

# INF 4130 Problem Set for Dino's Lecture 2

## Proposed solutions

### Problem 9 from the compendium

Let

$L_1 = \{M \mid M \text{ writes a } \$ \text{ for every input}\}$

$L_2 = \{M \mid M \text{ writes a } \$ \text{ for input '010'}\}$

$L_3 = \{M \mid \text{There is no } y \text{ such that } M \text{ writes a } \$ \text{ for input } y\}$

Show that  $L_1$ ,  $L_2$  and  $L_3$  are undecidable.

All three proofs are simple modifications of the standard proof given at Pages 74-75 (Lecture 3, Slide 12) in the compendium.

For  $L_1$  and  $L_2$  the unmodified standard reduction will work – observe that the  $M'$  that this reduction produces does not look at its input; it simply halts for every input (and in particular for input '010') if the corresponding instance of the Halting Problem is a positive one ( $M$  halts on input  $x$ ).

For  $L_3$  we only need to exchange the YES and NO in standard reduction.

### Problem A

Consider  $L = \{M : M \text{ skriver } \$ \text{ etter } < 100 \text{ skritt for ethvert input}\}$  Is  $L$  undecidable? Justify your answer (produce an informal proof).

$L$  is decidable. The decision algorithm  $M_L$  is a modification of the Universal Turing Machine.  $M_L$  generates all possible inputs of length  $< 100$  (notice that there are finitely many, since by convention the size of the input alphabet is constant). For each input  $M_L$  simulates  $M$  for at most 100 steps and answers YES and halts if  $M$  halts. If  $M$  does not halt for any of the inputs,  $M_L$  halts and answers NO.