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# Polymorphism and Type Inference

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**Based on John C. Mitchell's slides (Stanford U.) ,
adapted by Gerardo Schneider, UiO.**

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# ML lectures

1. 04.09: The Algol Family and ML (Mitchell's chap. 5 + more)
2. 11.09: More on ML & types (chap. 5 and 6)
3. **18.09: More on Types, Type Inference and Polymorphism (chap. 6)**
4. 25.09: Control in sequential languages, Exceptions and Continuations (chap. 8)

# Outline

- **Polymorphism*s***
    - **parametric polymorphism**
    - ***ad hoc* polymorphism**
    - **subtype polymorphism**
- Type inference
- Type declaration

# Polymorphism: three forms

## ◆Parametric polymorphism

- Single function may be given (infinitely) many types
- The type expression involves *type variables*

Example: in ML the identity function is polymorphic

```
- fn x => x;
val it = fn : 'a -> 'a
```

This pattern is called *type scheme*

*Type variable* may be replaced by *any* type

An *instance* of the type scheme may give:

int→int, bool→bool, char→char,
int*string*int→int*string*int, (int→real)→(int→real), ...

# Polymorphism: three forms

## ◆Parametric polymorphism

- Single function may be given (infinitely) many types
- The type expression involves *type variables*

Example: polymorphic sort

```
sort : ('a * 'a -> bool) * 'a list -> 'a list

- sort((op<),[1,7,3]);
val it = [1,3,7] : int list
```

# Polymorphism: three forms (cont.)

◆ Ad-hoc polymorphism (or Overloading)
  - A single symbol has two (or more) meanings (it refers to more than one algorithm)
  - Each algorithm may have different type
  - Overloading is resolved at compile time
  - Choice of algorithm determined by type context

Example: In ML, **+** has 2 different associated implementations: it can have types int*int→int and real*real→real, no others

# Polymorphism: three forms (cont.)

- Subtype polymorphism
  - The subtype relation allows an expression to have many possible types
  - Polymorphism not through type parameters, but through subtyping:
    - If method *m* accept any argument of type *t* then *m* may also be applied to any argument from any subtype of *t*

REMARK 1: In OO, the term "polymorphism" is usually used to denote subtype polymorphism (ex. Java, OCAML, etc)

REMARK 2: ML does not support subtype polymorphism!

# Parametric polymorphism

- Explicit: The program contains type variables
  - Often involves explicit instantiation to indicate how type variables are replaced with specific types
  - Example: C++ templates

- Implicit: Programs do not need to contain types
  - The type inference algorithm determines when a function is polymorphic and instantiate the type variables as needed
  - Example: ML polymorphism

# Parametric Polymorphism: ML vs. C++

◆ C++ function template

- Declaration gives type of funct. arguments and result
- Place inside template to define type variables
- Function application: type checker does instantiation

◆ ML polymorphic function

- Declaration has no type information
- Type inference algorithm
  - Produce type expression with variables
  - Substitute for variables as needed

ML also has module system with explicit type parameters

# Example: swap two values

- C++

```
void swap (int& x, int& y){
    int tmp=x;  x=y;  y=tmp;
}
```

```
template <typename T>
void swap(T& x, T& y){
   T tmp=x; x=y; y=tmp;
}
```

- Instantiations:
  - int i,j;  ...  swap(i,j); //use swap with T replaced with int
  - float a,b;... swap(a,b); //use swap with T replaced with float
  - string s,t;... swap(s,t); //use swap with T replaced with string

# Example: swap two values

- ML

```
- fun swap(x,y) =
      let val z = !x in x := !y; y := z end;
val swap = fn : 'a ref * 'a ref -> unit

val a = ref 3 ; val b = ref 7 ;
- val a = ref 3 : int ref
- val b = ref 7 : int ref
swap(a,b) ;
- val it = () : unit
- !a ;
val it = 7 : int
```

Remark: Declarations look similar in ML and C++,
but compile code is very different!

# Parametric Polymorphism: Implementation

- C++
  - Templates are instantiated at program link time
  - Swap template may be stored in one file and the program(s) calling swap in another
  - Linker duplicates code for each type of use
- ML
  - Swap is compiled into one function (no need for different copies!)
  - Typechecker determines how function can be used

# Parametric Polymorphism: Implementation

- Why the difference?
  - C++ arguments passed by reference (pointer), but local variables (e.g. tmp, of type T) are on stack
    - Compiled code for swap depends on the size of type T => Need to know the size for proper addressing
  - ML uses pointers in parameter passing (*uniform data representation*)
    - It can access all necessary data in the same way, regardless of its type
- Efficiency
  - C++: more effort at link time and bigger code
  - ML: run more slowly

# ML overloading

- Some predefined operators are overloaded
  - + has types int*int→int and real*real→real
- User-defined functions must have unique type
  - fun plus(x,y) = x+y; (compiled to int or real function, not both)

In SML/NJ:

```
- fun plus(x,y) = x+y;
 val plus = fn : int * int -> int
```

If you want to have plus = fn : real * real -> real you must provide the type:

```
- fun plus(x:real,y:real) = x+y;
```

# ML overloading (cont.)

- Why is a unique type needed?
  - Need to compile code implies need to know which **+** (different algorithm for distinct types)
  - Overloading is resolved at compile time
    - Choosing one algorithm among all the possible ones
    - Automatic conversion is possible (**not** in ML!)
  - Efficiency of type inference – overloading complicates type checking
  - Overloading of user-defined functions is not allowed in ML!

# Outline

- Polymorphism*s*
- **Type inference**
- Type declaration

# Type checking and type inference

- Type checking: The process of checking whether the types declared by the programmer “agrees” with the language constraints/ requirement
- Type inference: The process of determining the type of an expression based on information given by (some of) its symbols/sub-expressions

ML is designed to make type inference tractable

(one of the reason for not having subtypes in ML!)

# Type checking and type inference

- Standard type checking

```
int f(int x) { return x+1; };
int g(int y) { return f(y+1)*2;};
```

  - Look at body of each function and use declared types of identifies to check agreement.

- Type inference

~~int~~ f(~~int~~ x) { return x+1; };<br>
~~int~~ g(~~int~~ y) { return f(y+1)*2;};

  - Look at code without type information and figure out what types could have been declared.

# Type inference algorithm: Some history

- Usually known as Milner-Hindley algorithm
- **1958:** Type inference algorithm given by H.B. Curry and R. Feys for the *typed lambda calculus*
- **1969:** R. Hindley extended the algorithm and proved it gives the most general type
- **1978:** R. Milner -independently of Hindley- provided an equivalent algorithm (for ML)
- **1985:** L. Damas proved its completeness and extended it with polymorphism

# ML Type Inference

- Example

```
- fun f(x) = 2+x;
 val f = fn : int → int
```

- How does this work?
  - + has two types: int*int → int, real*real→real
  - 2 : int, has only one type
  - This implies + : int*int → int
  - From context, need x: int
  - Therefore f(x:int) = 2+x has type int → int

Overloaded + is unusual. Most ML symbols have unique type.

In many cases, unique type may be polymorphic.

# Another presentation

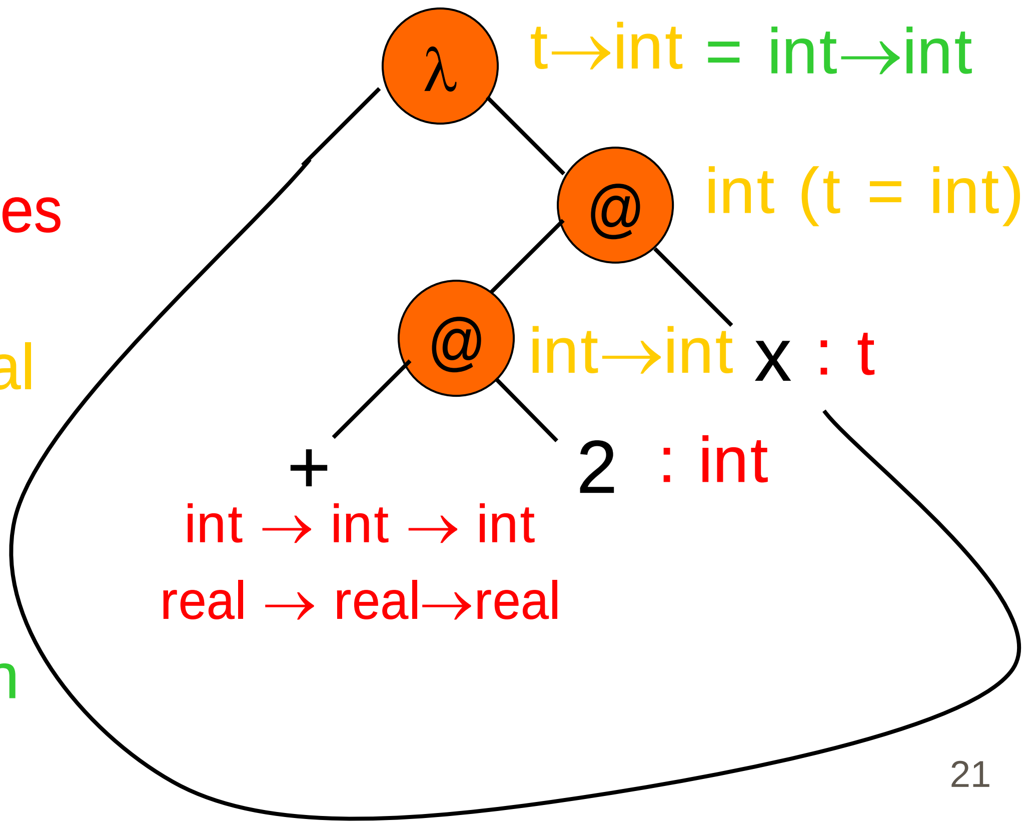
f(x) = 2+x equiv f = λx. (2+x) equiv f = λx. ((plus 2) x)

Graph for λx. ((plus 2) x)

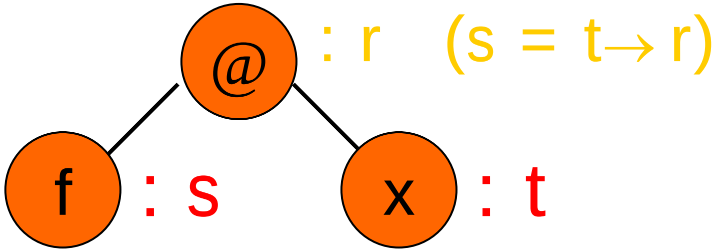
- Example
  - fun f(x) = 2+x;
  - (val f = fn x => 2+x ;)

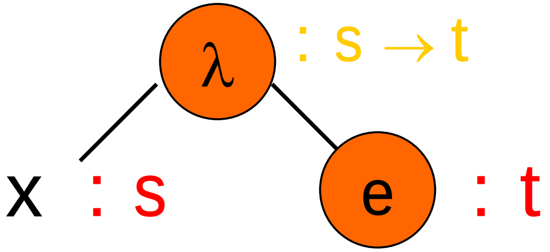
  val f = fn : int → int
- How does this work?

1. Assign types to leaves
2. Propagate to internal nodes and generate constraints
3. Solve by substitution



# Application and Abstraction





◆ Application

- f(x)
- f must have function type domain→ range
- domain of f must be type of argument x
- result type is range of f

◆ Function expression

- λx.e (fn x => e)
- Type is function type domain→ range
- Domain is type of variable x
- Range is type of function body e

# Types with type variables

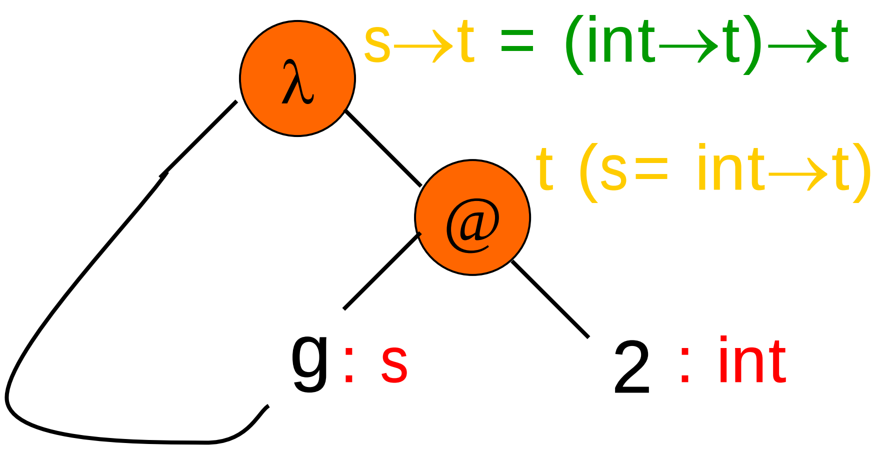
'a is syntax for "type variable" (t in the graph)

- Example
  - fun f(g) = g(2);

    val f = fn : (int→'a)→'a
- How does this work?

1. Assign types to leaves
2. Propagate to internal nodes and generate constraints
3. Solve by substitution

Graph for λg. (g 2)



# Use of Polymorphic Function

- Function

```
- fun f(g) = g(2);
 val f = fn : (int→'a)→'a
```

- Possible applications

g may be the function:

```
- fun add(x) = 2+x;
 val add = fn : int → int
```

Then:

```
- f(add);
 val it = 4 : int
```

g may be the function:

```
- fun isEven(x) = ...;
 val it = fn : int → bool
```

Then:

```
- f(isEven);
  val it = true : bool
```

# Recognizing type errors

- Function

  - fun f(g) = g(2);

   val f = fn : (int→'a)→'a

- Incorrect use

  - fun not(x) = if x then false else true;

   val not = fn : bool → bool

  - f(not);

  Why?

  Type error: cannot make bool → bool = int → 'a

# Another type inference example

- Function Definition
  - fun f(g,x) = g(g(x));

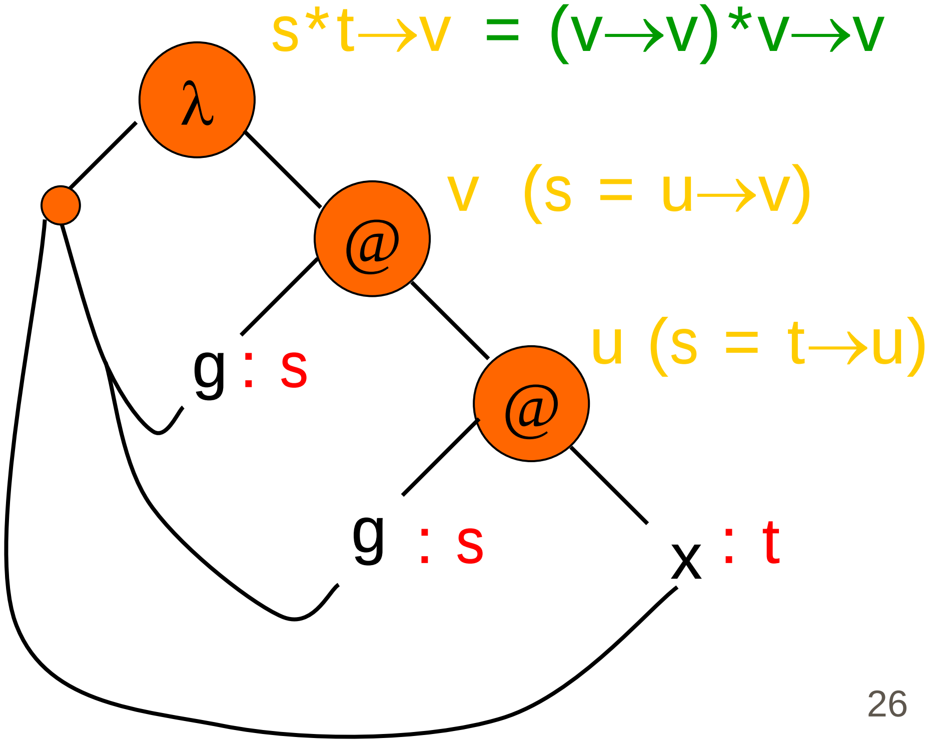
    val f = fn : ('a→'a)*'a → 'a
- Type Inference

  Assign types to leaves

  Propagate to internal nodes and generate constraints

  Solve by substitution

Graph for λ⟨g,x⟩. g(g x)



# Polymorphic datatypes

- Datatype with type variable

```
- datatype 'a list = nil | cons of 'a*('a list);
 nil : 'a list
 cons : 'a*('a list) → 'a list
```

- Polymorphic function

```
- fun length nil = 0
    |   length (cons(x,rest)) = 1 + length(rest);
  length : 'a list → int
```

- Type inference
  - Infer separate type for each clause
  - Combine by making two types equal (if necessary)

# Main points about type inference

- Compute type of expression
  - Does not require type declarations for variables
  - Find *most general type* by solving constraints
  - Leads to polymorphism
- Static type checking without type specifications
- May lead to better error detection than ordinary type checking
  - Type may indicate a programming error even if there is no type error (example following slide).

# Information from type inference

- An interesting function on lists

```
fun reverse (nil) = nil
|     reverse (x::lst) = reverse(lst);
```

- Most general type

```
reverse : ’a list → ’b list
```

- What does this mean?

  Since reversing a list does not change its type, there must be an error in the definition

  x is not used in “reverse(lst)”!

# Outline

- Polymorphism*s*
- Type inference
- **Type declaration**

# Type declaration

- Transparent: alternative name to a type that can be expressed without this name

- Opaque: new type introduced into the program, different to any other

ML has both forms of type declaration

# Type declaration: Examples

- Transparent ("type" declaration)

```
- type Celsius = real;
- type Fahrenheit = real;
```

```
- fun toCelsius(x) = ((x-32.0)*0.5556);
 val toCelsius = fn : real → real
```

More information:

```
- fun toCelsius(x: Fahrenheit) = ((x-32.0)*0.5556): Celsius;
  val toCelsius = fn : Fahrenheit → Celsius
```

- Since Fahrenheit and Celsius are synonyms for real, the function may be applied to a real:

```
- toCelsius(60.4);
val it = 15.77904 : Celsius
```

# Type declaration: Examples

- Opaque ("datatype" declaration)

```
- datatype A = C of int;
- datatype B = C of int;
```

- A and B are different types
- Since B declaration follows A decl.: C has type int→B

Hence:

```
- fun f(x:A) = x: B;
Error: expression doesn't match constraint [tycon mismatch]
 expression: A constraint: B
 in expression:  x: B
```

- *Abstract types* are also opaque (Mitchell's chapter 9)

# Equality on Types

Two forms of type equality:

- Name type equality: Two type names are equal in type checking only if they are the same name
- Structural type equality: Two type names are equal if the types they name are the same

Example: Celsius and Fahrenheit are structurally equal although their names are different

# Remarks – Further reading

- More on subtype polymorphism (Java): Mitchell’s Section 13.3.5

# ML lectures

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4. **25.09: Control in sequential languages, Exceptions and Continuations (chap. 8) ++?**