INF3410/4411, Fall 2018

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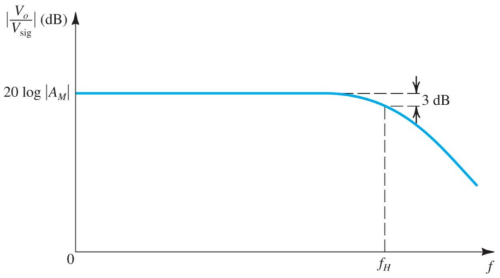
Excerpt of Sedra/Smith Chapter 9: Frequency Response of Basic CMOS Amplifiers

Content

Content

MOSFET IC transfer function



BW and GB(P)

$$\text{GB}=\text{BW} * A_M$$

GB: gain bandwidth product, BW: bandwidth, A_M: mid-band gain. There is usually a trade-off between BW and A_M. If this trade off is inversely proportional, the GB is constant, e.g. in opamp feedback configurations.

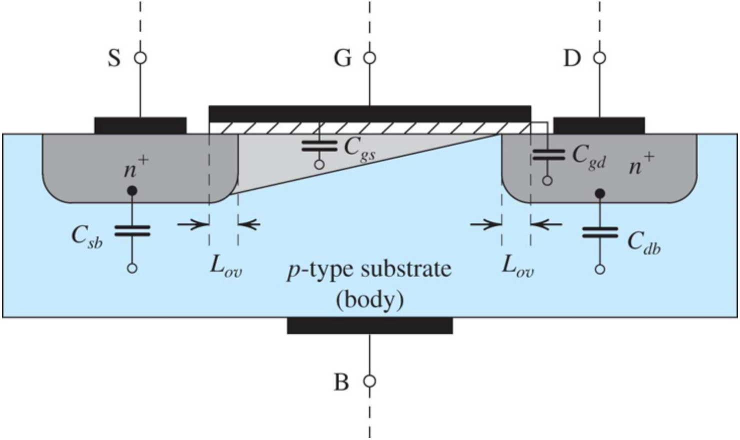
BW and GB(P) for CMOS integrated Circuits

For integrated circuits which normally have a pure low-pass characteristics (i.e. no explicit AC-coupling at the input) you can substitute A_M with A_{DC}, i.e. the gain at DC. And:

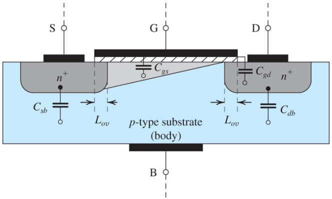
$$\text{BW} = f_H = f_{-3\text{dB}}$$

Where f_H is the high frequency cutoff and is the frequency at which point A_M i reduced by -3dB, i.e. the signal power is reduced by $\frac{1}{2}$, as $10 \log_{10} \frac{1}{2} = 3.0$

MOSFET 'Parasitic' Capacitances Illustration



MOSFET 'Parasitic' Capacitances Equations

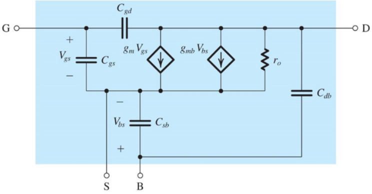


$$C_{gs} = C_{ox} W(\frac{2}{3} L + L_{ov}) \quad (9.22)$$

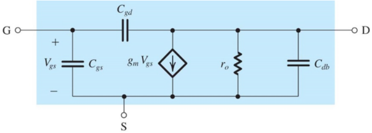
$$C_{gd} = C_{ox} W L_{ov} \quad (9.23)$$

$$C_{sb/db} = \frac{C_{sb0/db0}}{\sqrt{1 + \frac{V_{SB/DB}}{V_0}}} \quad (9.24/9.25)$$

High Frequency Small Signal Model (1/2)



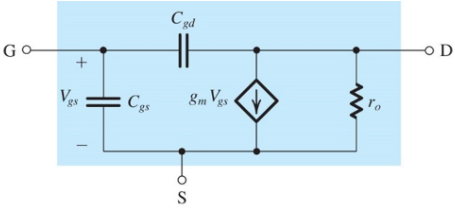
(a)



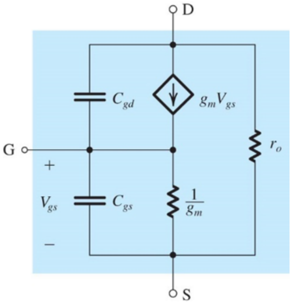
(b)

High Frequency Small Signal Model (2/2)

With only the two most relevant parasitic capacitors.



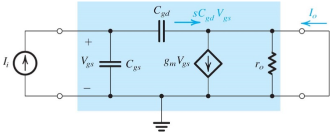
(c)



(d)

Unity Gain Frequency f_T

Short circuit current gain. A measure for the best case transistor speed.



Neglecting the current through C_{gd}:

$$\frac{i_o}{i_i} = \frac{g_m}{s(C_{gs} + C_{gd})} \quad (9.28)$$

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})} \quad (9.29)$$

Trade-off f_T vs A_o (i.e. GB)

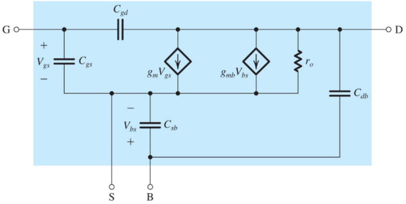
$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})} \quad (9.29)$$
$$\approx \frac{3\mu_n V_{ov}}{4\pi L^2}$$

$$A_0 = g_m r_o \quad (7.40)$$
$$\approx \frac{2}{\lambda V_{ov}}$$
$$= \frac{2L}{\underbrace{\lambda L}_{\text{const}} V_{ov}}$$

Summary CMOS HF Small Signal Model

Table 9.1 The MOSFET High-Frequency Model

Model



Model Parameters

$$g_m = \mu_n C_{ox} \frac{W}{L} |V_{OV}| = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = \frac{2I_D}{|V_{OV}|}$$

$$g_{mb} = \chi g_m, \quad \chi = 0.1 \text{ to } 0.2$$

$$r_o = |V_A| / I_D$$

$$C_{gs} = \frac{2}{3} WLC_{ox} + WL_{ov}C_{ox}$$

$$C_{gd} = WL_{ov}C_{ox}$$

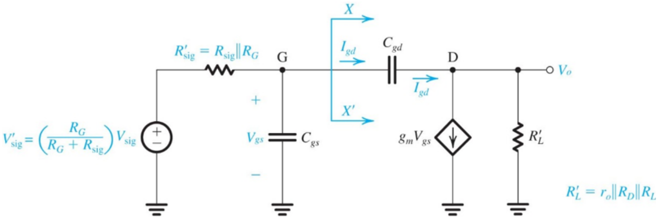
$$C_{sb} = \frac{C_{sb0}}{\sqrt{1 + \frac{|V_{SB}|}{V_0}}}$$

$$C_{db} = \frac{C_{db0}}{\sqrt{1 + \frac{|V_{DB}|}{V_0}}}$$

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

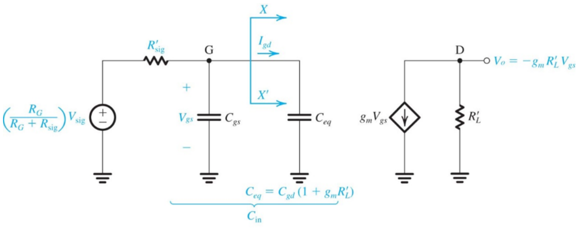
Content

CS Amplifier HF small signal model



(b)

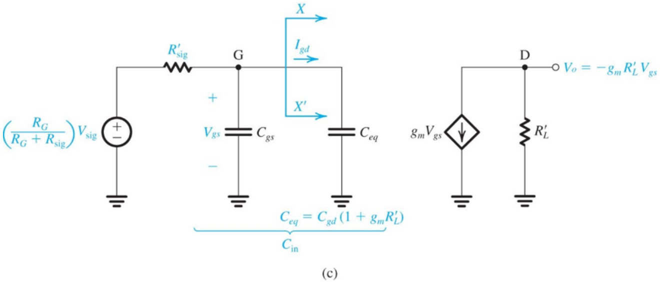
Using the Miller Effect



(c)

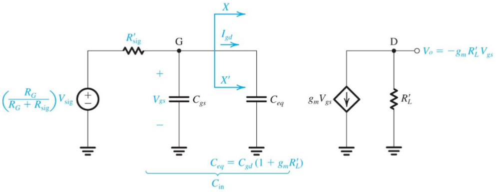
Note the simplyfying assumption that $v_o = g_m r_o v_{gs}$, i.e. neglecting feed forward contributions of i_{gd} which will still be very small around f_H and makes the following a quite exact approximation of the dominant pole's frequency $f_P \approx f_H$

CS transfer function dominant R_{sig}(1/4)



$$v_{gs}\left(s(C_{gs} + C_{eq}) + \frac{1}{R'_{sig}}\right) \quad = \quad v_{sig}\,\frac{1}{R'_{sig}}$$

CS transfer function dominant R_{sig}(2/4)

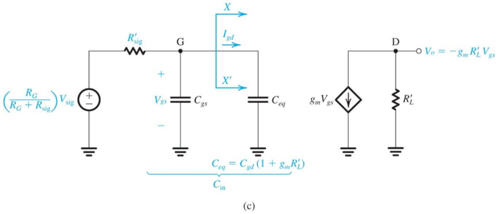


(c)

$$\frac{v_{gs}}{v_{sig}} = \frac{\frac{1}{R'_{sig}}}{s(C_{gs} + C_{eq}) + \frac{1}{R'_{sig}}}$$

$$\frac{v_{gs}}{v_{sig}} = \frac{1}{1 + sR'_{sig}(C_{gs} + C_{eq})}$$

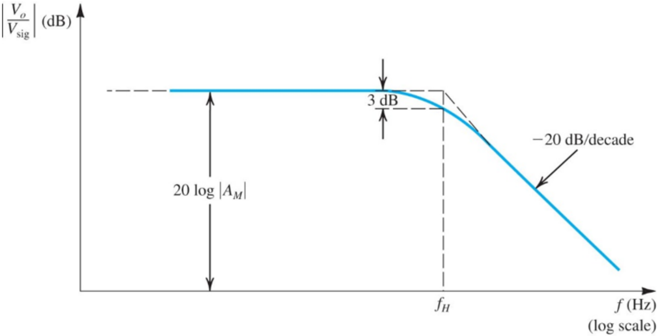
CS transfer function dominant R_{sig} (3/4)



$$\frac{v_o}{v_{sig}} = \frac{A_M}{1 + \frac{s}{\omega_0}} \quad (9.51)$$

$$\omega_H = \frac{1}{R'_{sig}(C_{gs} + C_{eq})} \quad (9.53)$$

CS transfer function dominant R_{sig}(4/4)



(d)

Interrupt: Transfer Function and Bode Plot

Transfer Function

The transfer function $H(s)$ of a linear filter is

- the Laplace transform of its impulse reponse $h(t)$.
- the Laplace transform of the differential equation describing the I/O realtionship that is then solved for $\frac{V_{out}(s)}{V_{in}(s)}$
- (this lecture!!!) the circuit diagram solved quite normally for $\frac{V_{out}(s)}{V_{in}(s)}$ by putting in impedances $Z(s)$ for all linear elements according to some simple rules.

Impedances of Linear Circuit Elements

resistor:	R
capacitor:	$\frac{1}{sC}$
inductor:	sL

Ideal sources (e.g. the $i_d = g_m v_{gs}$ sources in small signal models of FETs) are left as they are.

Transfer Function in Root Form

Transfer functions $H(s)$ for linear electronic circuits can be written as dividing two polynomials of s.

$$H(s) = \frac{a_0 + a_1 s + \ldots + a_m s^m}{1 + b_1 s + \ldots + b_n s^n}$$

$H(s)$ is often written as products of first order terms in both nominator and denominator in the following *root form*, which is conveniently showing some properties of the Bode-plots. More of that later.

$$H(s) = a_0 \frac{(1 + \frac{s}{z_1})(1 + \frac{s}{z_2})\ldots(1 + \frac{s}{z_m})}{(1 + \frac{s}{\omega_1})(1 + \frac{s}{\omega_2})\ldots(1 + \frac{s}{\omega_n})}$$

Bode Plots

Plots of magnitude (e.g. $|H(s)|$) in dB and phase e.g. $\angle H(s)$ or ϕ) vs. $\log(\omega)$.

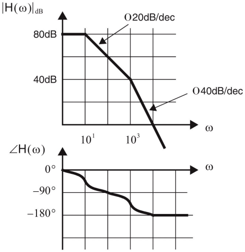


Figure 4.4
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In general for transfer functions with only real poles and no zeros (pure low-pass): a) $\omega \to 0_+$ $|H(s)|$ is constant at the low frequency gain and $\angle H(s) = 0^o$ b) for each pole as ω increases the slope of $|H(s)|$ increses by $-\frac{20dB}{\text{decade}}$ c) each pole contributes -90^o to the phase, but in a smooth transistion so that at a frequency exactly at the pole it is exactly -45^o

General rules of thumb to use real zeros and poles for Bode plots (1/3)

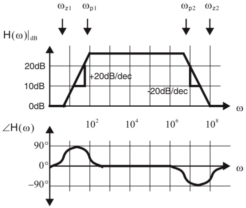

Figure 4.8

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a) find a frequency ω_{mid} with equal number k of zeros and poles where

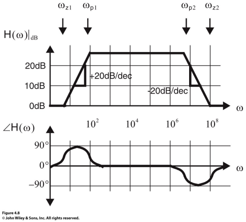
$z_1, \ldots, z_k, \omega_1, \ldots, \omega_k < \omega_{mid} \Rightarrow$

$$|H(s)| \approx K \frac{|z_{(k+1)} \cdots |z_m|}{|\omega_{(k+1)}| \ldots |\omega_n|}$$

$$\angle H(s) \approx 0^o$$

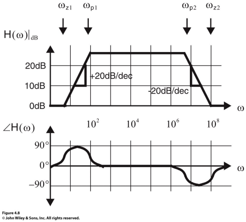
and the gradient of both $|H(s)|$ and $\angle H(s)$ is zero

General rules of thumb to use real zeros and poles for Bode plots (2/3)



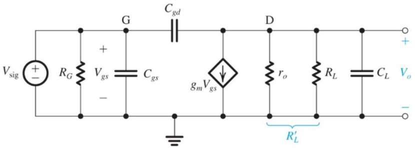
b) moving from ω_{mid} in the magnitude plot at each $|\omega_i|$ add -20dB/decade to the magnitude gradient and for each $|z_i|$ add +20dB/decade
c) moving from ω_{mid} in the phase plot to higher frequencies at each ω_i add -90^o to the phase in a smooth transition (respectively -45^o right *at* the poles) and vice versa towards lower frequencies.

General rules of thumb to use real zeros and poles for Bode plots (3/3)



d) For the zeros towards higher frequencies if the nominater is of the form $(1 + \frac{s}{z_i})$ add $+90^o$ and if its of the form $(1 - \frac{s}{z_i})$ (refered to as right half plain zero as the solution for s of $0 = (1 - \frac{s}{z_i})$ is positive) add -90^o to the phase in a smooth transition (i.e. respectively $\pm 45^o$ right at the zeros) and vice versa towards lower frequencies.

CS Frequency Response, Dominant C_L (1/2)



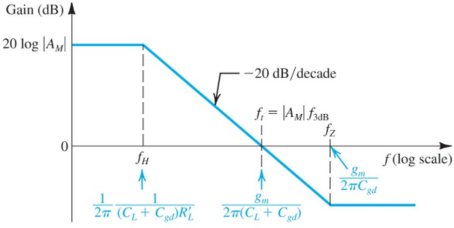
(a)

$$v_o(s(C_{gd} + C_L) + G_L) + g_m v_{gs} = v_{gs} s C_{gd}$$

$$\frac{v_o}{v_{gs}} = \frac{sC_{gd} - g_m}{s(C_{gd} + C_L) + G_L}$$

$$= -g_m R_L \frac{1 - s\frac{C_{gd}}{g_m}}{s(C_{gd} + C_L)R_L + 1} \quad (9.65)$$

CS Frequency Response, Dominant C_L (2/2)



(c)

$$\omega_z = \frac{g_m}{C_{gd}} \quad (9.66)$$

$$\omega_p = \frac{1}{(C_{gd} + C_L)R_L} \quad (9.67)$$

$$\frac{\omega_z}{\omega_p} = g_m R_L \left(1 + \frac{C_L}{C_{gd}}\right) \quad (9.68)$$

$$\omega_t = \frac{g_m}{C_L + C_{gd}} \quad (9.69)$$

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Notation in this Book

$$A(s) = A_M F_H s$$

$$F_H(s) = \frac{(1 + \frac{s}{\omega_{z1}})...(1 + \frac{s}{\omega_{zn}})}{(1 + \frac{s}{\omega_{p1}})...(1 + \frac{s}{\omega_{pm}})}$$

Dominant Pole Approximation

If $\omega_{p1} < 4\omega_{p2}$ and $\omega_{p1} < 4\omega_{z1}$ then

$$A(S) \approx \frac{1}{1 + \frac{s}{\omega_{p1}}}$$

$$\omega_H \approx \omega_{p1}$$

An Approximation Without a Dominant Pole

2^{nd} order example

$$
\begin{aligned}
|F_H(\omega_H)|^2 = \frac{1}{2} &= \frac{(1+\frac{\omega_H^2}{\omega_{z1}^2})(1+\frac{\omega_H^2}{\omega_{z2}^2})}{(1+\frac{\omega_H^2}{\omega_{p1}^2})(1+\frac{\omega_H^2}{\omega_{p2}^2})} \\
&= \frac{1+\omega_H^2\left(\frac{1}{\omega_{z1}^2}+\frac{1}{\omega_{z2}^2}\right)+\omega_H^4\left(\frac{1}{\omega_{z1}^2\omega_{z2}^2}\right)}{1+\omega_H^2\left(\frac{1}{\omega_{p1}^2}+\frac{1}{\omega_{p2}^2}\right)+\omega_H^4\left(\frac{1}{\omega_{p1}^2\omega_{p2}^2}\right)} \\
\Rightarrow \omega_H &\approx \frac{1}{\sqrt{\frac{1}{\omega_{p1}^2}+\frac{1}{\omega_{p2}^2}-\frac{2}{\omega_{z1}^2}-\frac{2}{\omega_{z2}^2}}} \quad (9.76)
\end{aligned}
$$

An Approximation Without a Dominant Pole

general:

$$\omega_H \approx \frac{1}{\sqrt{\frac{1}{\omega_{p1}^2} + \frac{1}{\omega_{p2}^2} \cdots + \frac{1}{\omega_{pm}^2} - \frac{2}{\omega_{z1}^2} - \frac{2}{\omega_{z2}^2} \cdots - \frac{2}{\omega_{zn}^2}}} \quad (9.77)$$

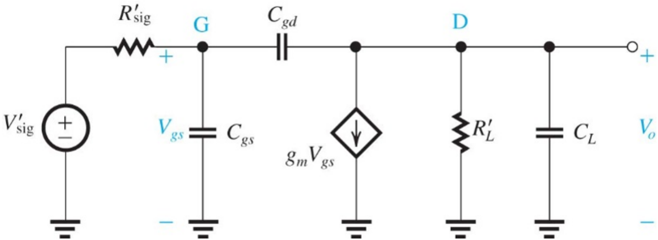
If ω_{p1} is much smaller than all other pole- and zero-frequencies this reduces to the dominant pole approximation.

Open-Circuit Time Constants Method

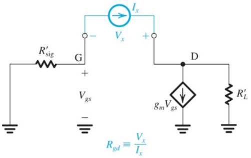
$$\omega_H \approx \frac{1}{\sum_i C_i R_i}$$

Where C_i are all capacitors in the circuit and R_i is the resistance seen by C_i when the input signal source is zeroed and all other capacitors are open circuited.

Open-Circuit Time Constants Method Example CS Amp



The Difficult One is R_{gd}



(b)

$$
\begin{aligned}
i_x &= -\frac{v_{gs}}{R_{sig}} \\
&= \frac{v_{gs} + v_x}{R_L} + v_{gs} g_m \\
&= \frac{v_x}{R_L} - i_x R_{sig} \left(\frac{1}{R_L} + g_m \right)
\end{aligned}
$$

$$
R_{gd} = \frac{v_x}{i_x} = [R_L + R_{sig} (1 + g_m R_L)]
$$

Open Circuit Time Constant

$$\tau_H = R_{sig} C_{gs} + R_L C_L + [R_L + R_{sig}(1 + g_m R_L)] C_{gd}$$
$$= R_{sig} \left[C_{gs} + (1 + g_m R_L) C_{gd}\right] + R_L \left[C_{gd} + C_L\right] \quad (9.88)$$

Previously:

$$\omega_H = \frac{1}{R'_{sig}(C_{gs} + (1 + g_m R_L) C_{gd})} \quad (9.53)$$

$$\omega_H = \frac{1}{(C_{gd} + C_L) R_L} \quad (9.67)$$

Comparing Approximations

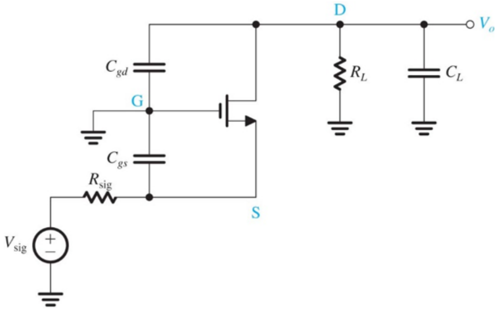
If you combine the prviously transfer functions for $\frac{v_{gs}}{v_{sig}}$ derived from (9.46) and $\frac{v_o}{v_{gs}}$ from (9.65) as $A(s) = \frac{v_{gs}}{v_{sig}} \frac{v_o}{v_{gs}}$ you get both of these previous ω_H as poles and can compute the combined ω_H according to (9.77):

$$\tau_H = \frac{1}{\omega_H} \approx \sqrt{[R'_{sig}(C_{gs} + C_{eq})]^2 + [(C_{gd} + C_L)R_L]^2} \quad (9.77)$$

So the geometric mean rather than the sum ...

Content

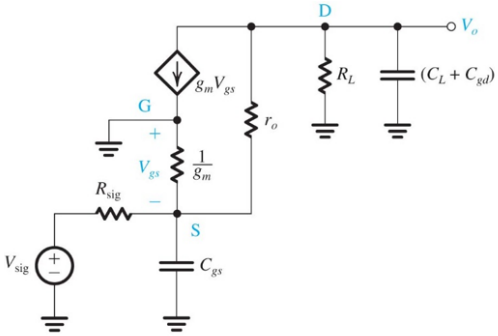
CG Amplifier HF Response



(a)

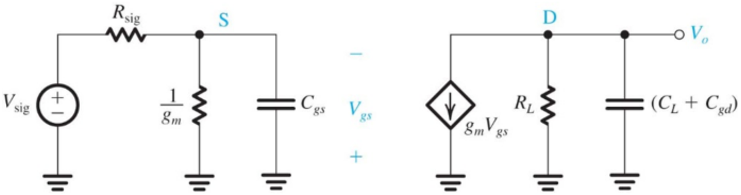
NOTE: no Miller effect!

CG Amplifier HF Response T-model



(b)

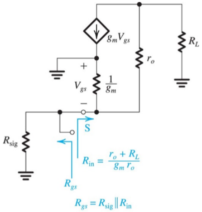
CG Amplifier HF Response without r_o



(c)

$$\tau_{p1} = C_{gs}\left(R_{sig} || \frac{1}{g_m}\right) \quad \tau_{p2} = (C_{gd} + C_L)R_L$$

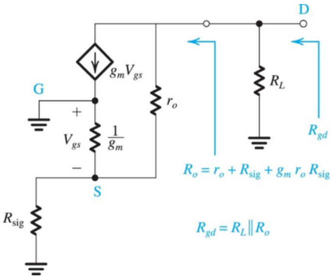
CG Amplifier open circuit time-constant with r_o for C_{gs}



(d)

$$\tau_{gs} = C_{gs}\left(R_{sig} || \frac{r_o + R_L}{g_m r_o}\right)$$

CG Amplifier open circuit time-constant with r_o for $C_{gd} + C_L$



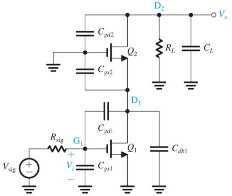
(e)

$$\tau_{gd} = (C_{gd} + C_L)\left(R_{sig}||(r_o + R_{sig} + g_m r_o R_{sig})\right)$$

CG Amplifier HF Response Conclusion

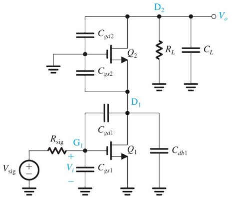
No Miller effect that would cause low impedance at high frequencies, but due to low input resistance the impedance is already low at DC $\Rightarrow$ low A_M for $R_{sig} > 0$

Cascode Amplifier HF Response



$$\begin{aligned}
\tau_{gs1} &= C_{gs1} R_{sig} \\
\tau_{gd1} &= C_{gd1}\left[(1 + g_{m1} R_{d1}) R_{sig} + R_{d1}\right] \\
&\quad \text{where } R_{d1} = r_{o1} || \frac{r_{o2} + R_L}{g_{m2} r_{o2}} \\
\tau_{gs2} &= (C_{gs2} + C_{db1}) R_{d1} \\
\tau_{gd2} &= (C_L + C_{gd2})(R_L || (r_{o2} + r_{o1} + g_{m2} r_{o2} r_{o1})) \\
\tau_h &\approx \tau_{gs1} + \tau_{gd1} + \tau_{gs2} + \tau_{gd2}
\end{aligned}$$

Cascode Amplifier HF Response

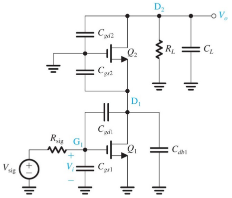


Rearranging τ_h grouping by the three nodes' resistors:

$$\begin{aligned}\tau_h \approx\ & R_{sig}\left[C_{gs1} + C_{gd1}(1 + g_{m1}R_{d1})\right] \\ & + R_{d1}(C_{gd1} + C_{gs2} + C_{db1}) \\ & + (R_L||R_o)(C_L + C_{gd2})\end{aligned}$$

Thus, if $R_{sig} > 0$ and terms with R_{sig} are dominant one can either get larger bandwidth at the same DC gain than a CS amplifier when $R_L \approx r_o$ or get more DC gain at the same bandwith than a CS amplifier when $R_L \approx g_m r_o^2$ or increase both bandwith and DC gain to less than their maximum by tuning R_L somewhere inbetween.

Cascode Amplifier HF Response



Rearranging τ_h grouping by the three nodes' resistors:

$$\begin{aligned} \tau_h \quad \approx \quad & R_{sig}\left[C_{gs1} + C_{gd1}(1 + g_{m1}R_{d1})\right] \\ & + R_{d1}(C_{gd1} + C_{gs2} + C_{db1}) \\ & + (R_L || R_o)(C_L + C_{gd2}) \end{aligned}$$

With $R_{sig} \approx 0$ one can trade higher BW for reduced A_{DC} or higher A_{DC} for reduced BW compared to a CS amp, keeping the unity gain frequency (i.e. the GB) constant.

Cascode vs CS

CS:

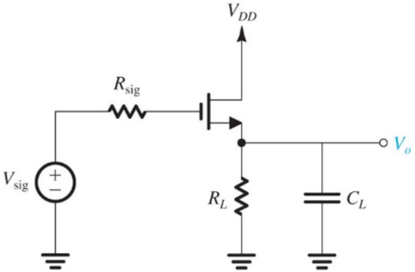
$$
\begin{aligned}
A_{DC} &= -g_m(r_o||R_L) \\
\tau_H &= R_{sig}\left[C_{gs} + (1 + g_m(r_o||R_L))\, C_{gd}\right] + (r_o||R_L)\left[C_{gd} + C_L\right]
\end{aligned}
$$

Cascode:

$$
\begin{aligned}
A_{DC} &= (-g_{m1}(R_O||R_L) \\
&\quad \text{where } R_O = g_{m2} r_{o2} r_{o1} \\
\tau_H &= R_{sig}\left[C_{gs1} + C_{gd1}(1 + g_{m1} R_{d1})\right] \\
&\quad + R_{d1}(C_{gd1} + C_{gs2} + C_{db1}) \\
&\quad + (R_L||R_o)(C_L + C_{gd2}) \\
&\quad \text{where } R_{d1} = r_{o1}||\frac{r_{o2} + R_L}{g_{m2} r_{o2}}
\end{aligned}
$$

Content

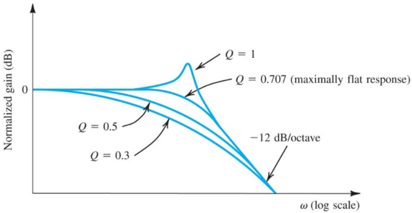
Source Follower HF Response



(a)

$$A(s) = A_M \frac{1 + \left(\frac{s}{\omega_z}\right)}{1 + b_1 s + b_2 s^2} = A_M \frac{1 + \left(\frac{s}{\omega_z}\right)}{1 + \frac{1}{Q}\frac{s}{\omega_0} + \frac{s^2}{\omega_0^2}}$$

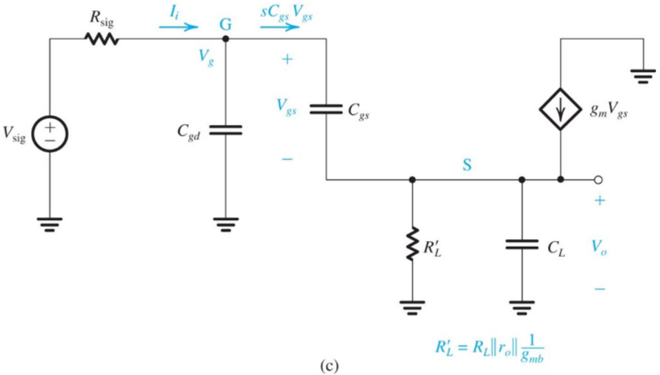
Source Follower Frequency Response Possibilities



(b)

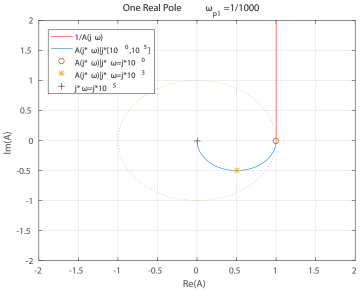
$$\omega_{p1,p2} = \frac{-\frac{1}{Q\omega_0} \pm \sqrt{\frac{1}{\omega_0^2 Q^2} - 4\frac{1}{\omega_0^2}}}{2}$$

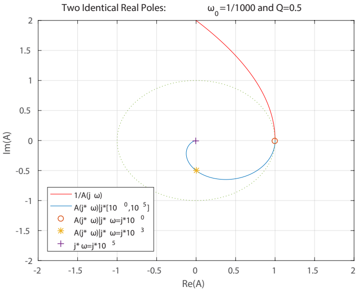
Intuition for Resonance/Instability



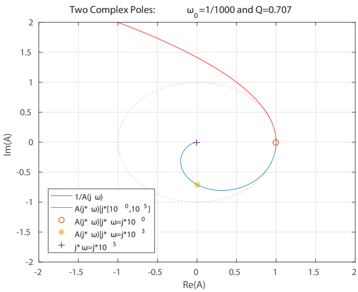
(c)

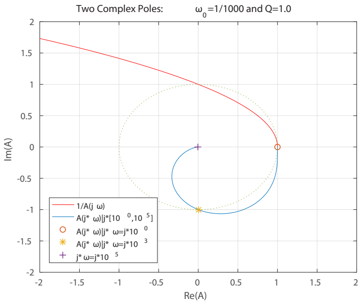
Dependence on Q-factor (1/2)





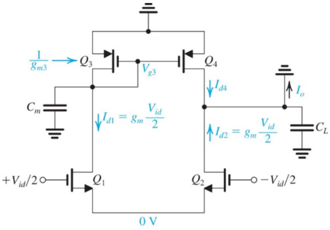
Dependence on Q-factor (2/2)





Content

HF Analysis of Current-Mirror-Loaded CMOS Amp (1/2)



(a)

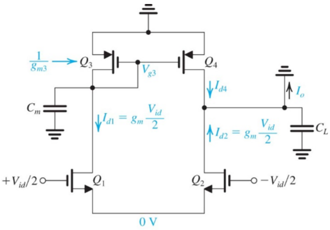
Neglecting r_o in current mirror:

$$G_M = g_m \frac{1 + s\frac{C_m}{2g_{m3}}}{1 + s\frac{C_m}{g_{m3}}}$$

$$\omega_{p2} = \frac{g_{m3}}{C_m}$$

$$\omega_z = \frac{2g_{m3}}{C_m}$$

HF Analysis of Current-Mirror-Loaded CMOS Amp (2/2)



(a)

$$v_o = v_{id} G_M Z_o$$

$$\frac{v_o}{v_{id}} = g_{m3} R_o \left(\frac{1 + s\frac{C_m}{2g_{m3}}}{1 + s\frac{C_m}{g_{m3}}} \right) \left(\frac{\quad}{R_o + \cdots} \right.$$

$$\omega_{p1} = \frac{1}{C_L R_o}$$

And ω_{p1} is usually clearly dominant.