of mountains) are smaller than they are at the ground. Layover: extreme case of foreshortening when foreslope is "reversed" in the range dimension.

Doppler effect

1. $R(t) = \sqrt{R_0 + (vt)^2} = R_0\left[1 + \left(\frac{vt}{R_0}\right)^2\right]^{1/2}$

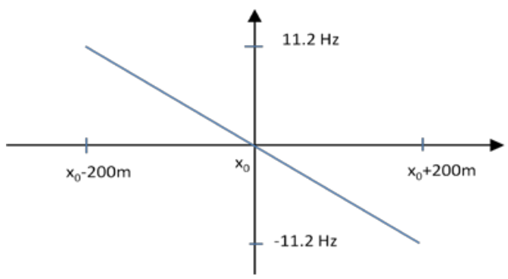
2. $R(t) = R_0\left[1 + \frac{1}{2}\left(\frac{vt}{R_0}\right)^2 - \frac{1}{8}\left(\frac{vt}{R_0}\right)^4 + \dots\right] \approx R_0 + \left(\frac{v^2}{2R_0}\right)t^2$

3. $\phi(t) = 2\pi\frac{2R(t)}{\lambda} = \frac{4\pi}{\lambda}\left[R_0 + \left(\frac{v^2}{2R_0}\right)t^2\right] = \frac{4\pi}{\lambda}R_0 + \frac{2\pi}{\lambda}\left(\frac{v^2}{R_0}\right)t^2$

   $f_d(t) = -\frac{1}{2\pi}\frac{d\Phi}{dt} = -\frac{1}{\lambda}\frac{2v^2}{R_0}t$

   Quadratic relation between phase and time, linear relation between Doppler shift and time

4. Using $x = vt$, $f_d(t) = -\frac{1}{2\pi}\frac{d\Phi}{dt} = -\frac{1}{\lambda}\frac{2v}{R_0}x = -\frac{1}{0.05}\frac{140}{10000}x = 11.2\text{Hz}(x = -200\text{m})\text{ or } = -11.2\text{Hz}(x = 200\text{m})$



5. The total path is covered in 2.9s, thus the Doppler resolution is 0.37 Hz ($\Delta f_d = 1/T$).
   From the former equation, the Doppler resolution can also be expressed as:

$$\Delta f_d = -\frac{1}{\lambda}\frac{2v}{R_0}\Delta x \Rightarrow \Delta x = -\frac{\lambda R_0}{2v}\Delta f_d = -\frac{\lambda R_0}{2vT} = -\frac{0.05 \times 10000}{2 \times 400} = 60\text{cm}$$

   Note that the minimum cross range resolution of 50 cm is achieved when the platform covers a distance equal to the size of the antenna beam at R_0 (here 500m).

Image interpretation

Image of a mountaneous area (Udine, Italy). We see foreshortening and layover distortion effects