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Given two disjoint co-Turing-recognizable languages there exists a decidable language separating them.

Let A and B be two disjoint co-Turing-recognizable languages so that $A \cap B = \emptyset$, $coTM_A$ recognizes $\overline{A}$ and $coTM_B$ recognizes $\overline{B}$. We construct TM_C that decides C, a language separating A and B.

TM_C = "on input ω:

1. Simulate running $coTM_A$ on ω and $coTM_B$ on ω in parallel (alternating between $coTM_A$ and $coTM_B$).
2. If at any time $coTM_A$ accepts, $REJECT$. If at any time $coTM_B$ accepts, $ACCEPT$."

C satisfies the criteria for separating A and B:

- $A \subseteq C$: On input $\omega \in A$ $coTM_A$ will loop or $REJECT$, $coTM_B$ will $ACCEPT$ since $\omega \notin B$ and TM_C $ACCEPT$s.
- $B \subseteq \overline{C}$: On input $\omega \in B$ $coTM_B$ will loop or $REJECT$, $coTM_A$ will $ACCEPT$ since $\omega \notin A$ and TM_C $REJECT$s.

TM_C is a decider: if $\omega \notin A \cup B$ then $coTM_A$ and $coTM_B$ will race. Both machines eventually accept, but the machine that finishes computation first decides whether TM_C accepts or rejects.