# INF3410/4411, Fall 2018

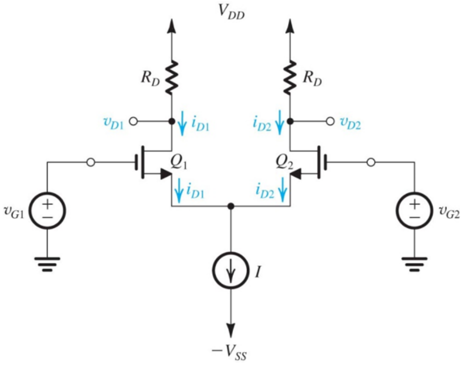
Philipp Häfliger
hafliger@ifi.uio.no

Excerpt of Sedra/Smith Chapter 8: Differential and Multistage CMOS Amplifier Basics

# Content

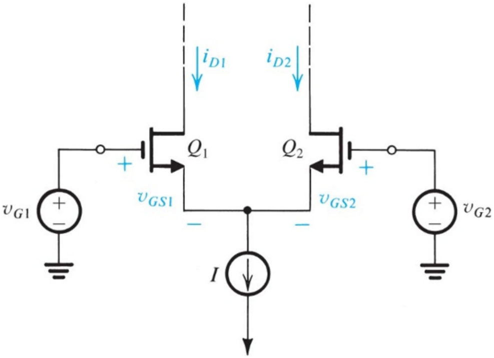
# Content

# The differential pair with resistive loads

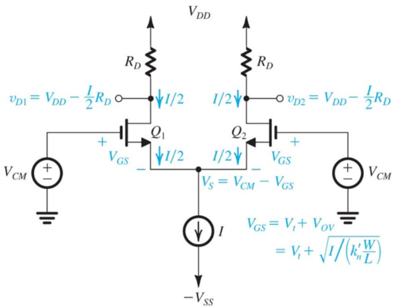



The resistors turn $i_d$ linearly into voltage

# Easier (in my opinion): look simply at $i_d$



# Analysis for Common Mode Input



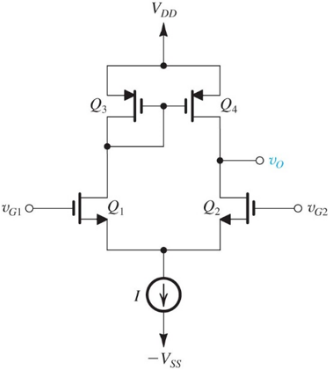

With ideal current source: common mode voltage $V_{CM}$ has no effect, but beware the range of operation!!!

$$V_{CMmax} = V_t + V_{DD} - \frac{I}{2}R_D \quad (8.7)$$

$$V_{CMmin} = -V_{SS} + V_{CS} + V_t + V_{OV} \quad (8.8$$

(Note that the book always writes $-V_{SS}$ at the actual terminal, i.e. always expresses $V_{SS}$ as a positive number...)

# With current mirror



(a)

When the output is connected to a voltage source, the output current becomes the difference of the two $i_d$

# Large Signal, Weak Inversion

Simpler analysis in weak inversion:

$$I_b = I_1 + I_2 = I_S e^{\frac{-V_t - nV_S}{nV_T}} \left( e^{\frac{V_1}{nV_T}} + e^{\frac{V_2}{nV_T}} \right)$$

$$I_{out} = I_1 - I_2 = I_S e^{\frac{-V_t - nV_S}{nV_T}} \left( e^{\frac{V_1}{nV_T}} - e^{\frac{V_2}{nV_T}} \right)$$

$$\frac{I_{out}}{I_b} = \frac{e^{\frac{V_1}{nV_T}} - e^{\frac{V_2}{nV_T}}}{e^{\frac{V_1}{nV_T}} + e^{\frac{V_2}{nV_T}}}$$

$$I_{out} = I_b \frac{e^{\frac{V_1}{nV_T}} - e^{\frac{V_2}{nV_T}}}{e^{\frac{V_1}{nV_T}} + e^{\frac{V_2}{nV_T}}} = I_b \tanh \frac{V_1 - V_2}{2nV_T}$$

# Small Signal, Weak Inversion

Since the slope of $\tanh x$ for $x = 0$ is 1, the slope of $I_b \tanh \frac{\Delta V}{2nV_T}$ with respect to $\Delta V$ (the transconductance $g$ of this transamp) is:

$$g = \frac{I_b}{2nV_T}$$

# Large Signal, Strong Inversion (1/2)

$$I_b = I_1 + I_2 = k_n(V_{OV1}^2 + V_{OV2}^2)$$

$$I_{out} = I_1 - I_2 = k_n(V_{OV1}^2 - V_{OV2}^2)$$

$$\frac{I_{out}}{I_b} = \frac{V_{OV1}^2 - V_{OV2}^2}{V_{OV1}^2 + V_{OV2}^2}$$

$$I_{out} = I_b \frac{V_{OV1}^2 - V_{OV2}^2}{V_{OV1}^2 + V_{OV2}^2}$$

# Large Signal, Strong Inversion (2/2)

Rewrite with $\hat{V}_{OV} = \frac{V_{OV1}+V_{OV2}}{2}$ and $\Delta V_{OV} = V_{OV1} - V_{OV2}$

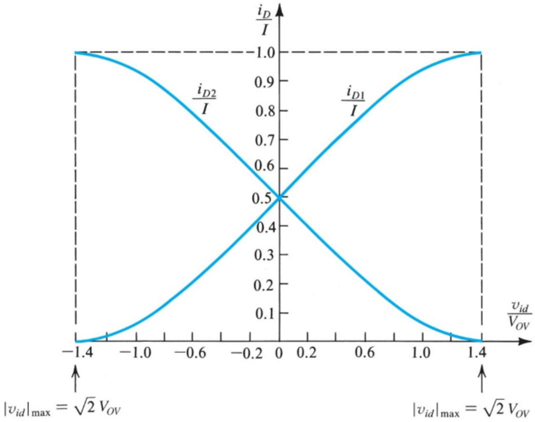
$$I_{out} = I_b \frac{2\Delta V_{OV} \hat{V}_{OV}}{\frac{1}{2}\left(\Delta V_{OV}^2 + (2\hat{V}_{OV})^2\right)}$$

Note that this is not yet a closed solution as i the large signal world $V_S$, and thus $\hat{V}_{OV}$ depends on $\Delta V_{OV}$.
Extrema where one transistor conducts the entire $I_B$ is where $\Delta V_{OV} = 2\hat{V}_{OV}$ (since then one branch has $V_{OV2} = 0$) and $I_B = k_n \Delta V_{OV}^2$. It follows that:

$$\Delta V_{OV} = \sqrt{\frac{I_B}{k_n}} = \sqrt{2} V_{OV}$$

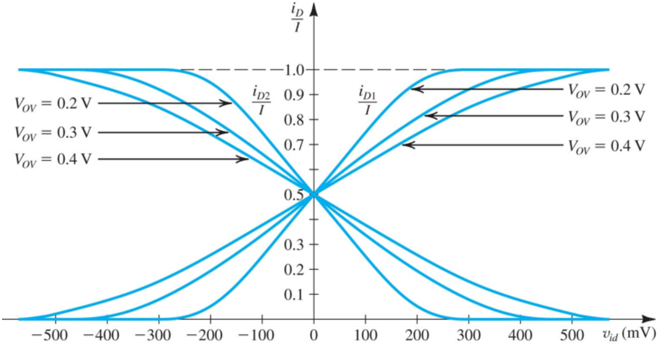
Where like in the book $V_{OV}$ is the overdrive voltage for $\Delta V_{OV} = 0$

# Normalized I/V Curves and Ranges

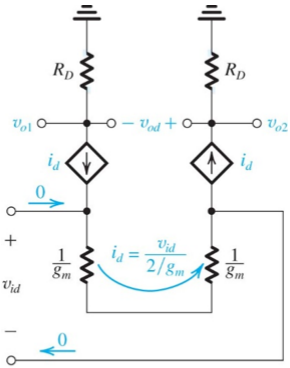


# I/V Curves for different $V_{OV}$ respectively $\frac{W}{L}$

(This is only valid for strong inversion)



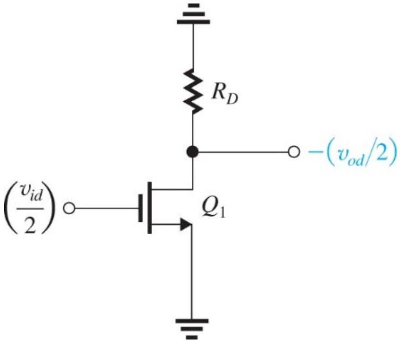
# Small Signal Analysis on the Half Circuit (1/2)




(b)

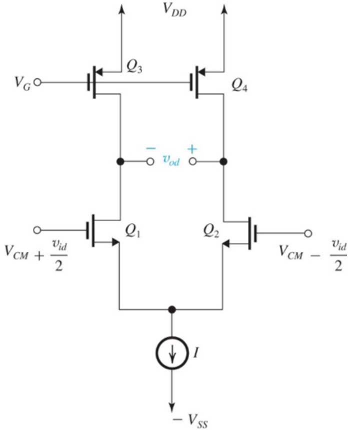
Assuming a 'balanced' input, i.e. $v_{g1} = -v_{g2} = \frac{v_{id}}{2}$.
This results in a virtual small signal Gnd at the source of the transistors.

# Small Signal Analysis on the Half Circuit (2/2)

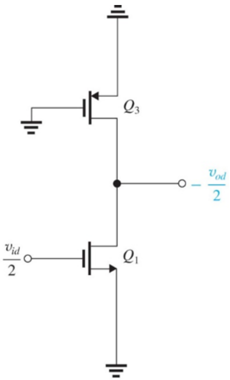



Thus one can look at the branches individually: It's the good old common source amp.

# Current Source Load Differential Amplifier

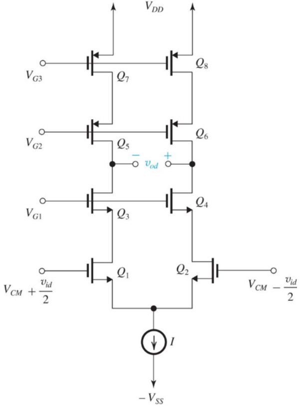



(a)

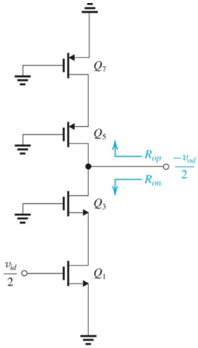



(b)

# Cascode Differential Amplifier



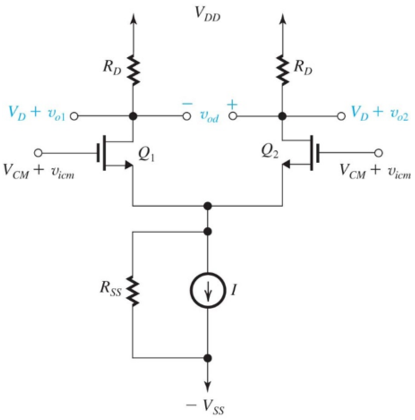

(a)




(b)

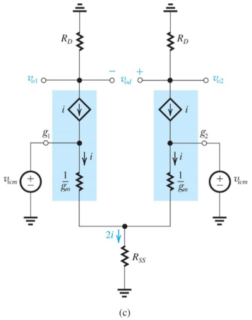
# Content

# Common Mode Rejection



(a)

# Common Mode Rejection



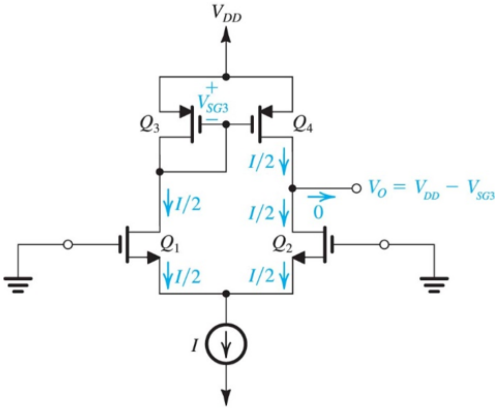

$$v_{icm} = \frac{i}{g_m} + 2iR_{SS}$$

$$i = \frac{v_{icm}}{\frac{1}{g_m0} + 2R_{SS}} \approx \frac{v_{icm}}{2R_{SS}}$$

And $R_D$ converts $i$ into the two outout voltages $v_{o1}$ and $v_{o2}$. Note that the *difference* of currents is still 0, i.e. not affected by the comon mode input. However, since a change in $i$, respectively a change in $I_b$, affects the transconductance, $v_{icm}$ will influence the output difference if the differential input is not zero, and mismatch will lead to common mode gain, i.e. a DC offset with zero input difference that varies with $v_{icm}$.

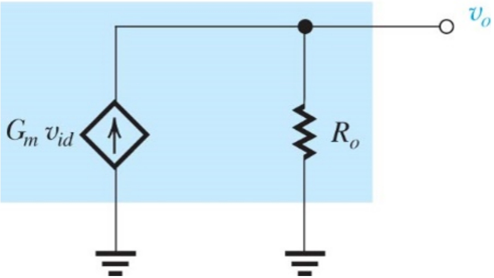
# Content

# Systematic DC offset with current mirror load



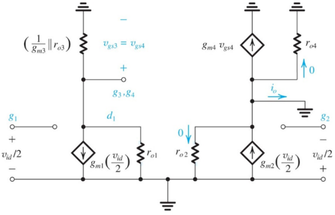

(b)

# Output equivalent circuit




$$A_d = \frac{v_o}{v_{id}} = G_m R_o = g_{m1,2}(r_{o2} || r_{o4})$$

# A more careful deduction of $G_m$



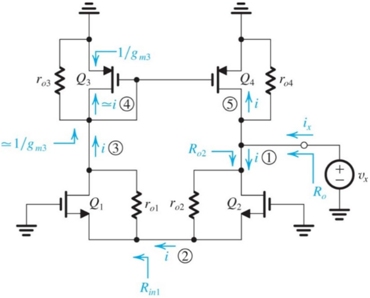

(b)

$$i_o = g_{m2}\frac{v_{id}}{2} - g_{m4}v_{gs4} \quad (8.132)$$

$$\begin{aligned} v_{gs3} &= -g_{m1}\frac{v_{id}}{2}\left(\frac{1}{g_{m3}}||r_{o3}||r_{o1}\right) \\ &\approx -\frac{g_{m1}}{g_{m3}}\frac{v_{id}}{2} \quad (8.134) \end{aligned}$$

$$i_o \approx g_m v_{id} \;\Rightarrow\; G_m = g_m$$

# A more careful deduction of $R_o$

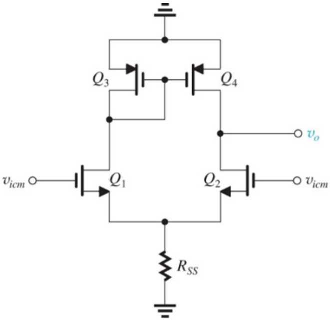



$$i = \frac{v_x}{R_{o2}}$$

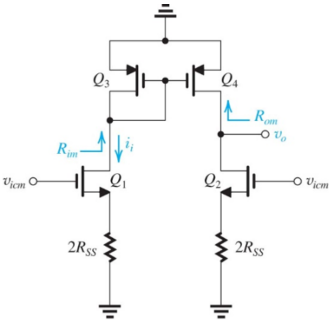
$$R_{in1} = \frac{r_{o1} + R_L}{g_{m1} r_{o1}} \approx \frac{1}{g_{m1}}$$

$$\begin{aligned} R_{o2} &= R_{in1} + r_{o2} + g_{m2} r_{o2} R_{in1} \\ &\approx 2 r_{o2} \quad (8.135) \end{aligned}$$

# Common Mode Gain (1/2)

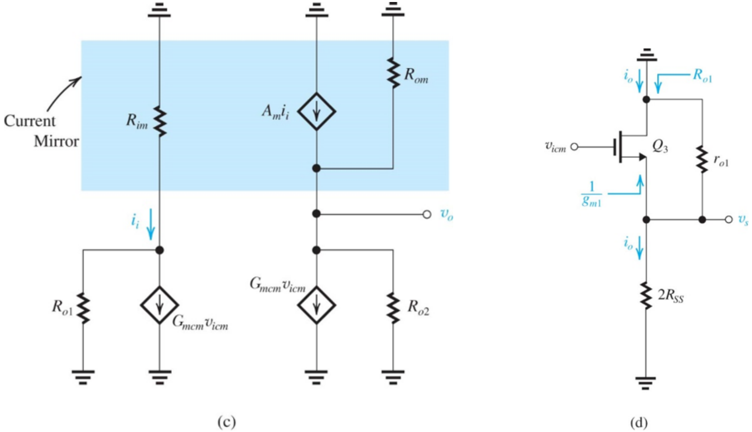



(a)

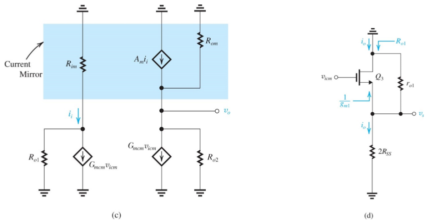



(b)

# Common Mode Gain (2/2)

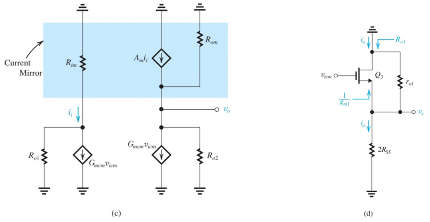


# For General Current Gain




$$A_{cm} = \frac{v_o}{v_{icm}} = -(1 - A_m)G_{mcm}(R_{om}||R_{o2})$$

# For Simple Current Mirror

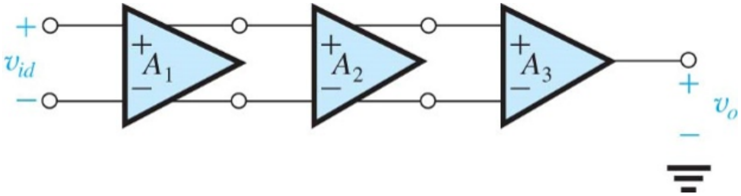



$$A_m i_i = v_{gs3} g_{m4} \quad v_{gs3} = i_i R_{im} \quad R_{im} = \frac{1}{g_{m3}} || r_{o3}$$

$$A_m = \frac{1}{1 + \frac{1}{g_{m3} r_{o3}}}$$

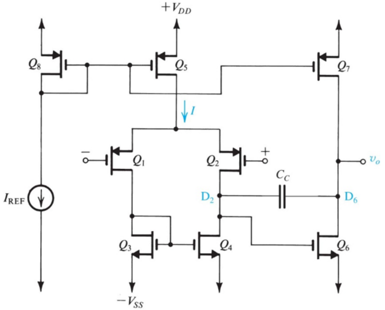
# Content

# Internally All Differential Example




Has some advantages, foremost a better CMRR. And with single ended stages you have to care about 'hitting' the right input DC level of the next stage.

# Two Stage CMOS op-amp Example




$$A = A_1 A_2 = g_{m1}(r_{o2}||r_{o4})g_{m6}(r_{o6}||r_{o7})$$

$$\frac{(W/L)_6}{(W/L)_4} = 2\frac{(W/L)_7}{(W/L)_5}$$