INF2270, exercise on combinational logic: solution

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Exercise 1:

(A-a) Boolean function

$$[\mathrm{a} \oplus \mathrm{b}] + [(\mathrm{b} \cdot \mathrm{c}) \oplus (\mathrm{c} \cdot \mathrm{a}))]$$

(A-b) Truth table:

a	b	c	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

(A-c) Karnaugh map:

bc\a	0	1
00	0	1
01	0	1
11	1	0
10	1	0

(A-d) Resulting Boolean function:

$$(\mathrm{a} \cdot \bar{\mathrm{b}}) + (\bar{\mathrm{a}} \cdot \mathrm{b})$$

(B-a) Boolean function

$$[((\mathrm{a} \cdot \mathrm{b}) + \mathrm{c}) + (\mathrm{c} \cdot (\mathrm{b} + \mathrm{d}))] \oplus [(\mathrm{c} \cdot (\mathrm{b} + \mathrm{d})) \oplus (\overline{(\mathrm{b} + \mathrm{d}) + \mathrm{a}})]$$

(B-b) Truth table:

a	b	c	d	F
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

(B-c) Karnaugh map:

cd\ab	00	01	11	10
00	1	0	1	0
01	0	0	1	0
11	0	0	0	0
10	0	0	0	1

(B-d) Resulting Boolean function:

$$(\mathrm{a} \cdot \mathrm{b} \cdot \bar{\mathrm{c}}) + (\mathrm{a} \cdot \bar{\mathrm{b}} \cdot \mathrm{c} \cdot \bar{\mathrm{d}}) + (\bar{\mathrm{a}} \cdot \bar{\mathrm{b}} \cdot \bar{\mathrm{c}} \cdot \bar{\mathrm{d}})$$

The K-map gives us the minimal 'sum' of 'products'. However, 'advanced users' can sometimes reduce the gate-count some more using some of the rules for Boolean expressions and/or using other gates. The following deduction is purely for illustartion: do not attempt to do this at the exam as it will not give extra points! Here, the two last products both contain the term $\bar{\mathrm{b}}\bar{\mathrm{d}}$. Using the distributivity rule we can thus move that term outside of the brackets:

$$(\mathrm{ab}\bar{\mathrm{c}}) + [(\bar{\mathrm{b}}\bar{\mathrm{d}})(\bar{\mathrm{a}}\bar{\mathrm{c}} + \mathrm{ac})] \tag{1}$$

Then the term $(\bar{a}\bar{c} + ac)$ is actually the XNOR function of a and c, and $(\bar{b}\bar{d})$ can be expressed as $\overline{b + d}$ to get rid of some inverters.

$$(ab\bar{c}) + [(\overline{b + d})(\overline{a \oplus c})] \tag{2}$$

If this result really is simpler than the direct result of the K-map () approach is arguable. As mentioned before the standard format of the sum of products can be very beneficial for simple implementation and can thus be preferable. However, the transistor count is actually smaller in (2).

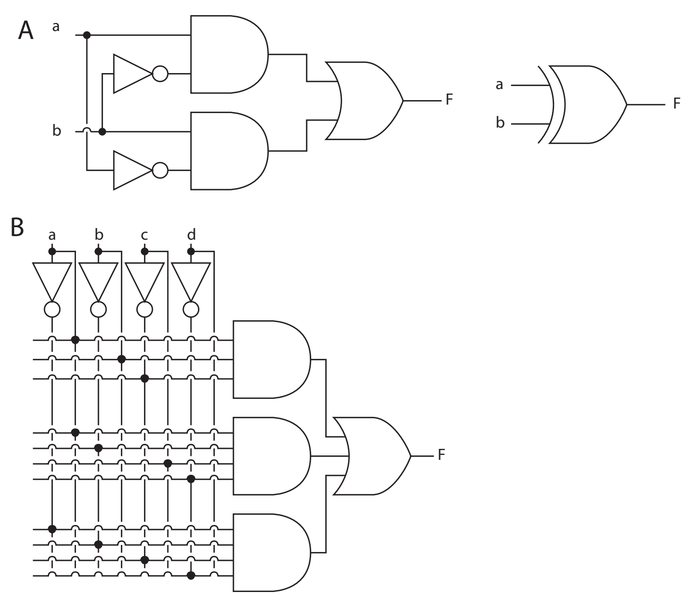

Figure 1: The resulting simplified circuits

Exercise 2: Adder

(a) 1-bit adder:

The 1-bit adder described here is also known as half-adder.

a	b	Sum
0	0	0
0	1	1
1	0	1
1	1	0

With the input a=1, b=1 we would get a overflow bit which is handled as carry out.

a	b	Sum	Cout
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

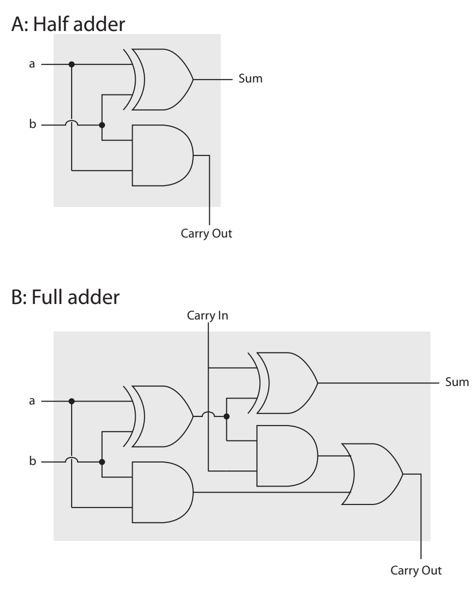

Figure 2: Half and full adder

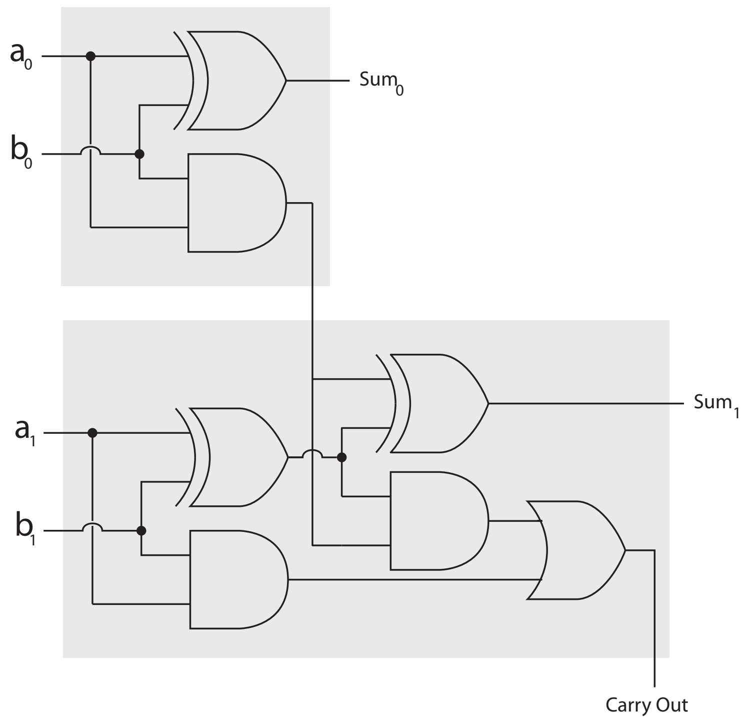

Figure 3: 2-bit adder.

(b) 2-bit adder

This can be constructed with one half adder and one full adder. Shown in Figure 3.

Exercise 3: Gates

Exercise 4: Simplify Expressions

(a) x

(b) x

(c) y

(d) 0

(e) B

(f) z(x+y)

(g) x'y'

(h) x(w+y)

(i) 0

Reduce the following Boolean expressions to the indicated number of literals:

(a) AB+C'

(b) x+y+z

(c) B

(d) A'(B+C'D)

Find the complement of F = x+yz then show that FF'=0 and F+F' = 1.

List the truthtable for the function: F = xy + xy' + y'z

Solution: F(x,y,z) = $\sum(1, 4, 5, 6, 7)$

Note: $\sum$ represents the minterms, that is where the given function is 'true' or '1'. For those of you who are interested in further reading can also search for Maxterms. There is a relation between the min- and Maxterms by using deMorgans theorem.