

INF 4130 Oppgavesett 7, 19/10-2010

Løsningsforslag

Oppgave 9 fra kompendiet

Let

$L_1 = \{M \mid M \text{ writes a } \$ \text{ for every input}\}$

$L_2 = \{M \mid M \text{ writes a } \$ \text{ for input '010'}\}$

$L_3 = \{M \mid \text{There is no } y \text{ such that } M \text{ writes a } \$ \text{ for input } y\}$

Show that L_1 , L_2 and L_3 are undecidable.

All three proofs are simple modifications of the standard proof given at Pages 74-75 (Lecture 3, Slide 12) in the compendium.

For L_1 and L_2 the unmodified standard reduction will work – observe that the M' that this reduction produces does not look at its input; it simply halts for every input (and in particular for input '010') if the corresponding instance of the Halting Problem is a positive one (M halts on input x).

For L_3 we only need to exchange the YES and NO in standard reduction.

Oppgave A

Anta $L = \{M \mid M \text{ skriver } \$ \text{ etter } < 100 \text{ skritt for ethvert input}\}$

Er L uavgjørbar? Begrunn svaret.

L is decidable. The decision algorithm M_L is a modification of the Universal Turing Machine. M_L generates all possible inputs of length < 100 (notice that there are finitely many, since by convention the size of the input alphabet is constant). For each input M_L simulates M for at most 100 steps and answers YES and halts if M halts. If M does not halt for any of the inputs, M_L halts and answers NO.