This document contains exercises made for INF3580. Please send any comments, errors, bug or improvement reports to this exercise set to martige@ifi.uio.no. Feedback is most welcome! Alphabetically thanks to Audun Stolpe, Espen H. Lian, Martin Giese and Rune Dahl for feedback.


Keep all the work you do for these exercises in a safe place. Setting up a version control system like cvs, svn or git for the work you do is smart. You can create a svn repository on IfI’s svn server[^2], see their help section[^3] for more information. There is also a walk-through[^4] from old INF3120 on how to set up a svn repository and connect it to Eclipse, but news is that you’ll need the plug-in subclipse[^5] to make it work. Please contact me if you have any smart tips to share.

[^1]: mailto:martige@ifi.uio.no
[^2]: https://wwws.ifi.uio.no/system/svn/
[^3]: https://wwws.ifi.uio.no/system/svn/help.cgi
[^4]: http://www.uio.no/studier/emner/matnat/ifi/INF3120/h06/studentarbeider/Prosjektoppgave/SVN_i_Eclipse.pdf
[^5]: http://subclipse.tigris.org/
12 Repetition

12.1 Sets and relations

In the exercises in this section let the following be sets:

- $A = \{a, b, c, s, d, f\}$
- $B = \{d, f, r, k, t, g, t, e, t, p, t, z, e, e, r\}$
- $C = \{r, e, m\}$
- $D = \{q, l\}$
- $E = \{\}$
- $\Delta$ is the universal set.

12.1.1 Exercise

What is the cardinality of the sets given above?

12.1.2 Exercise

List all the elements in the following sets:

1. $A \cup B$.
2. $A \cup (B \cap C)$.
3. $(A \cap B) \cup (C \cap A)$.
4. $B - C$.
5. $C - B$.
6. $D \cap -E$.
7. $D \cup -E$.

12.1.3 Exercise

Let $F$ and $G$ be two arbitrary sets and $\Delta$ the universal set. Draw Venn diagrams containing the sets $F$, $G$ and $\Delta$ and shade the area representing the following sets:

1. $-F$.
2. $-G$.
3. $-(F \cup G)$.
4. $-F \cap -G$.
5. $-(F \cap G)$.
6. $-F \cup -G$.

12.1.4 Exercise

Let $R = \{(d,d), (d,k), (t,e), (t,p), (f,t), (p,k), (k,f)\}$ be a relation on $B$. With $R$ as starting point make new relations, labelled $R_1$ to $R_5$, by adding or removing pairs such that they meet the following requirements:

1. $R \subseteq R_1$ is reflexive.
2. $R \subseteq R_2$ is symmetric.
3. $R \subseteq R_3$ is transitive.
4. $R_4 \subseteq R$ is irreflexive.
5. $R_5 = R_4 \circ R_3$, i.e., $R_5$ is the composition of $R_4$ and $R_4$.

For each relation, list the pairs in the relation.

12.2 Semantics

Recall the notions validity, consistency from the lectures. We say that a knowledge base $K$

- is valid iff $I \models K$ for all interpretations/models $I$ of $K$;
- is consistent iff $I \models K$ for at least one interpretation/model $I$ of $K$.

12.2.1 Exercise

Let $K$ be a $\mathcal{ALCQ}$ knowledge base $K = \{B \sqsubseteq A, C \sqsubseteq B, B(e)\}$. Answer the following questions, and create a model to prove your answer when appropriate.

1. Is $K$ consistent?
2. Does $K$ entail $A(e)$?
3. Does $K$ entail $C(e)$?
4. Does there exist a model $I_1$ such that $I_1 \models K$ and $I_1 \not\models A(e)$?
5. Does there exist a model $I_2$ such that $I_2 \models K$ and $I_2 \not\models C(e)$?

12.2.2 Exercise

Let $S$ be the set of sentences

$$S = \{\text{TwoCV} \sqsubseteq \text{Car}, \text{TwoCV} \sqsubseteq \forall \text{driveAxle}.\text{FrontAxle}\}.$$ 

Let $I$ be an interpretation of $S$ where

- $\Delta^I = \{a, b\}$. 
• \( \text{CAR}^I = \{a\} \)
• \( \text{TwoCV}^I = \{a\} \)
• \( \text{FrontAxle}^I = \{b\} \)
• \( \text{driveAxle}^I = \{(a, b)\} \)

This is the model is illustrated in the figure below, and is the same setup as used in the walkthrough in the lecture slides.

![Diagram](image)

Figure 2: An interpretation of \( S \).

Exercises:

1. Show that the interpretation \( I \) satisfies the set of sentences \( S \).

2. List the elements in the following sets:
   
   (a) \( \text{FrontAxle}^I \)
   
   (b) \( (\exists \text{driveAxle.FrontAxle})^I \)
   
   (c) \( (\text{CAR} \cap \exists \text{driveAxle.FrontAxle})^I \)
   
   (d) \( (\text{CAR} \cap \text{FrontAxle})^I \)