Mathematical foundations

Read the relevant lecure slides.

1 Sets

1.1 Exercise

What is the difference between $\emptyset$ and $\{\emptyset\}$?

1.2 Exercise

In this exercise we will use the following sets:

- $A = \{a, b, c, d\}$
- $B = \{d, f, e, r, k\}$
- $C = \{r, e, m\}$
- $D = \{q, l\}$
- $E = \{\}$
- Δ is the universal set.

What is the cardinality of each of these sets?

List all the elements in the following sets:

1. $A \cup B$.
2. $A \cup (B \cap C)$.
3. $(A \cap B) \cup (C \cap A)$.
4. $B \setminus C$.
5. $C \setminus B$.
6. $D \cap \overline{E}$.
7. $D \cup \overline{E}$.

1.3 Exercise

Let F and G be two arbitrary sets and Δ the universal set. Draw Venn diagrams containing the sets F, G and Δ and shade the area representing the following sets:

1. $\overline{F}$.
2. $\overline{G}$.
3. $\overline{(F \cup G)}$.
4. $\overline{F} \cap \overline{G}$.
5. $\overline{(F \cap G)}$.
6. $\overline{F} \cup \overline{G}$.

1.4 Exercise

Create three sets A, B and C such that the following hold:

- The union of A and B is $\{1, 2, 3, 4\}$.
- The intersection of A and C is $\{3\}$.
- The union of B and C is $\{3, 4, 5, 6\}$.
- The intersection of B and C is $\{4\}$.

1.5 Exercise

Let $A = \{1, 2, \{1, 2\}, \{1, 3\}, \{1, 2, 3\}\}$ and decide if the following hold

- $1 \in A$
- $2 \in A$
- $3 \in A$
- $\emptyset \in A$
- $\{1\} \in A$
- $\{1, 3\} \in A$
- $\{1, 2, \{1, 2\}\} \in A$
- $\emptyset \subseteq A$
- $\{1\} \subseteq A$
- $\{1, 3\} \subseteq A$
- $\{1, 2, \{1, 2\}\} \subseteq A$
- $\{\{1, 2, 3\}\} \in A$

2 Relations

2.1 Exercise

Let A be the set $A = \{a, b, c, d, e, f\}$. Create non-empty relations R_i on A such that the conditions below hold.

1. $R_1 = A \times A$
2. R_2 is reflexive.
3. R_3 is symmetric.
4. R_4 is transitive.
5. R_5 is irreflexive.

2.2 Exercise

Assume the normal intended interpretation. Which of the following relations are reflexive, transitive and/or symmetric?

- hasSister
- hasSibling
- hasFather
- hasParent
- hasAge
- hasSpouse
- likes

3 Propositional logic

3.1 Exercise

Let ϕ be the propositional formula $(P \wedge Q) \vee R \rightarrow S \wedge Q$.

- Create an interpretation $\mathcal{I}_1$ such that $\mathcal{I}_1 \models \phi$.
- Create an interpretation $\mathcal{I}_2$ such that $\mathcal{I}_2 \not\models \phi$.

3.2 Exercise

- Find the truth table to the formula $(P \rightarrow Q) \rightarrow P$
- Find the truth table to the formula $(P \rightarrow Q) \vee (Q \rightarrow P)$
- What is there to note about the two formulae?

3.3 Exercise

Decide the following entailment questions. If the answer is yes, then produce a proof, e.g., a truth table, which shows why the answer is yes. If the answer is no, then produce a countermodel, i.e., an interpretation which makes the first formula true and the second false.

- Does $P \vee Q$ entail Q?

- Does $P \wedge Q$ entail $P \vee Q$?
- Does $P \to (P \to Q)$ entail Q?
- Does $P \wedge \neg P$ entail Q?