Homework #5

For Friday, February 25

⋆ 1. Show that if k is a natural number and $\varphi_1, \ldots, \varphi_k$ are propositional formulas, then $[\![\varphi_1 \wedge \ldots \wedge \varphi_k]\!]_v = 1$ if and only if $[\![\varphi_i]\!]_v = 1$ for each i from 1 to k. Remember that, for example, $\varphi_1 \wedge \varphi_2 \wedge \varphi_3$ is an abbreviation for $((\varphi_1 \wedge \varphi_2) \wedge \varphi_3)$. Do this carefully, using only the definition of $[\![\cdot]\!]_v$.

⋆ 2. Show that if $\varphi_1, \ldots, \varphi_k$ and ψ are in PROP, then the following is true:

$$\{\varphi, \ldots, \varphi_k\} \models \psi \text{ if and only if } \models \varphi_1 \wedge \ldots \wedge \varphi_k \to \psi.$$

Once again, do this carefully, using the definition of semantic entailment.

⋆ 3. Show that if $\{\varphi\} \models \psi$ and $\{\psi\} \models \theta$ then $\{\varphi\} \models \theta$.

⋆ 4. Do problem 1a on page 20 of van Dalen. (In particular, compute the truth table.)

⋆ 5. Do problems 2, 3, 5, and 6 on page 21 of van Dalen.

⋆ 6. Use our semantic definitions to prove or find a counterexample to each of the following:

a. For every set of formulas Γ, every formula φ, and every formula ψ, if $\Gamma \models \varphi \wedge \psi$, then $\Gamma \models \varphi$ and $\Gamma \models \psi$.

b. For every set of formulas Γ, every formula φ, and every formula ψ, if $\Gamma \models \varphi \vee \psi$, then $\Gamma \models \varphi$ or $\Gamma \models \psi$.

c. $\{p_1 \wedge p_2, \neg p_2\} \models \neg p_1$

d. $\bot \models \phi$ for any $\phi \in$ PROP.

e. $\phi \models \top$ for any $\phi \in$ PROP.