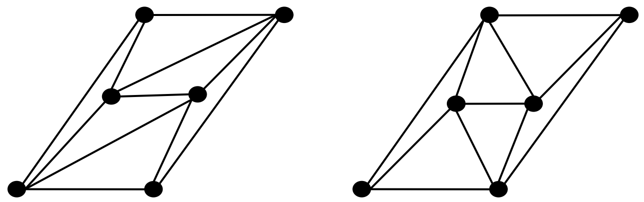
INF 4130 Exercise set 11

Exercise 1

Go through (in some detail) the process illustrated on slide 15 from the lecture: adding a new vertex and restoring the Delaunay property. Make sure you understand the process.

Exercise 2

We are given six points (in $\mathbf{R}^2$), and two possible triangulations of them, as shown in the figure below.



Answer the following questions:

a) Which of the two triangulations is a Delaunay-triangulation?

b) Draw the Voronoi-diagram for the given points, and show that it matches your answer for question a.

c) Show that you can get from the triangulation that is non-Delaunay to one that is, by doing the "Delaunay-trick" one or more times.

d) Verify that your answer for question a also is correct with the "angel-definition" (max-of-min) of a Delaunay-triangulation.

Exercise 3

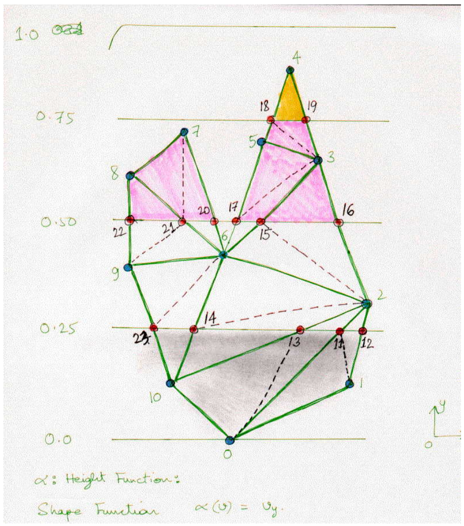
We are checking whether two adjacent triangles that make up a convex quadrilateral (a convex polygon with four sides) locally has the Delaunay property, or if we have to exchange the diagonal with the opposite one. On the slides it says that we have to calculate a determinant to check this. One could, however, wonder if always choosing the shortest diagonal will work. In most cases it is this one we chose. Find a counterexample where choosing the shortest diagonal does not work.

Hint: Start with four points placed somewhat irregularly on a circle, and then move one of the points.

Exercise 4

Assume you have a reasonably gathered set of points P in the plane ($\mathbf{R}^2$), and a point q far away from P. Show that the triangulation of $P \cup \{q\}$ becomes a triangulation of P if you remove q and all the edges connected to it.

Exercise 5



a) Is the polygon to the right a convex polygon or not? Justify your answer.

b) If it isn't convex (it would then be *concave*), can it be divided into a number of convex (sub-)polygons? And if so, what is the smallest number of polygons you need?

Exercise 6

Given a set of points *Q*, show that the pair of points with the largest pairwise distance *must* be corners in CH(*Q*) – the convex hull of *Q*.

Exercise 7

Solve Exercise 8.32 in the textbook: Show that finding the convex hull of *n* points cannot be done faster than $O(n \log n)$. Or more correctly: Show that finding the convex hull is $\Omega(n \log n)$.

Hint: Show that sorting *n* values cannot be faster than finding the convex hull of *n* points.

[END]