## Question 1 Runtime-systems, scoping, types (weight 40\%)

```
1a (25%)
```



## 1 b (15\%)

a)

The call $f(r P)$ via the call rc.mc() will imply that $g$ is called with $r p$ that denotes a Point object, and $g$ attempts to access the $c$ attribute.
b)

No. The problem arises from the assignment of $r P$ to the formal parameter cp as a result of the call $f(r p)$ via the call rc.mc (). An explicit casting should have been $f(($ ColorPoint $) r p)$, but that would rule out calls $f(r p)$ where $r p$ denotes a Point object, and that should be allowed in cases where $f$ is given an actual parameter with a same parameter type, i.e. Point.
c)

Yes. $\mathrm{rP}=$ new ColorPoint()

## Question 2 ML (weight 40\%)

## 2a Type inference (15\%)

## a)

The function fl takes two lists as input and returns a list of pairs with corresponding elements. (Extra: The function will fail if the lists are of different lengths.) (The function is also known as the zip function)
b)

The type of fl is 'a list * 'b list -> ('a * 'b) list
The input $\mathrm{x}:$ :xs and $\mathrm{y}:$ :ys are lists, which we see by the : : operator (and the [] in the second clause). We call the types of the two arguments 'a list and 'b list. The output is also a list, which we see by the : : operator(or the []) in the right hand side. By the first clause we see that it must be a list of pairs (2-tuples). The type of the first member of the pair must match the type of the elements in the first input list (' $a$ ) and the second must match the type of the elements in the second list ('b). Hence the output of the function is a value of type ('a*'b) list.
c)

The type is ('a int) * 'a $\rightarrow$ int

1. Assign types to the subexpressions in the tree, using variables ( $\mathrm{r}, \mathrm{s}, \mathrm{t}$, etc. ) where the type is unknown.

2. generate a set of constraints on the types (using the rules for abstraction and application):

$$
\begin{aligned}
& \mathrm{r}=\mathrm{s} \rightarrow \mathrm{t} \\
& \text { int } \rightarrow \text { int }=\mathrm{t} \rightarrow \mathrm{u}
\end{aligned}
$$

$\mathrm{v}=\mathrm{r} * \mathrm{~s} \rightarrow \mathrm{u}$
3. Solve the constraints by unification/substitution

1. int $\rightarrow$ int $=\mathrm{t} \rightarrow \mathrm{u} \quad \Rightarrow \quad \mathrm{t}=$ int, $\mathrm{u}=$ int
2. $\mathrm{r}=\mathrm{s} \rightarrow \mathrm{t} \quad \Rightarrow \quad \mathrm{r}=\mathrm{s} \rightarrow$ int (by 1.)
3. $\mathrm{v}=\mathrm{r} * \mathrm{~s} \rightarrow \mathrm{u} \quad \Rightarrow \quad \mathrm{v}=(\mathrm{s} \rightarrow \mathrm{int}) * \mathrm{~s} \rightarrow$ int (by 1 and 2$)$

Use ' a for s and the resulting type is: ( $\mathrm{a} \rightarrow$ int) * 'a $\rightarrow$ int

## 2b Programming with lists (15\%)

a)

```
fun getEquals(( \(x, y\) )::ps)= if \(x=y\) then ( \(x, y\) )::getEquals(ps) else getEquals(ps)
    getEquals(nil) = nil ;
fun sumPairs((x,y)::ps) = (x+y)::sumPairs(ps)
    | sumPairs(nil) = nil ;
```

Lots of other variants are also possible. F.ex.

```
fun getEquals(nil) = nil
    | getEquals(p::ps) =
        if #1(p) = #2(p) then (#1p,#2p) :: getEquals(ps) else getEquals(ps) ;
```

b)
fun getEquals(ps) = filter (op=) ps ;
fun sumPairs(ps) = map (op+) ps ;
c)
fun $\operatorname{snoc}(x, x s)=$
case $x$ s of nil $=>\quad[\mathrm{x}]$
| y::ys => y::(snoc(x,ys)) ;
or alternatively
fun $\operatorname{snoc}(x,(y:: y s))=y::(\operatorname{snoc}(x, y s))$
$\mid \operatorname{snoc}(x, n i l)=[x]$;

## 2c Records

```
fun listToRec(rs:(state list), {il=is,jl=js}) =
    case rs of (r::rs') =>
    listToRec(rs', { il=snoc((#i r),is) , jl=snoc((#j r),js) })
            | nil => {il=is,jl=js} ;
```

Other solutions are possible, but the lists in the resulting record should come out in the same order as the input lists and not reversed.

## Question 3 Prolog (weight 20\%)

3a
royal(X,male,_,_).
( $1 \%$ without the _)
3b

- (male(X) :- royal(X,male,_,_).
- female(X) :- royal(X,female,_,_).
- $\quad \operatorname{child}(X, Y):-\operatorname{royal}\left(X,, Y, \_\right)$.
- descendant $(X, X)$.
descendant(X,Y) :- child(X,Z), descendant(Z,Y).
- older(X,Y) :- royal(X,_,,YearX), royal(Y,_,_YearY), YearX < YearY.

3c
candidate $(\mathrm{X}):-\operatorname{regent}(\mathrm{K})$, descendant $(\mathrm{X}, \mathrm{K})$, ( male(X); female(X), \+ born_before(X,1971) ).

3d
yca( $\mathrm{X}, \mathrm{X}, \mathrm{X}$ ).
yca(X,Y,A) :- older(X,Y), child(Y,P), yca(X,P,A).
yca(X,Y,A) :- $\backslash+\operatorname{older}(X, Y)$, child(X,P), yca(P,Y,A).

