Weekly Assignment: Z-Transform (Part - 01)

September 9, 2014

****

All questions are compulsory.

All steps must be shown. Writing answers directly will not be awarded any marks.

For all problems, always assume the problem to be of bilateral nature, unless otherwise stated.

No calculator or computing software is allowed unless it is mentioned specifically.

You are suppose to remember the Z transform of basic functions such as $u[n]$, $\delta[n]$ and $\gamma^n u[n]$.

If you are not sure about your hand-writing, use LaTex to typeset your answers.

The numbers in bracket at the end of the question represent the marks the question carries.

Total marks = 70. Divide your obtained score by 7 to scale it out of 10.

****

Q1. Find the Z transformation of $x[n] = \delta[n-m]$, m is some constant delay. Plot relevant ROC and show poles, if any. Give their order as well. Given that $\mathcal{Z}\{\delta[n]\} = 1$, check if the answer you obtained for $\delta[n-m]$ is correct. $(2+1+0.5+0.5) = 4$

Q2. Find the Z transformation of $x[n] = u[n-m]$, m is some constant delay. Plot relevant ROC and show poles, if any. Give their order as well. Given that

$$\mathcal{Z}\{\mathrm{u}[n]\} = \tfrac{z}{z-1}$$

,

check if the answer you obtained for $u[n-m]$ is correct. $(2+1+1+1) = 5$

Q3. Find the Z transformation of $x[n] = n\ u[n]$. Plot relevant ROC and show poles and zeroes, if any. Give their order as well. $(2+1+1) = 4$

Q4. Find the Z transformation of $x[n] = \gamma^n\ u[n]$, γ is some constant. Plot relevant ROC and show poles and zeroes, if any. Give their order as well. $(2+1+1) = 4$

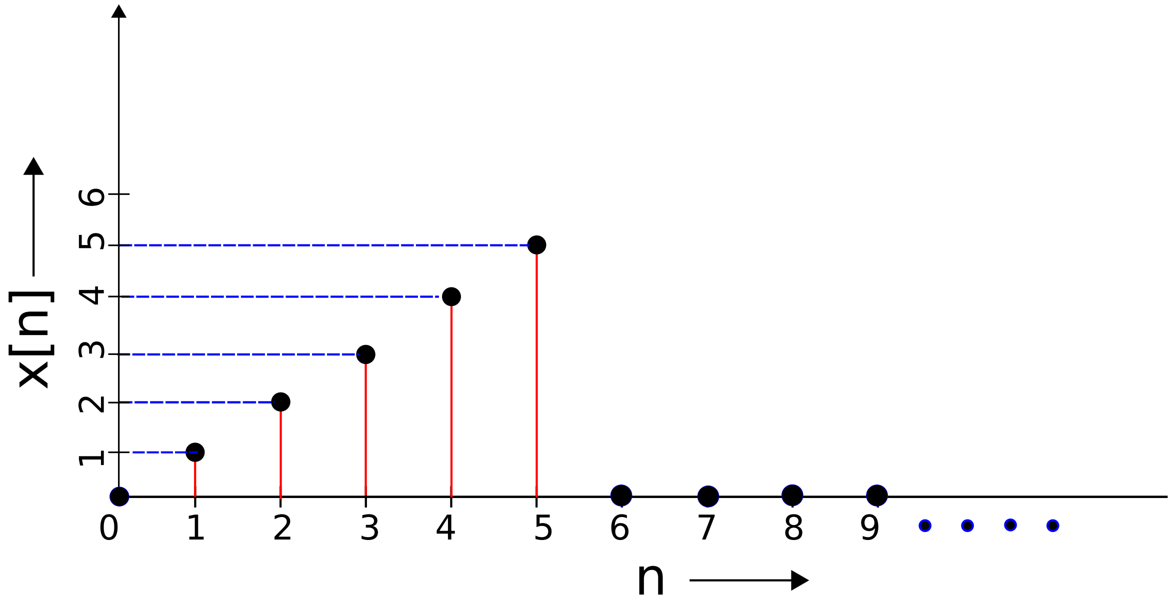
Q5. Find the Z transformation of $x[n] = cos(n\ \beta)u[n]$. (3)

Q6. Find the Z transformation of $x[n] = \gamma^n u[n+m]$, γ and m are some constant. Plot relevant ROC and show poles and zeroes, if any. Give their order as well. Given that

$$\mathcal{Z}\{\gamma^n \mathrm{u}[n]\} = \frac{\mathrm{z}}{\mathrm{z}\text{-}\gamma}$$

check the answer you obtained for $x[n] = \gamma^n u[n+m]$ is correct. $(2+1+1+1) = 5$

Q7. Find the Z transformation of the given plot. You might (not necessarily, though!) need to use right shift theorem. (4)



Q8. From the uploaded file: Oppgave 2 (3)

Q9. If $x[n] = 1$, can you find its Z transform, i.e., $\mathcal{Z}\{x[n]\}$? Justify your answer through relevant calculation. (4)

Q10. From the uploaded file: Oppgave 3 (5)

Q11. For the function $x(n) = a^n u(n)$, $a = constant$, use initial value theorem to find the value of $x(0)$. (2)

Q12. Establish the relationship $-z\ X'[z] = \mathcal{Z}\{\mathrm{n\ x}[n]\}$. Here $X'[z]$ and $\mathcal{Z}$ are first derivative of Z transform and Z transform operator respectively.

Use the relation you have proved above to obtain the Z transformation of $x[n] = n\ u[n]$, $x[n] = n\ \delta[n]$ and $x[n] = n\gamma^n u[n]$. $(2+1+1+1) = 5$

Q13. This question will illustrate the applicability of different methods to find inverse Z transformation. It will require effort, concentration and patience.

You are permitted to use scientific calculator for this problem. Note: Be very careful while calculating argument (polar angle) in the complex plane.

Given that $x[n]$ is the signal and $X[z]$ is its respective Z transform, i.e., $\mathcal{Z}\{x[n]\} = X[z]$.

You have to apply different methods to find $x[n]$. Given,

$$X[z] = \frac{6z^2 + 34z}{(z-1)(z^2 - 6z + 25)}$$

.

The inverse Z transformation of this function in closed form is

$$x[n] = [2 + 3.2\,(5)^n \cos(0.927n - 2.246)]\, u[n]\,.$$

a) Method 1: As you can notice the denominator in $X[z]$ has a quadratic term, hence find inverse Z transform of this function by directly implementing partial decomposition relevant to quadratic terms.

You will need this standard relation:

$$\mathcal{Z}\{[r\,|\gamma|^n\, cos\,[\beta n + \theta]]\mathrm{u}[n]\} = \frac{z\,(Az+B)}{z^2 + 2az + |\gamma|^2}$$

$$Here\ r = \sqrt{\frac{A^2|\gamma|^2 + B^2 - 2AaB}{|\gamma|^2 - a^2}},\ \beta = cos^{-1}\left(\frac{-a}{|\gamma|}\right)\ and\ \theta = tan^{-1}\left(\frac{Aa - B}{A\sqrt{|\gamma|^2 - a^2}}\right).$$

b) Method 2: Find all poles of the given function. Show all poles in the Argand (complex) plane.

After you have completely factorized the denominator, apply the concept of residues and Heaviside cover up method to find the inverse Z transformation.

c) Method 3: Since you already have a completely factorized denominator (from Method 2), apply partial decomposition (but NOT of quadratic form) to find $x[n]$. This is a lengthy method!

d) Method 4: Obtain the values of first four samples of the signal by finding the respective power series expansion of the Z transform. Use synthetic division.

Check if the values you have obtained by method 4 matches with the closed form solution mentioned above.

e) Which method do you find more robust than others and why?

f) Which method do you find more efficient than others and why?

$(3 + (2+2) + 4 + 3 + 1 + 1) = 16$

Q14. From the uploaded file: Oppgave 4 (3)

Q15. From the uploaded file: Oppgave 6 (3)