INF3410/4411, Fall 2018

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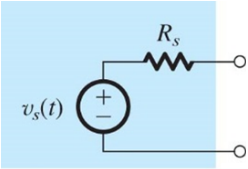
Excerpt of Sedra/Smith Chapter 1: Signals and Amplifier Concept

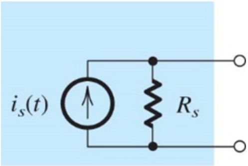
Content

Content

Signal Sources



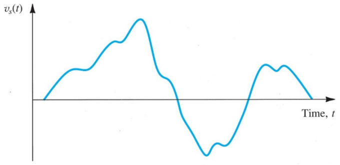
(a)



(b)

Two *equivalent* models of signal sources (e.g. a sensor or other *transducer*): a) is called the The Thévenin form and b) the Norton form. Note: Outputresistance R_S is the same in both models while $v_S(t) = i_S(t)R_S$. (A more general model would consider an output*impedance* (i.e. including output capacitance/inductance) rather than just a resistance, but we'll ignore that for now.)

Arbitrary Signals

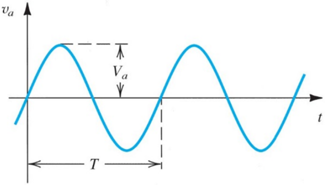


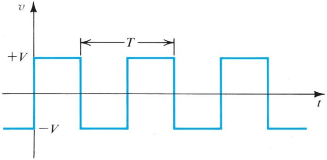
Any arbitrary signal can be expressed as an (infinite) sum or (infinite) integral of sine wave signals of different frequencies and phases by means of the *inverse Fourier transform*:

$$v_S(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{v}_s(\omega)e^{i\omega t}d\omega$$

Where the Fourrier transform $\hat{v}_s$ is a complex number describing the *frequency spectrum* of the signal. Note that for a real signal $v_s(t)$, $\hat{v}_s(-\omega) = -\hat{v}_s(\omega)$, i.e. if you add them they will also be real. You can conveniently plot $\hat{v}_s$ in a *Bode plot* (more on that later) consisting of a frequency dependent magnitude and phase plot.
A little matlab animation for illustration to show how this is a sum of sine-waves shall be shown here.

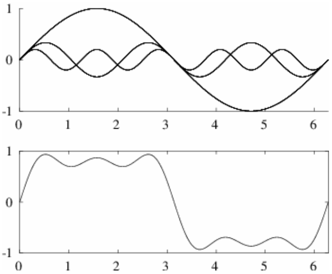
Cyclic Signals





Cyclic signals can be expressed as discreet sums of sine wave signals, more precisely with harmonics of the cycle frequency.

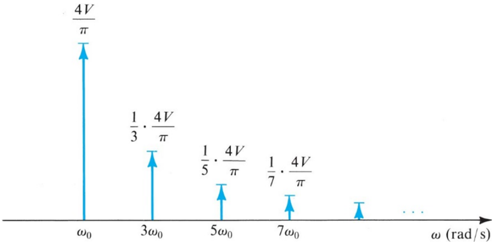
Example: Approximating Square Wave



Here is how a square wave can be approximated with the sum of 3 sine waves.

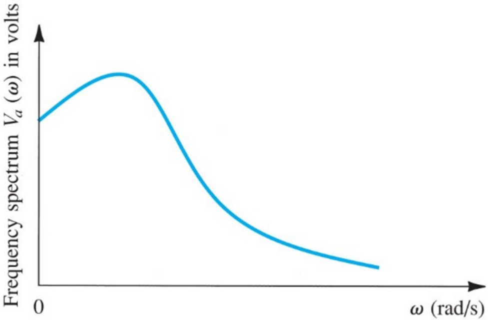
$$v(t) = \frac{4V}{\pi}\left(\sin \omega_0 t + \frac{1}{3}\sin 3\omega_0 t + \frac{1}{5}\sin 5\omega_0 t + ...\right) \quad (1.2)$$

Spectrum of Square Wave



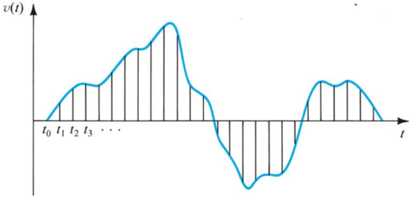
Thus the spectrum of a square wave (or any cyclic signal) is non-zero at the harmonics (multiples of the fundamental frequency) only. It can thus be expressed as a sum.

Spectrum of Arbitrary Function

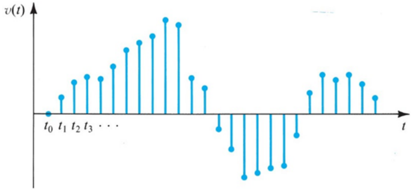


whereas the spectrum of a non-cyclic function can be non-zero for any frequency, e.g. the function in figure 1.3.

Discrete Time/Sampled



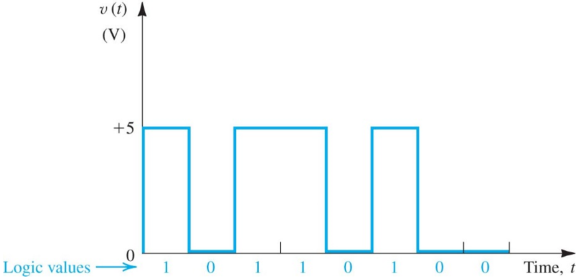
(a)



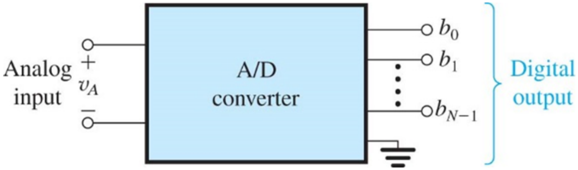
(b)

Discrete Value/Digital

(for example binary)

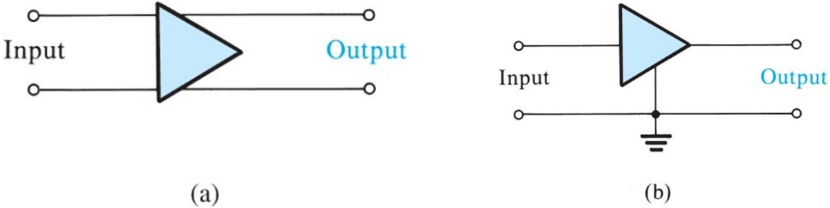


ADC

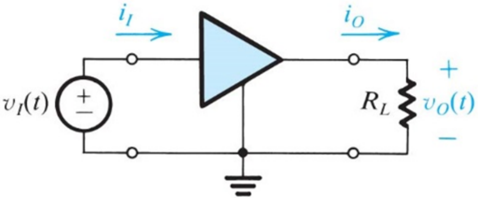


Content

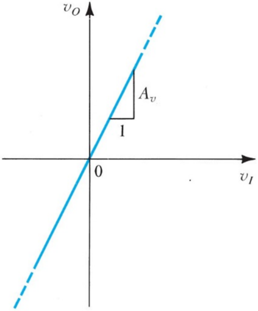
Amplifier Basic Concept



Example: Voltage Amplifier with Load Resistance



(a)



(b)

Gain in Decibels

$$A_v = \frac{v_O}{v_I}$$

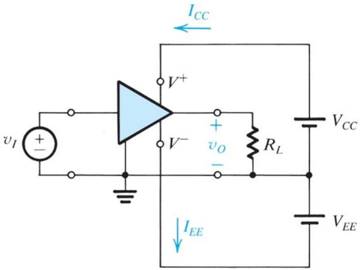
$$A_p = \frac{p_O}{p_L} = \frac{v_O i_O}{v_I i_I} = A_v A_i$$

$$A_{p,dB} = 10 \log_{10} A_p \text{ [dB]}$$

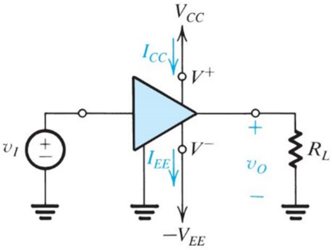
$$A_{v,dB} = 20 \log_{10} A_v \text{ [dB]}$$

Increasing Signal Power

(and thus in need of a power supply)



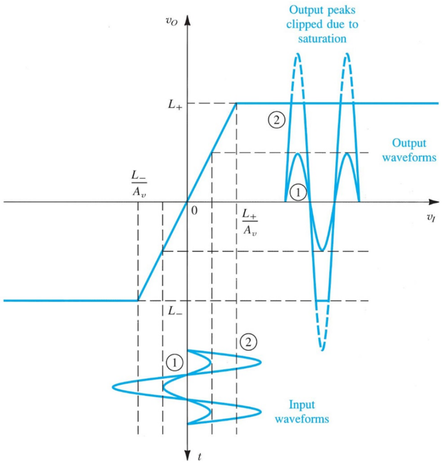
(a)



(b)

$$\eta = \frac{P_L}{P_{dc}} \times 100 \quad (1.10)$$

Non-Ideal Behaviour (1/many)

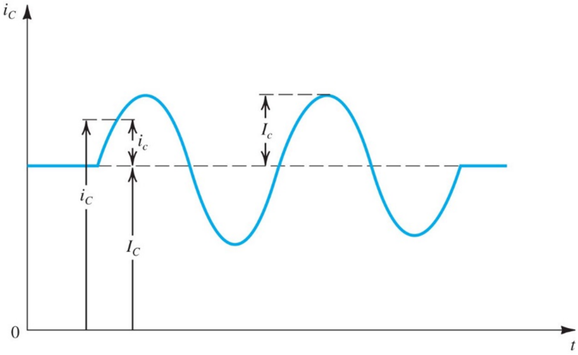


Linear range:

$$\frac{L_-}{A_v} \leq v_I \leq \frac{L_+}{A_v}$$

(near Fig. 1.14)

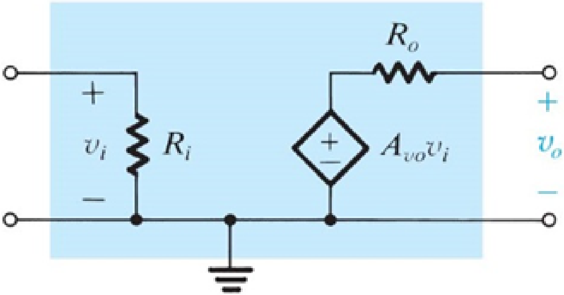
Symbol Convention



$$i_C(t) = I_C + i_c(t) \quad (1.11)$$

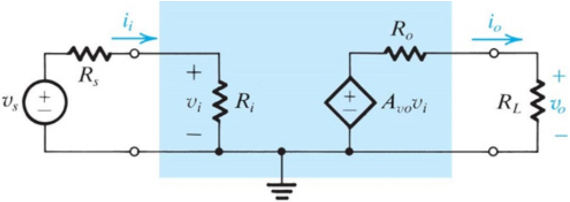
A First Voltage Amplifier Model

(slightly less ideal, i.e. adding input and output resistance)



(a)

Voltage Gain Dependence

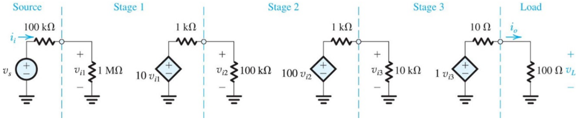


(b)

$$A_v \equiv \frac{v_o}{v_i} = A_{vo}\frac{R_L}{R_L + R_o} \quad (1.13)$$

$$\frac{v_o}{v_s} = A_{vo}\frac{R_i}{R_i + R_S}\frac{R_L}{R_L + R_o} \quad (\text{below } 1.13)$$

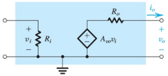
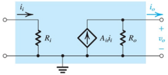
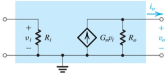
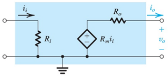
Example: Cascaded Amplifiers



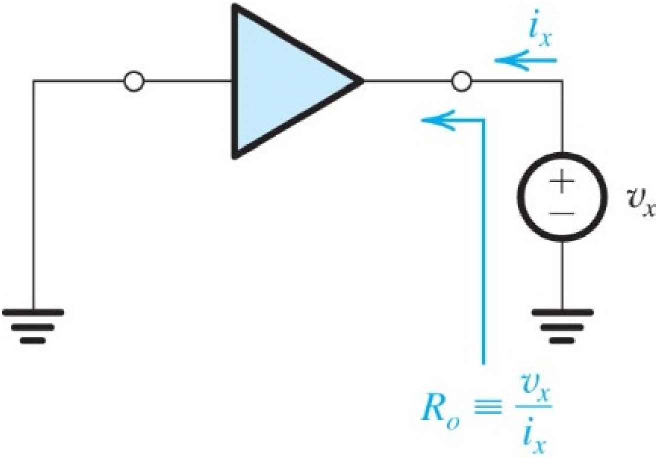
$$\frac{v_o}{v_s} = \frac{R_{i1}}{R_{i1} + R_S} A_{vo1} \frac{R_{i2}}{R_{i2} + R_{o1}} A_{vo2} \frac{R_{i3}}{R_{i3} + R_{o2}} \ldots A_{voN} \frac{R_L}{R_L + R_{oN}}$$

4 Equivalent Models

Table 1.1 The Four Amplifier Types

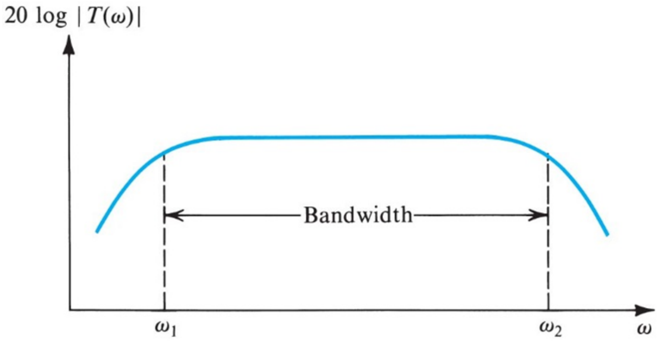
Type	Circuit Model	Gain Parameter	Ideal Characteristics
Voltage Amplifier		Open-Circuit Voltage Gain $A_{vo} \equiv \left.\frac{v_o}{v_i}\right\|_{i_o=0}$ (V/V)	$R_i = \infty$ $R_o = 0$
Current Amplifier		Short-Circuit Current Gain $A_{is} \equiv \left.\frac{i_o}{i_i}\right\|_{v_o=0}$ (A/A)	$R_i = 0$ $R_o = \infty$
Transconductance Amplifier		Short-Circuit Transconductance $G_m \equiv \left.\frac{i_o}{v_i}\right\|_{v_o=0}$ (A/V)	$R_i = \infty$ $R_o = \infty$
Transresistance Amplifier		Open-Circuit Transresistance $R_m \equiv \left.\frac{v_o}{i_i}\right\|_{i_o=0}$ (V/A)	$R_i = 0$ $R_o = 0$

Determining R_i and R_o



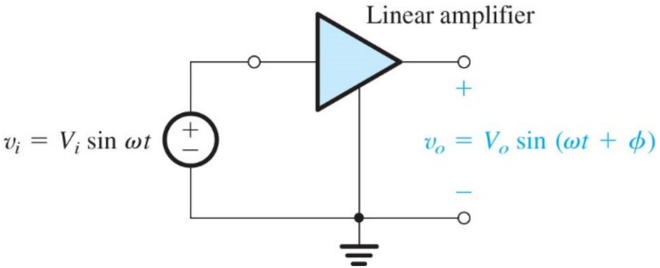
Frequency Response

(This behaviour is not explained by the simple model! Capacitors and/or inductors are needed.)

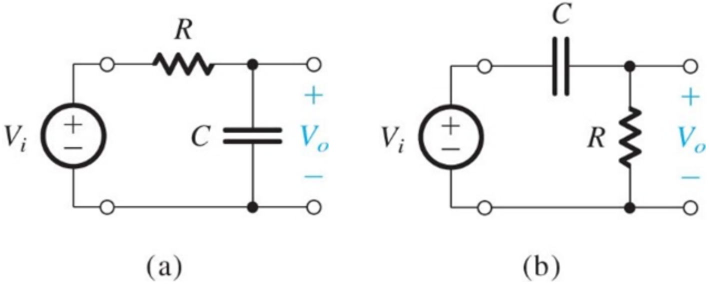


Linear Amplifier

Linear here means that there is no distortion of a fixed frequency sinusoid. Equivalet in math-speak: the amplifier/filter output can be modelled as a linear differential equation of the input signal. An amplifier composed of but linear elements will behave like that, including somewhat more complicated models than our first purely resistive model ...



Single Time Constant Networks



STC netwoks are circuits that can be expressed as a first order linear differential equation of the input. When the input voltage source provides a signal the STC network is a *filter* with a specific *transfer function*, i.e. a frequency dependent complex number that describes how the *spectrum* of the input is modified at each ferequency.

Transfer Function

Transfer functions $T(s)$ for linear electronic circuits can be written as dividing two polynomials of s (for us s is simply short for $j\omega$).

$$T(s) = \frac{a_0 + a_1 s + ... + a_m s^m}{1 + b_1 s + ... + b_n s^n}$$

$T(s)$ is often written as products of first order terms in both nominator and denominator in the following *root form*, which is conveniently showing some properties of the Bode-plots. More of that later.

$$T(s) = a_0 \frac{(1 + \frac{s}{z_1})(1 + \frac{s}{z_2})...(1 + \frac{s}{z_m})}{(1 + \frac{s}{\omega_1})(1 + \frac{s}{\omega_2})...(1 + \frac{s}{\omega_n})}$$

Transfer Function

The transfer function $T(s)$ of a linear filter is

- the Laplace transform of its impulse reponse $h(t)$.
- the Laplace transform of the differential equation describing the I/O realtionship that is then solved for $\frac{V_{out}(s)}{V_{in}(s)}$
- (easiest!!!) the circuit diagram solved quite normally for $\frac{V_{out}(s)}{V_{in}(s)}$ by putting in impedances $Z(s)$ for all linear elements according to some simple rules (next page).

Impedances of Linear Circuit Elements

resistor:	R
capacitor:	$\frac{1}{sC}$
inductor:	sL

Ideal linearly dependent sources (e.g. the $i_d = g_m v_{gs}$ sources in small signal models of FETs) are left as they are.

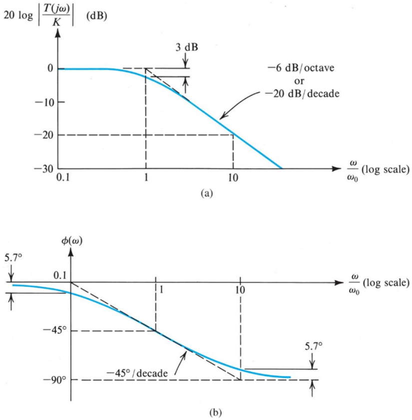
Single Time Constant Transfer Functions

Table 1.2 Frequency Response of STC Networks

	Low-Pass (LP)	High-Pass (HP)
Transfer Function $T(s)$	$\dfrac{K}{1+(s/\omega_0)}$	$\dfrac{Ks}{s+\omega_0}$
Transfer Function (for physical frequencies) $T(j\omega)$	$\dfrac{K}{1+j(\omega/\omega_0)}$	$\dfrac{K}{1-j(\omega_0/\omega)}$
Magnitude Response $\lvert T(j\omega)\rvert$	$\dfrac{\lvert K\rvert}{\sqrt{1+(\omega/\omega_0)^2}}$	$\dfrac{\lvert K\rvert}{\sqrt{1+(\omega_0/\omega)^2}}$
Phase Response $\angle T(j\omega)$	$-\tan^{-1}(\omega/\omega_0)$	$\tan^{-1}(\omega_0/\omega)$
Transmission at $\omega = 0$ (dc)	K	0
Transmission at $\omega = \infty$	0	K
3-dB Frequency	$\omega_0 = 1/\tau$; $\tau \equiv$ time constant $\tau = CR$ or L/R	
Bode Plots	in Fig. 1.23	in Fig. 1.24

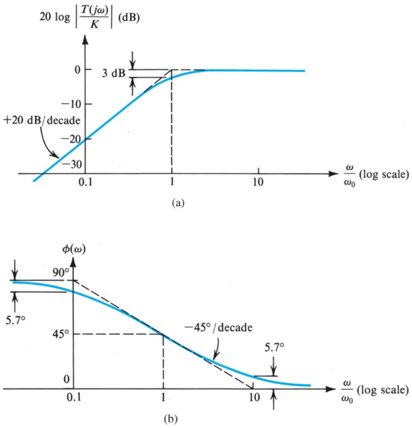
Bode Plot

1^{st} Order Low-Pass Filter

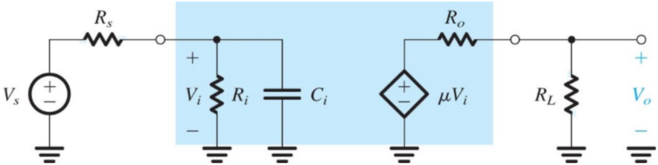


Bode Plot

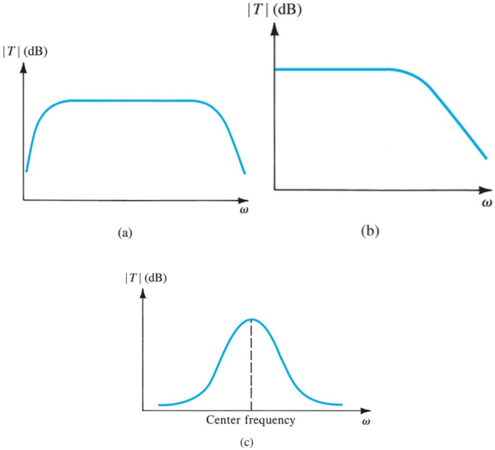
1^{st} Order High-Pass Filter



Example



Characterizing Amplifiers by Transfer Characteristics



Capacitively Coupled Two Stage Amplifiers