INF3580 Notes on formal semantics and entailment

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1 Reference

Notation and definitions collected from the lecture slides. (Simple interpretations)

Triple pattern	Triple instance	Abbreviation
`indi prop indi .`	$i_1\ r\ i_2$	$r(i_1, i_2)$
`indi rdf:type class .`	i_1 `rdf:type` C	$C(i_1)$
`class rdfs:subClassOf class .`	C `rdfs:subClassOf` D	$C \sqsubseteq D$
`prop rdfs:subPropertyOf prop .`	r `rdfs:subPropertyOf` s	$r \sqsubseteq s$
`prop rdfs:domain class .`	r `rdfs:domain` C	$\mathsf{dom}(r, C)$
`prop rdfs:range class .`	r `rdfs:range` C	$\mathsf{rg}(r, C)$

1.1 Interpretation

An *interpretation*[1] $\mathcal{I}$ consists of:

- A set $\Delta^{\mathcal{I}}$, called the *domain* $\mathcal{I}$
- For each individual URI i, an element $i^{\mathcal{I}} \in \Delta^{\mathcal{I}}$
- For each class URI C, a subset $C^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
- For each property URI r, a relation $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$

Given an interpretation $\mathcal{I}$, define $\mathcal{I} \models T$ (read: "$\mathcal{I}$ models T"/"$\mathcal{I}$ is a valid interpretation of T") as follows:

[1] Also called model or (Norwegian:) tolkning.

1. $\mathcal{I} \models r(i_1, i_2)$ iff $\langle i_1^{\mathcal{I}}, i_2^{\mathcal{I}} \rangle \in r^{\mathcal{I}}$
2. $\mathcal{I} \models C(i)$ iff $i^{\mathcal{I}} \in C^{\mathcal{I}}$
3. $\mathcal{I} \models C \sqsubseteq D$ iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
4. $\mathcal{I} \models r \sqsubseteq s$ iff $r^{\mathcal{I}} \subseteq s^{\mathcal{I}}$
5. $\mathcal{I} \models \mathsf{dom}(r, C)$ iff $\mathsf{dom}\ r^{\mathcal{I}} \subseteq C^{\mathcal{I}}$
6. $\mathcal{I} \models \mathsf{rg}(r, C)$ iff $\mathsf{rg}\ r^{\mathcal{I}} \subseteq C^{\mathcal{I}}$

Where $\mathsf{dom}\ r^{\mathcal{I}}$ is defined as all x such that $\langle x, y \rangle \in r^{\mathcal{I}}$, and $\mathsf{rg}\ r^{\mathcal{I}}$ as all y such that $\langle x, y \rangle \in r^{\mathcal{I}}$. Less formally: all subjects of r and all objects of r respectively.

1.2 Entailment

Given a set of triples A and other triple T we say "T is *entailed* by A" or "T follows from A" (written $A \models T$) if:

For all possible interpretations $\mathcal{I}$ where $\mathcal{I} \models A$ is true, $\mathcal{I} \models T$ is also true.

Said in another way: $A \models T$ if all interpretations that model A also model T.

Note that the same symbol as in 1.1 is used, but here the left hand side is a set of statements, **not** an interpretation. Ie. the symbol $\models$ means different things depending on its arguments.

1.3 Example

Define A as the rdfs graph/set of triples/set of statements below.

Listing 1: A

```
:Lisa  :hasSister  :Maggie
:hasSister  rdfs:range  :Woman
:hasSister  rdfs:domain  :Person
:Woman  rdfs:subClassOf  :Person
```

Then the following is a valid interpretation ($\mathcal{I}$) of A. Ie. $\mathcal{I} \models A$.

$$\begin{aligned}
\Delta^{\mathcal{I}} &= \{x, y\} \\
:\text{Lisa}^{\mathcal{I}} &= x \\
:\text{Maggie}^{\mathcal{I}} &= y \\
:\text{Woman}^{\mathcal{I}} &= \{x, y\} \\
:\text{Person}^{\mathcal{I}} &= \{x, y\} \\
:\text{hasSister}^{\mathcal{I}} &= \{\langle x, y \rangle\}
\end{aligned}$$

Why? We simply check that the rules in 1.1 holds for each triple in A. eg. A states that $:\text{Woman} \sqsubseteq :\text{Person}$ so according to rule 3 $:\text{Woman}^{\mathcal{I}} \subseteq :\text{Person}^{\mathcal{I}}$ must hold - and it does. Similarily we can check that the rest of the rules hold. (exercise)

Also note that **all** URIs are mapped to some element in the domain ($\Delta^{\mathcal{I}}$).

Counter-model

Does it follow (*entails*) from A that that Lisa is a woman? Ie. is the following true?

$$A \models :\text{Lisa}\ \text{rdf}:\text{type}\ \ :\text{Woman}$$

Thinking a bit we can intuitvely conclude that it's not - eg. since the domain of $:\text{hasSister}$ isn't $:\text{Woman}$.

To show this formally we need to demonstrate that there exist an interpretation that is valid for A and **not** valid for $:\text{Lisa}\ \text{rdf}:\text{type}\ \ :\text{Woman}$, ie. an *counter model*.

If we modify $\mathcal{I}$ by redefining $:\text{Woman}^{\mathcal{I}} = \{y\}$, Lisa is surly not a woman since $:\text{Woman}^{\mathcal{I}}$ list all women that exist in this interpretation, and Lisa is mapped to x. Is this modifed $\mathcal{I}$ still valid for A? Yes (exercise). We thus have a counter model for the statement $:\text{Lisa}\ \text{rdf}:\text{type}\ \ :\text{Woman}$.

2 Why semantics?

> Introducing a formal semantics for RDF(S) became necessary because the previous informal RDF(S) specification – though successful in conveying some intuition – left plenty of room for interpretation about what conclusions can be drawn from a given specification. Indeed, first implementations of RDF(S) storage and reasoning tools (so-called *triple stores*) provided differing results to posed queries, a situation severely obstructing interoperability of tools and interchangeability of specifications, aims the RDF(S) standard actually was designed for.
>
> While providing sets of examples for valid and invalid conclusions might clarify some singular cases, this can never ensure that each of the infinitely many entailments in question will be agreed upon. The most convenient way to resolve this problem is to avoid the vagueness of an informal specification by providing a well-defined formal semantics.
>
> - From *Foundations of Semantics Web Technologies*

Some simple examples of such ambiguities is the *open world assumption*, and the *unique name assumption.* These are of course possible to formulate more directly - in fact it requires some thought to realize the semantics imply them[2]. But there are more subtle ambiguities, and using a precise mathematical model covers all these and piggybacks on years of mathematical research.

3 Intuition

Rdf(s) use the *open world assumption*. This means that a rdf(s) graph doesn't describe one exact world. It merly put some restrictions on the "shape" the world. Infinity many worlds might fit the shape.

When we create an interpretation we can think of it as "closing the world". An (valid) interpretation is *one example* of a world that has a "shape" that fit the rdfs graph.

Inside the interpretation the *closed world assumption* holds. Only the stated things are true[3]/exists. eg. $:\text{Bicycle}^{\mathcal{I}}$ lists *every* bicycle that exist within *that* interpretation, so if $x \notin :\text{Bicycle}^{\mathcal{I}}$ we can conclude that x isn't a bicycle.

We can similarily say that the *unique name assumption* holds within the model. The domain list all things that exist, and each symbol/element in the domain has its own identity.

4 Full interpretations (INF4580) (Tentative)

V	Set of all URIs in our knowledge base
IR	Set of all resources in the model/interpretation
IP	Set of all properties in the model (usually a subset of IR)
I_s	Mapping from URIs to resources (more or less the same as $\cdot^{\mathcal{I}}$ in the simple interpretation)
I_{EXT}	Mapping from each property to it's definition (set of pairs)
I_{CEXT}	Mapping from each class to it's members.

4.1 Example

(Skipping LV etc.)

```
:Bart :hasSister :Maggie
:hasSister :range :Woman
:hasSister :domain :Person
:Woman :subClassOf :Person
```

[2]This is a good exercise

[3]Maybe not true in the strictest sense, but works for intuition

$$
\begin{aligned}
IR &= \{b, m, W, P, hs, do, ra, su\} \\
IP &= \{hs, do, ra, su\} \\
I_s(:\text{Bart}) &= b \\
I_s(:\text{Magge}) &= m \\
I_s(:\text{hasSister}) &= hs \\
I_s(:\text{Woman}) &= W \\
I_s(:\text{Person}) &= P \\
I_s(:\text{range}) &= ra \\
I_s(:\text{domain}) &= do \\
I_s(:\text{subClassOf}) &= su \\
I_{EXT}(hs) &= \{\langle b, m\rangle\} \\
I_{EXT}(ra) &= \{\langle hs, W\rangle\} \\
I_{EXT}(do) &= \{\langle hs, P\rangle\} \\
I_{EXT}(su) &= \{\langle W, P\rangle\} \\
I_{CEXT}(\text{W}) &= \{m\} \\
I_{CEXT}(\text{P}) &= \{b, m\}
\end{aligned}
$$