



INF3400 Del 6 Tidsforsinkelse i logiske kjeder

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Tidsforsinkelse i kjede av logiske porter

Logisk effort i kjede:

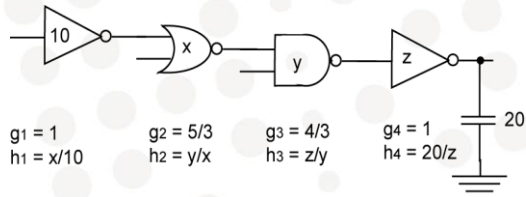
$$G = \prod_i g_i$$

Elektrisk effort i kjede:

$$H = \frac{C_{ekstern}}{C_{inngang}}$$

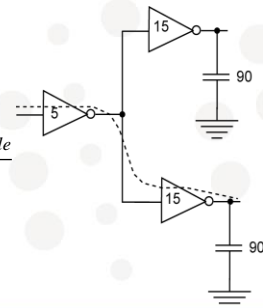
Forgreiningseffort i kjede:

$$B = \prod_i b_i$$



Forgreiningseffort:

$$b = \frac{C_{P\dot{A}\text{-kjede}} + C_{AV\text{-kjede}}}{C_{P\dot{A}\text{-kjede}}}$$



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Kjedeeffort:

$$\begin{aligned}
 F &= GBH \\
 &= 1 \cdot \left(\frac{15+15}{15}\right) \cdot \left(\frac{90}{5}\right) \\
 &= 1 \cdot 2 \cdot 18 \\
 &= 36
 \end{aligned}$$

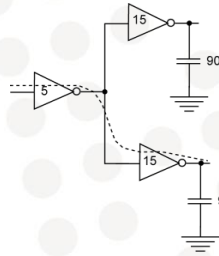
Kjedeforsinkelse:

$$\begin{aligned}
 D &= D_F + P \\
 &= \sum f_i + \sum P_i
 \end{aligned}$$

Kjedeforsinkelse vil ha en minimumsverdi når alle portene har lik effort forsinkelse f .

Dvs.:

$$\begin{aligned}
 f' &= f_i \\
 &= g, h_i \\
 &= F^{\frac{1}{N}}
 \end{aligned}$$



Minimum tidsforsinkelse:

$$D_{\text{minimum}} = NF^{\frac{1}{N}} + P$$

Transistorstørrelser:

$$\begin{aligned}
 f &= gh \\
 h &= \frac{C_{\text{ekstern}}}{C_{\text{inngang}}}
 \end{aligned}$$

Som gir:

$$C_{\text{inngang}} = \frac{C_{\text{ekstern}} g_i}{f'}$$



Logisk effort i kjede:

$$\begin{aligned}
 G &= \prod_i g_i \\
 &= \frac{4}{3} \cdot \frac{5}{3} \cdot \frac{5}{3} \\
 &= \frac{100}{27}
 \end{aligned}$$

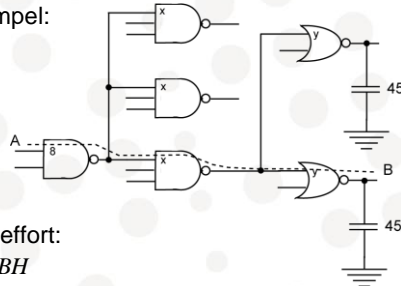
Elektrisk effort i kjede:

$$\begin{aligned}
 H &= \frac{C_{\text{ekstern}}}{C_{\text{inngang}}} \\
 &= \frac{45}{8}
 \end{aligned}$$

Forgreiningseffort i kjede: Optimal porteffort:

$$\begin{aligned}
 B &= \prod_i b_i \\
 &= \left(\frac{x+2x}{x}\right) \cdot \left(\frac{y+y}{y}\right) \\
 &= 6
 \end{aligned}$$

Eksempel:



Kjedens effort:

$$\begin{aligned}
 F &= GBH \\
 &= \frac{100}{27} \cdot 6 \cdot \frac{45}{8} \\
 &= 125
 \end{aligned}$$

Parasittisk tidsforsinkelse:

$$\begin{aligned}
 P &= 2 + 3 + 2 \\
 &= 7
 \end{aligned}$$

Minimum kjedeforsinkelse:

$$\begin{aligned}
 D &= N \cdot f' + P \\
 &= 3 \cdot 5 + 7 \\
 &= 22
 \end{aligned}$$



Starter ved utgangen og finner transistorstørrelser:

$$y = \frac{C_{ekstern}g}{f'}$$

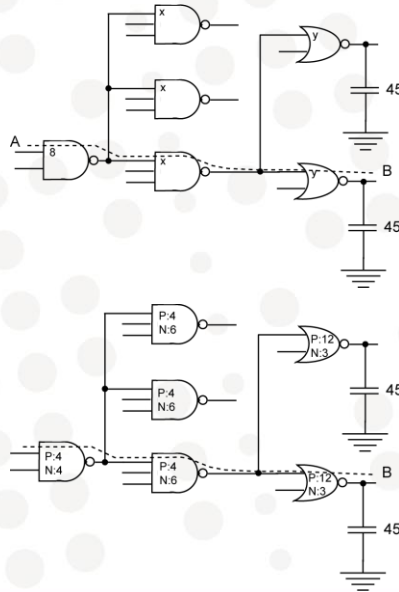
$$= \frac{45 \cdot 5}{3}$$

$$= 15$$

Beregner x:

$$x = \frac{5 + 15}{5}$$

$$= 10$$



Eksamensoppgave 2005



Finn logisk effort for portene og kjeden. Anta at utgangen Y skal drive 4 enhetsinvertere. Finn elektrisk effort for portene og kjeden. Hva blir kjedens effort F ?

Logisk effort i kjede:

$$G = \prod_i g_i$$

$$= \frac{4}{3} \cdot \frac{5}{3} \cdot 1 \cdot \frac{4}{3}$$

$$= \frac{80}{27}$$

$$\approx 3$$

Elektrisk effort i kjede:

$$H = \frac{4 \cdot 3}{x}$$

$$= \frac{12}{x}$$

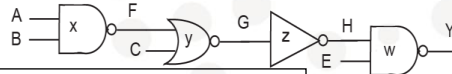
Kjedens effort:

$$F = GBH$$

$$= \frac{80}{27} \cdot 1 \cdot \frac{12}{x}$$



Eksamensoppgave 2005



Hva blir optimale port effort for kjeden? Finn kjedens parasittiske tidsforsinkelse og minimum kjedeforsinkelse. Anta videre at parasittisk tidsforsinkelse skal utgjøre halvparten av minimum kjedeforsinkelse. Finn en verdi for x slik at parasittisk tidsforsinkelse utgjør halvparten av minimum kjedeforsinkelse.

Setter $P = D/2$:

$$4 \cdot \left(\frac{80 \cdot 12}{27x} \right)^{\frac{1}{4}} = 7$$

Optimal porteffort:

$$f' = \left(\frac{80 \cdot 12}{27x} \right)^{\frac{1}{4}}$$

Minimum kjedeforsinkelse:

$$D = N \cdot f' + P$$

$$= 4 \cdot \left(\frac{80 \cdot 12}{27x} \right)^{\frac{1}{4}} + 7$$

$$x^{\frac{1}{4}} = \frac{7}{4} \left(\frac{80 \cdot 12}{27x} \right)^{\frac{1}{4}}$$

$$x \approx 4$$

Parasittisk tidsforsinkelse:

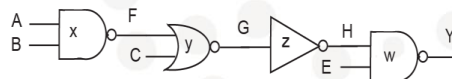
$$P = 2 + 2 + 1 + 2$$

$$= 7$$



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Eksamensoppgave 2005



Finn transistorstørrelser som gir minimum kjedeforsinkelse når parasittisk kjedeforsinkelse er halvparten av minimum kjedeforsinkelse.

Optimal porteffort:

$$f' = \left(\frac{80 \cdot 12}{27 \cdot 4} \right)^{\frac{1}{4}}$$

$$= 1.73$$

Starter ved utgangen og finner transistorstørrelser:

$$w = \frac{C_{ekstern} g}{f'}$$

$$= \frac{4 \cdot 3 \cdot \frac{4}{3}}{1.73}$$

$$\approx 9.3$$

Fortsetter med z:

$$z = \frac{9 \cdot 1}{1.73}$$

$$\approx 5.2$$

Fortsetter med y:

$$y = \frac{5 \cdot \frac{5}{3}}{1.73}$$

$$\approx 5$$

Kontrollerer for x:

$$x = \frac{5 \cdot \frac{4}{3}}{1.73}$$

$$\approx 4$$



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Prøveeksamen 2005

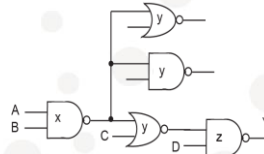
Finn logisk effort for portene og kjeden. Anta at utgangen Y skal drive 4 enhetsinvertere. Finn elektrisk effort for kjeden. Hva blir kjedens effort F ?

Logisk effort i kjede:

$$\begin{aligned} G &= \prod_i g_i \\ &= \frac{4}{3} \cdot \frac{5}{3} \cdot \frac{4}{3} \\ &= \frac{80}{27} \\ &\approx 3 \end{aligned}$$

Elektrisk effort i kjede:

$$\begin{aligned} H &= \frac{4 \cdot 3}{x} \\ &= \frac{12}{x} \end{aligned}$$



Kjedens forgreiningseffort:

$$\begin{aligned} B &= \frac{y + y + y}{y} \\ &= 3 \end{aligned}$$

Kjedens effort:

$$\begin{aligned} F &= GBH \\ &= \frac{2880}{27x} \end{aligned}$$



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Prøveeksamen 2005

Hva blir optimal port effort for kjeden? Finn kjedens parasittiske tidsforsinkelse og minimum kjedeforsinkelse. Anta videre at parasittisk tidsforsinkelse skal utgjøre halvparten av minimum kjedeforsinkelse. Finn en verdi for x slik at parasittisk tidsforsinkelse utgjør halvparten av minimum kjedeforsinkelse.

Optimal porteffort:

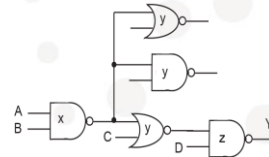
$$f' = \left(\frac{2880}{27x} \right)^{\frac{1}{3}}$$

Parasittisk tidsforsinkelse:

$$\begin{aligned} P &= 2 + 2 + 2 \\ &= 6 \end{aligned}$$

Minimum kjedeforsinkelse:

$$\begin{aligned} D &= N \cdot f' + P \\ &= 3 \cdot \left(\frac{2880}{27x} \right)^{\frac{1}{3}} + 6 \end{aligned}$$



Setter $P = D/2$:

$$\begin{aligned} 3 \cdot \left(\frac{2880}{27x} \right)^{\frac{1}{3}} &= 6 \\ x &\approx 13 \end{aligned}$$



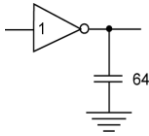
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Optimalt antall porter i kjede

Kjedens effort:

$$\begin{aligned} F &= GBH \\ &= 1 \cdot 1 \cdot \frac{64}{1} \\ &= 64 \end{aligned}$$

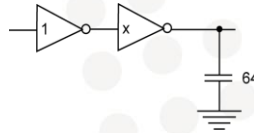
En port:



$$\begin{aligned} D &= f + 1 \\ &= 65 \end{aligned}$$



To porter:



Optimal porteffort:

$$N = 2$$

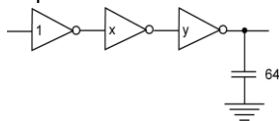
$$\begin{aligned} f' &= F^{\frac{1}{2}} \\ &= 8 \end{aligned}$$

Tidsforsinkelse:

$$\begin{aligned} D &= Nf' + P \\ &= 2 \cdot 8 + 1 + 1 \\ &= 18 \end{aligned}$$



Tre porter:



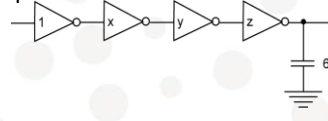
Optimal porteffort:

$$\begin{aligned} N &= 3 \\ f' &= F^{\frac{1}{3}} \\ &= 4 \end{aligned}$$

Tidsforsinkelse:

$$\begin{aligned} D &= Nf' + P \\ &= 3 \cdot 4 + 1 + 1 + 1 \\ &= 15 \end{aligned}$$

Fire porter:



Optimal porteffort:

$$\begin{aligned} N &= 4 \\ f' &= F^{\frac{1}{4}} \\ &= 2.8 \end{aligned}$$

Tidsforsinkelse:

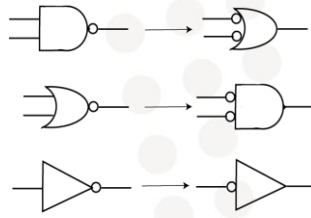
$$\begin{aligned} D &= Nf' + P \\ &= 4 \cdot 2.8 + 1 + 1 + 1 + 1 \\ &= 15.2 \end{aligned}$$



Bubble pushing

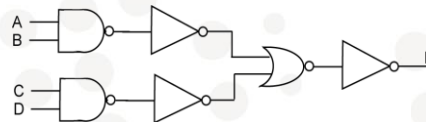
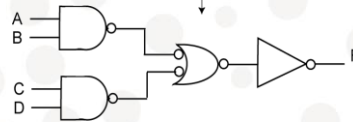
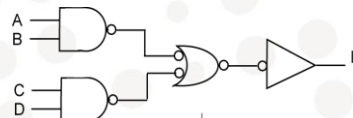
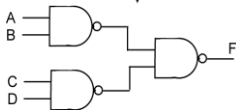
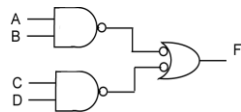
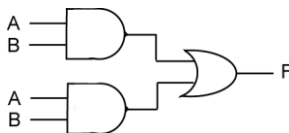
$$\overline{A \cdot B} = \overline{A} + \overline{B}$$

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$


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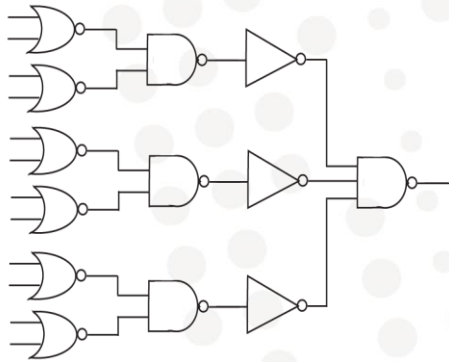
Eksempel:

$$F = AB + CD$$


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Oppgave 6.3

Tegn sjematikk for en 12inngangs OR port implementert med NAND og NOR porter med maksimalt 3 innganger hver.

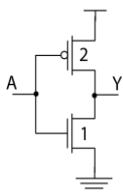



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Komplementær logikk

Enhetsinverter

$$Y = \overline{A}$$

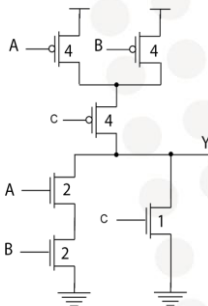
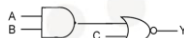


$$g_A = 3/3 = 1$$

$$p = 3/3 = 1$$

AOI21

$$Y = AB + C$$



$$g_A = 6/3 = 2$$

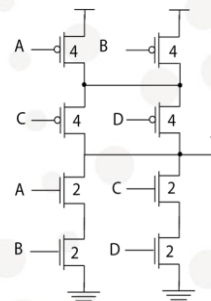
$$g_B = 6/3 = 2$$

$$g_C = 5/3$$

$$p = 7/3$$

AOI22

$$Y = AB + CD$$



$$g_A = 6/3 = 2$$

$$g_B = 6/3 = 2$$

$$g_C = 6/3 = 2$$

$$g_D = 6/3 = 2$$

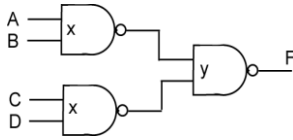
$$p = 12/3$$



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Eksempel:

$$F = AB + CD$$



Vi antar at:

$$H = \frac{100}{20} \\ = 5$$

Logisk effort:

$$G = \frac{4}{3} \cdot \frac{4}{3} \\ = \frac{16}{9}$$

Parasittisk tidsforsinkelse:

$$P = 2 + 2 \\ = 4$$

Kjedens effort:

$$F = GBH \\ = \frac{16}{9} \cdot 1 \cdot 5 \\ \approx 9$$

Optimal porteffort:

$$f' = 9^{\frac{1}{3}} \\ = 3$$

Vi beregner y:

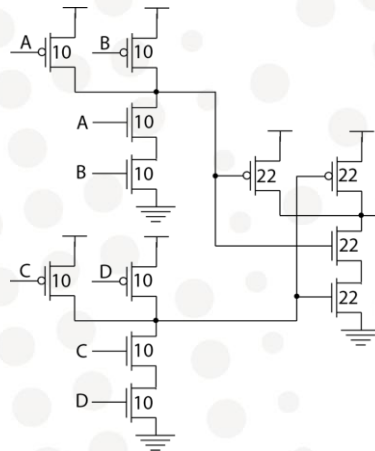
$$y = \frac{C_{ekstern} g_{NAND2}}{f'} \\ = \frac{100 \cdot \frac{4}{3}}{3} \\ = 44$$

Vi beregner x:

$$x = \frac{44 g_{NAND2}}{f'} \\ = \frac{44 \cdot \frac{4}{3}}{3} \\ \approx 20$$

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Løsning:

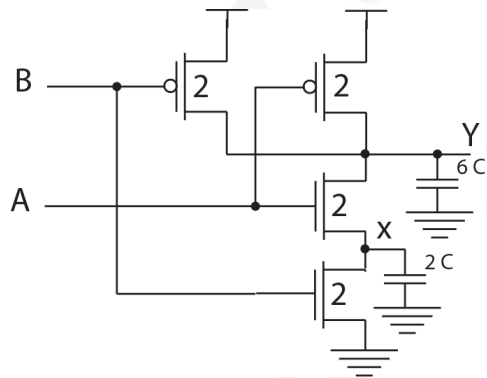


Tidsforsinkelse:

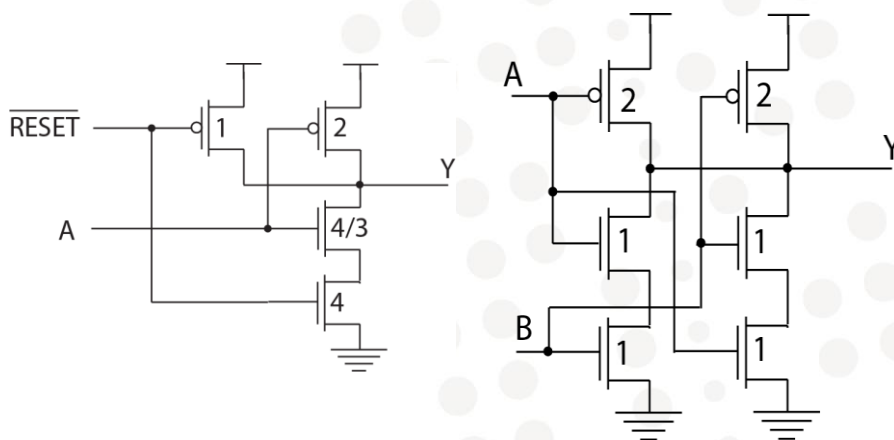
$$D = NF^{\frac{1}{N}} + P \\ = 2 \cdot 3 + 2 + 2 \\ = 10$$

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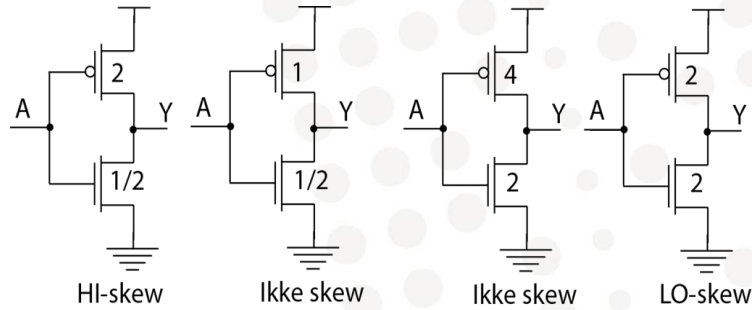
Rekkefølge på inngangssignaler



Asymmetriske porter



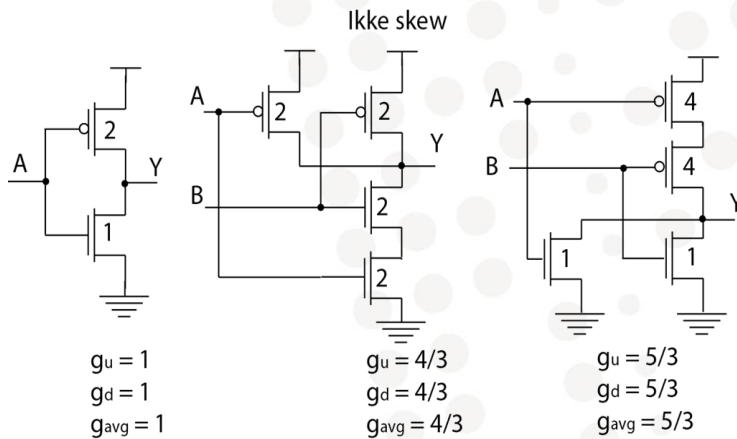
Porter med skew

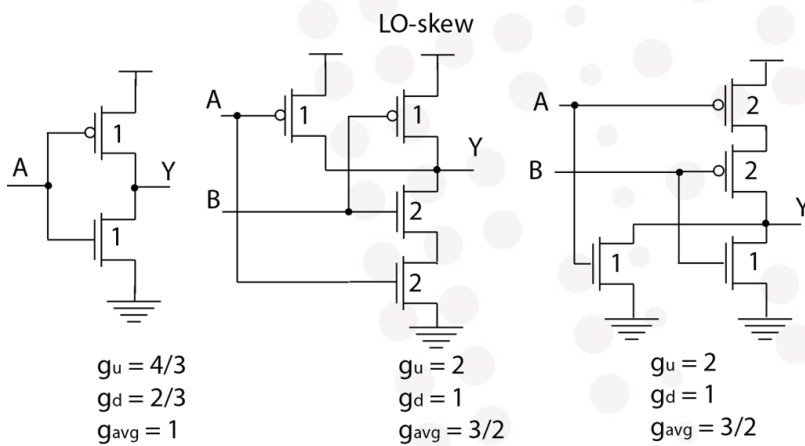
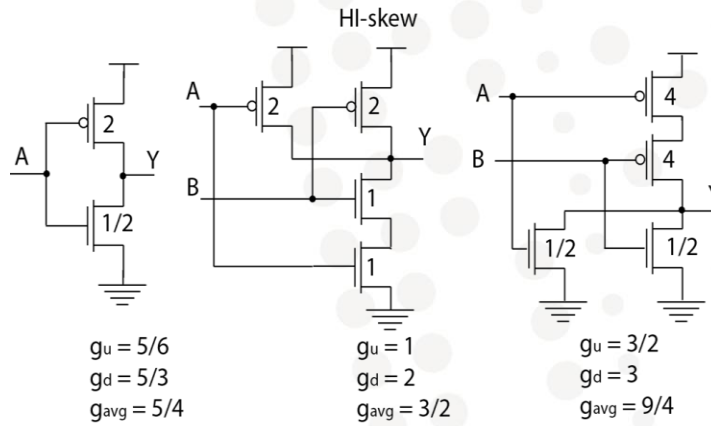


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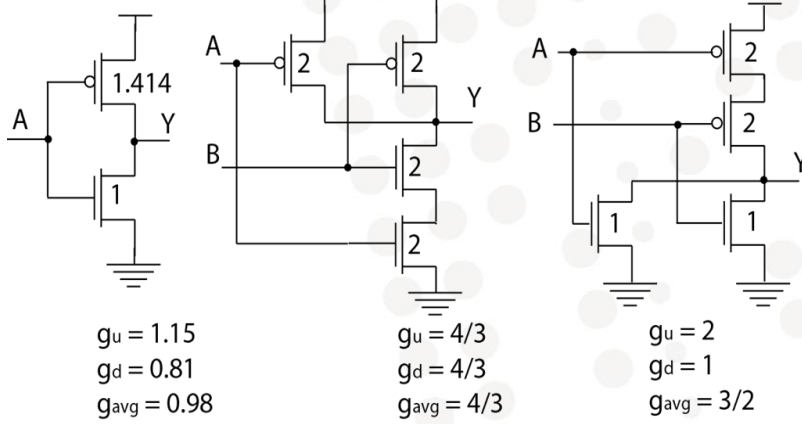
P/N forhold





Minimum tidsforsinkelse

HI-skew



Oppgave 6.10

Tegn sjematikk for HI-skew og LO-skew 3inngangs NAND og NOR porter. Hva er logisk effort for portene for kritisk transisjon?

