Assignment 1 - INF3170/4170 spring 2005
Due: 15pm, February 25th, 2005

Exercise 1

All formulas below are either (1) tautologies or (2) contradictions. Decide which formulas that are (1) or (2). Explain why by means of truth tables or a semantical argument.

1. $(\neg B \lor (C \to (A \land D))) \lor (B \to C)$
2. $((B \to C) \leftrightarrow D) \to ((A \land D) \to D)$
3. $((C \land (D \lor A)) \to B) \lor \neg B$
4. $B \land \neg((C \land A) \to B)$
5. $\neg(\neg C \to ((D \land B) \to D))$

Exercise 2

Find a formula $F(P, Q)$ with as few connectives as possible such that F has the following truth table. Use only connectives from the set $\{\land, \lor, \neg, \to, \leftrightarrow\}$.

P	Q	F
0	0	1
0	1	1
1	0	1
1	1	0

Exercise 3

Prove the following claim:
For all propositional formulas X: X is valid if and only if $\{\neg X\}$ is not satisfiable.

Exercise 4

Give tableau proofs for the following tautologies:

1. $\neg(P \wedge R) \vee P$
2. $(\neg Q \wedge P) \rightarrow P$
3. $R \vee (R \rightarrow \neg R)$
4. $(S \rightarrow (S \vee P)) \vee S$

Exercise 5

Consider the following claims:
(a) X is true if and only if Y is true.
(b) X is valid if and only if Y is valid.

Does (a) follow from (b)? Does (b) follow from (a)? Explain.

Exercise 6

True or False? Explain why.

1. If X is valid, then X is satisfiable.
2. If all tableaux for a set $\{X\}$ is such that no branch is closable (i.e. all brances are open), then the formula X is valid.
3. If X is not valid, then $\neg X$ is valid.
4. If $X \rightarrow Y$ is valid, then $(X \vee Z) \rightarrow Y$ is valid.
5. If $X \rightarrow Y$ is valid, then $(X \wedge Z) \rightarrow Y$ is valid.